$$C_n = n-1 + \frac{1}{n}\left(C_0 + C_{n-1} + C_1 + C_{n-2} + \cdots + C_{n-2} + C_1 + C_{n-1} + C_0\right)$$

$$= n-1 + \frac{2}{n}\left(C_0 + C_1 + \cdots + C_{n-1}\right)$$

$$nC_n = \boxed{n(n-1)} + 2\left(C_0 + C_1 + \cdots + C_{n-1}\right)$$

$$(n-1)C_{n-1} = \boxed{(n-1)(n-2)} + 2\left(C_0 + C_1 + \cdots + C_{n-2}\right)$$

$$nC_n - (n-1)C_{n-1} = \boxed{2(n-1)} + 2C_{n-1}$$

$$nC_n = (n+1)C_{n-1} + 2(n-1) \quad / \cdot \frac{1}{n(n+1)}$$

$$\underbrace{\frac{C_n}{n+1}}_{B_n} = \underbrace{\frac{C_{n-1}}{n}}_{B_{n-1}} + \frac{2(n-1)}{n(n+1)}$$

$$B_n = \underline{B_{n-1}} + \frac{2(n-1)}{n(n+1)} = \underbrace{B_{n-2} + \frac{2(n-2)}{(n-1)n}} + \frac{2(n-1)}{n(n+1)}$$

$$= \cdots = \frac{2(n-1)}{n(n+1)} + \frac{2(n-2)}{(n-1)n} + \cdots + \frac{2\cdot 1}{2\cdot 3} + \underset{0}{\underset{\shortparallel}{B_1}}$$

$$= 2\sum_{k=1}^{n-1} \frac{k}{(k+1)(k+2)}$$

4 18-13:58

$$\frac{k}{(k+1)(k+2)} = \frac{a}{k+2} + \frac{b}{k+1} \quad / \cdot (k+1)(k+2)$$

$$k = a(k+1) + b(k+2)$$

k: $1 = a + b$ $\quad b = -1$

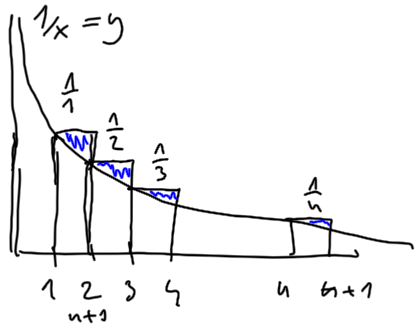
1: $0 = a + 2b$ $\quad a = 2$

$$B_n = \frac{C_n}{n+1} = 2\sum_{k=1}^{n-1}\left(\frac{2}{k+2} - \frac{1}{k+1}\right)$$

$$= 2\left(\frac{2}{3} + \frac{2}{4} + \cdots + \underbrace{\frac{2}{n+1}} - \underbrace{\frac{1}{2}} - \frac{1}{3} - \cdots - \frac{1}{n}\right)$$

$$= 2\left(\underbrace{\frac{1}{1} + \frac{1}{2}} + \underbrace{\frac{1}{3} + \cdots + \frac{1}{n}}_{\text{ok}} + \underbrace{\frac{1}{n+1}} \underbrace{-2} + \underbrace{\frac{1}{n+1}}\right) = 2\left(H_{n+1} - 2 + \frac{1}{n+1}\right)$$

4 18-14:19



$$H_n = \int_1^{n+1} \frac{1}{x}\,dx + \gamma_n = \left[\ln x\right]_1^{n+1} + \gamma_n$$

$$= \ln(n+1) + \gamma_n$$

$\lim \left(H_n - \ln(n+1)\right) \to \gamma$ Eulerova konst.

4 18-14:26

$$C_n = 2(n+1)H_{n+1} - \underline{4(n+1)} + 2$$

$$\sim 2(n+1)\left(\ln(n+2) + \gamma - \underline{2}\right) + 2$$

$$C_n = O(n \ln n)$$

nejde lépe než $n \log_2 n$

přitom $2 \cdot n \ln n = 2n \dfrac{\log_2 n}{\log_2 e}$

kde $\dfrac{2}{\log_2 e} \doteq 1{,}3$

4 18-14:32

011010 $\quad$ 3x 0, 3x 1

$$\frac{6!}{3!\,3!}$$

$a_1 \times 1, \ldots, a_n \times n$

$$\underbrace{0 \ldots 0}_{a_1} \,|\, \underbrace{0 \cdots 0}_{a_2} \,|\, \cdots \,|\, \underbrace{0 \cdots 0}_{a_n}$$

$k \times 0$ $\quad (n-1) \times |$

4 18-14:42

Zadání 2 ze 3 poslanců

A, B, C

$$\binom{15}{4} - \underset{\text{A,B spolu}}{\binom{13}{2}} - \underset{\text{B,C}}{\binom{13}{2}} - \underset{\text{C,A}}{\binom{13}{2}} + \underset{\text{A,B spolu, B,C spolu}}{\binom{12}{1}}$$

$$+ \underset{\text{A,B spolu, C,A spolu}}{\binom{12}{1}} + \underset{\text{B,C, C,A}}{\binom{12}{1}} - \underset{\text{A,B, B,C, C,A}}{\binom{12}{1}} = ?$$

4 18-14:47

$\binom{n+1}{2}$ ... výběry 2 prvků z $1,\dots,n+1$

— větší z nich je $n+1$ .... menší je jeden z $1,\dots,n$

— větší z nich je n ... menší je jeden z $1,\dots,n-1$

⋮

— 2 1

$$\binom{n+1}{2} = n + (n-1) + \dots + 1 + 0$$

4 18-14:52

$$(x-1)(x^n+\dots+x+1)$$
$$= x^{n+1} + x^n + \dots + x^2 + x$$
$$- x^n - \dots - x - 1$$
$$= x^{n+1} - 1$$

$$(x+y)^n = (x+y)\cdots(x+y)$$

⇝ roznásobením členy $x^k y^{n-k}$ ... kolikrát?

$\binom{n}{k}$ ... výběr závorek, ze kterých se bere x

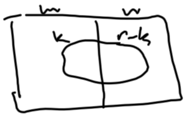
4 18-14:56

$\binom{n+1}{m+1}$ .... vybíráme $m+1$ čísel z $1,\dots,n+1$

– rozdělíme podle největšího

- $n+1$, zbylých m se vybírá z $1,\dots,n$ .... počet možností $\binom{n}{m}$
- $k+1$, ... $1,\dots,k$ .... $\binom{k}{m}$
- $m+1$, ... $1,\dots,m$ .... $\binom{m}{m}$

$$\binom{n+1}{m+1} = \binom{n}{m}+\dots+\binom{k}{m}+\dots+\binom{m}{m}$$

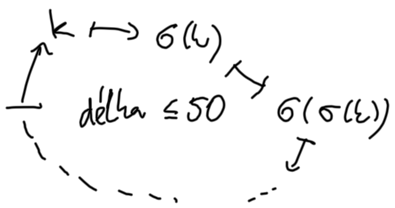


.... $\binom{m}{k}\cdot\binom{n}{r-k}$

4 18-15:01

k-tý vězeň otevře k-tou krabici, v ní je číslo $\sigma(k)$

$\sigma(k) \neq k$, dále otevře $\sigma(k)$-tou krabici, v ní je $\sigma(\sigma(k))$



4 18-15:10

prst úspěchu $= \dfrac{\text{počet permutací bez cyklu délky} > 50}{\text{počet permutací na 100 prvcích}}$

$\dfrac{100! - \dots}{100!} = 1 - \dfrac{\dots}{100!}$ | počet permutací s cyklem délky $n > 50$ je:

$$\binom{100}{n}(n-1)!\cdot(100-n)!$$
$$= \frac{100!}{n!\,(100-n)!}\,(n-1)!\,(100-n)! = \frac{100!}{n}$$

špatné případy: $100!\left(\frac{1}{51}+\frac{1}{52}+\dots+\frac{1}{100}\right)$

prst. špatného případu:

$$\frac{1}{51}+\frac{1}{52}+\dots+\frac{1}{100} = H_{100}-H_{50} \sim$$
$$\sim (\ln 101+\gamma) - (\ln 51 + \gamma)$$
$$= \ln\frac{101}{51} \sim \ln 2$$

4 18-15:14

$$(x+y)^\alpha = y^\alpha\left(\left(\frac{x}{y}\right)+1\right)^\alpha$$
$\|?$
$$\sum \binom{\alpha}{k} x^k y^{\alpha-k} = y^\alpha \sum\binom{\alpha}{k} x^k y^{-k}$$
$$= y^\alpha \sum \binom{\alpha}{k}\left(\frac{x}{y}\right)^k$$

$\frac{x}{y} = t$; chceme: $(t+1)^\alpha \overset{?}{=} \sum\binom{\alpha}{k}t^k$

funkce ↑ Taylorův rozvoj

$$f(t) = \sum_{k=0}^{\infty} \frac{1}{k!}\, f^{(k)}(0)\, t^k$$

$$((t+1)^\alpha)' = \alpha\cdot(t+1)^{\alpha-1}\overbrace{\cdot(t+1)'}^{?} \quad \xrightarrow{t\to 0} \alpha$$

$$((t+1)^\alpha)'' = \alpha(\alpha-1)\cdot(t+1)^{\alpha-2} \quad \xrightarrow{t\to 0} \alpha(\alpha-1)$$

⋮

$$(t+1)^\alpha = \sum \frac{1}{k!}\,\alpha(\alpha-1)\cdots(\alpha-k+1)\, t^k$$
$$= \sum \binom{\alpha}{k} t^k$$

4 18-15:24

$$(1+x)^n = \sum \binom{n}{k} x^k$$

$x=1$: $\quad 2^n = \sum \binom{n}{k}$

$()'$

$$n\cdot(1+x)^{n-1} = \sum \binom{n}{k} k \cdot x^{k-1}$$

$x=1$: $\quad n\cdot 2^{n-1} = \sum \underbrace{\binom{n}{k}\cdot k}_{= n\binom{n-1}{k-1}}$

4 18-15:33