Analysis of Large Graphs: Link Analysis, PageRank

Advanced Search Techniques for Large Scale Data Analytics

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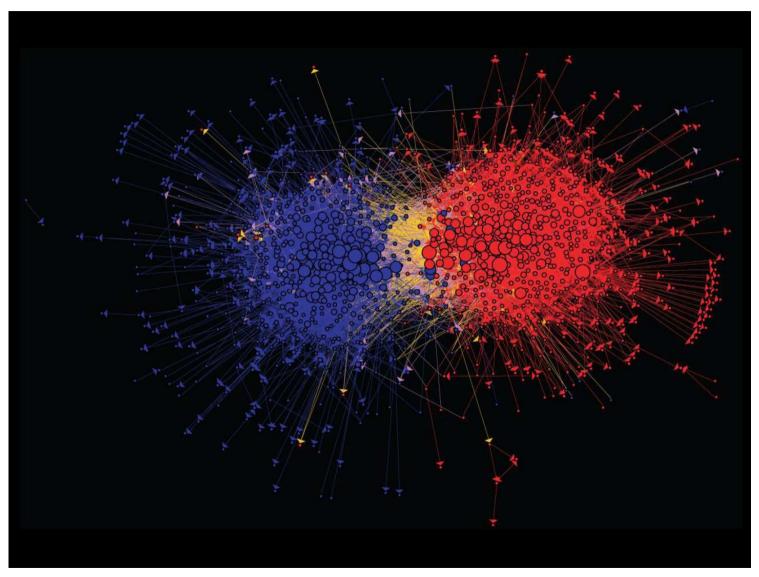
Graph Data: Social Networks



Facebook social graph

4-degrees of separation [Backstrom-Boldi-Rosa-Ugander-Vigna, 2011]

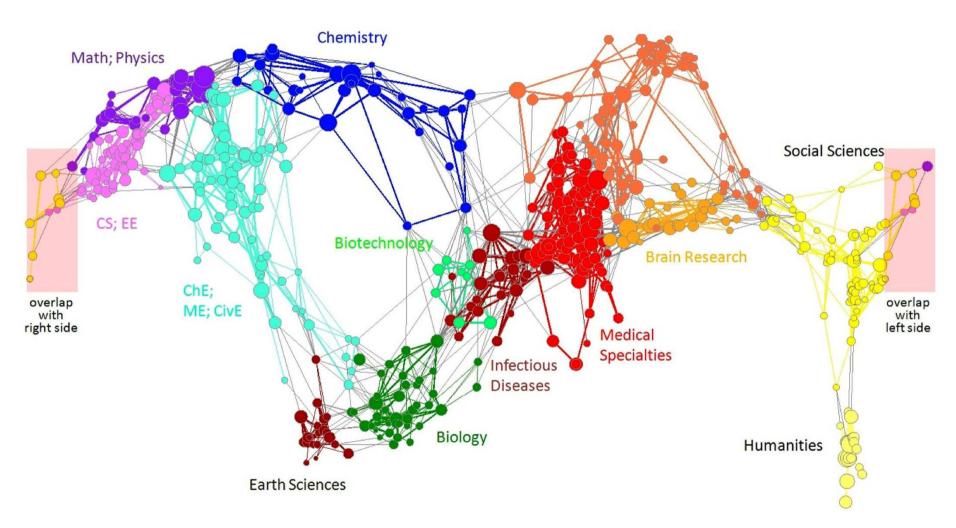
Graph Data: Media Networks



Connections between political blogs

Polarization of the network [Adamic-Glance, 2005]

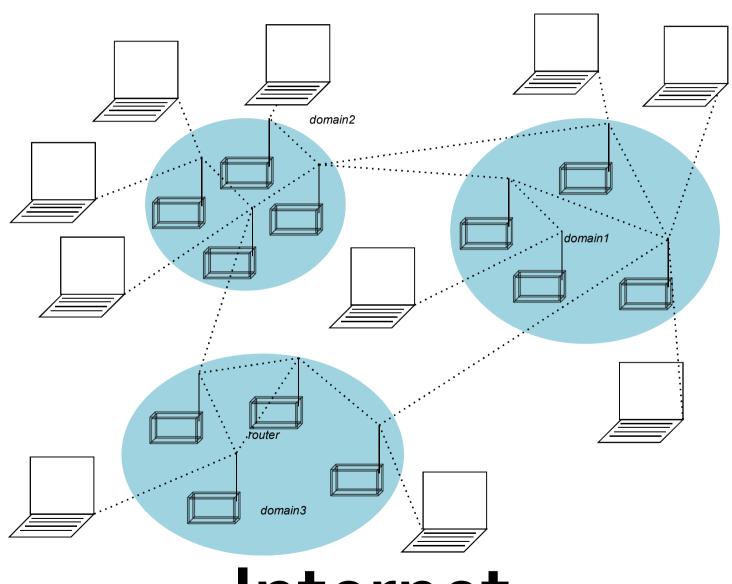
Graph Data: Information Nets



Citation networks and Maps of science

[Börner et al., 2012]

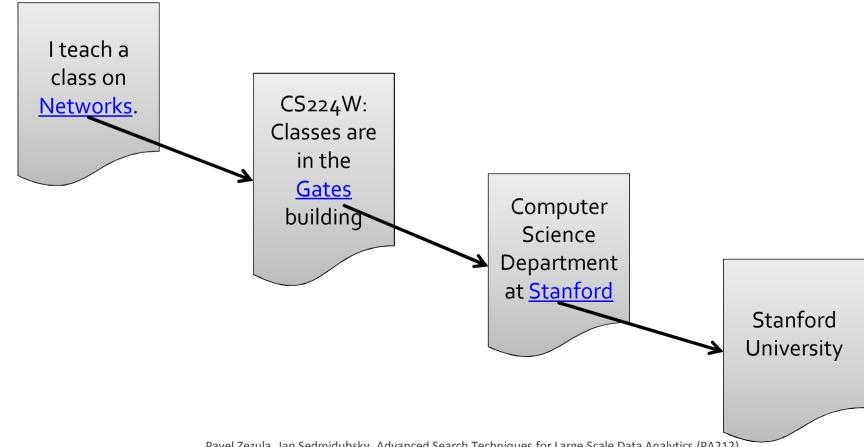
Graph Data: Communication Nets



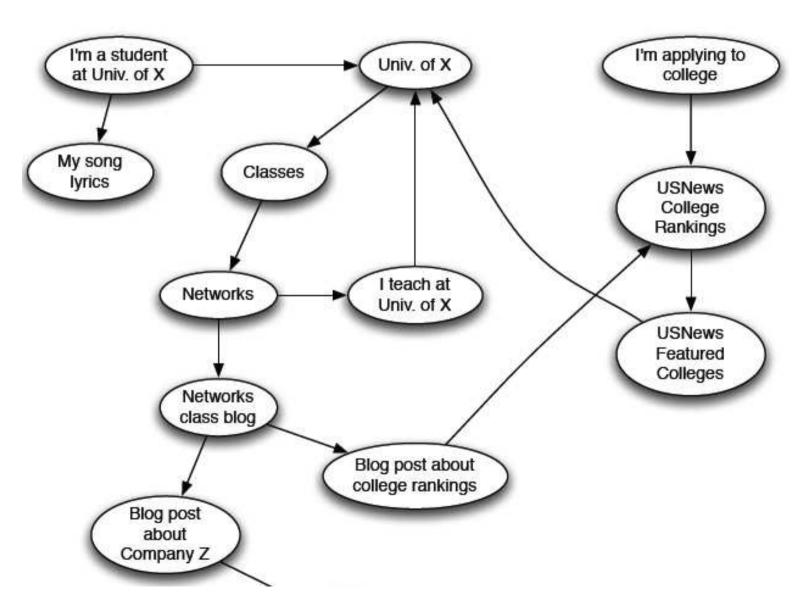
Internet

Web as a Graph

- Web as a directed graph:
 - Nodes: Webpages
 - **Edges: Hyperlinks**



Web as a Directed Graph



Broad Question

- How to organize the Web?
- First try: Human curated
 Web directories
 - Yahoo, DMOZ, LookSmart
- Second try: Web Search
 - Information Retrieval investigates:
 Find relevant docs in a small and trusted set
 - Newspaper articles, Patents, etc.
 - But: Web is huge, full of untrusted documents, random things, web spam, etc.



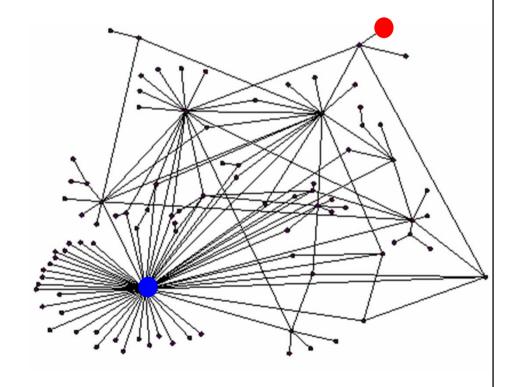
Web Search: 2 Challenges

- 2 challenges of web search:
- (1) Web contains many sources of information Who to "trust"?
 - Trick: Trustworthy pages may point to each other!
- (2) What is the "best" answer to query "newspaper"?
 - No single right answer
 - Trick: Pages that actually know about newspapers might all be pointing to many newspapers

Ranking Nodes on the Graph

 All web pages are not equally "important" www.joe-schmoe.com vs. www.stanford.edu

There is large diversity in the web-graph node connectivity.
 Let's rank the pages by the link structure!



Link Analysis Algorithms

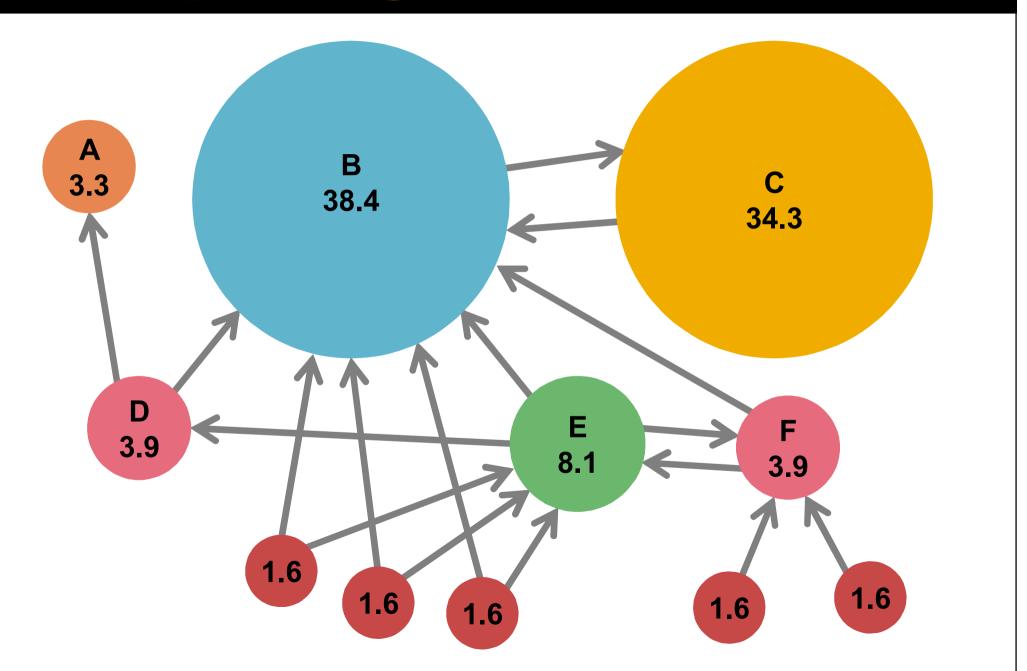
- We will cover the following Link Analysis approaches for computing importances of nodes in a graph:
 - Page Rank
 - Topic-Specific (Personalized) Page Rank
 - Web Spam Detection Algorithms

PageRank: The "Flow" Formulation

Links as Votes

- Idea: Links as votes
 - Page is more important if it has more links
 - In-coming links? Out-going links?
- Think of in-links as votes:
 - www.stanford.edu has 23,400 in-links
 - www.joe-schmoe.com has 1 in-link
- Are all in-links are equal?
 - Links from important pages count more
 - Recursive question!

Example: PageRank Scores



Simple Recursive Formulation

- Each link's vote is proportional to the importance of its source page
- If page j with importance r_j has n out-links, each link gets r_i / n votes
- Page j's own importance is the sum of the votes on its in-links

$$r_{j} = r_{i}/3 + r_{k}/4$$

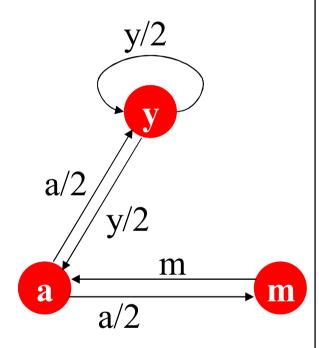
PageRank: The "Flow" Model

- A "vote" from an important page is worth more
- A page is important if it is pointed to by other important pages
- Define a "rank" r_j for page j

$$r_j = \sum_{i \to j} \frac{r_i}{d_i}$$

... out-degree of node

The web in 1839



"Flow" equations:

$$r_y = r_y/2 + r_a/2$$

$$r_a = r_y/2 + r_m$$

$$r_m = r_a/2$$

Solving the Flow Equations

- 3 equations, 3 unknowns, no constants
 - No unique solution
- Additional constraint forces uniqueness:

- Gaussian elimination method works for small examples, but we need a better method for large web-size graphs
- We need a new formulation!

Flow equations:

$$r_y = r_y/2 + r_a/2$$

$$r_a = r_y/2 + r_m$$

$$r_m = r_a/2$$

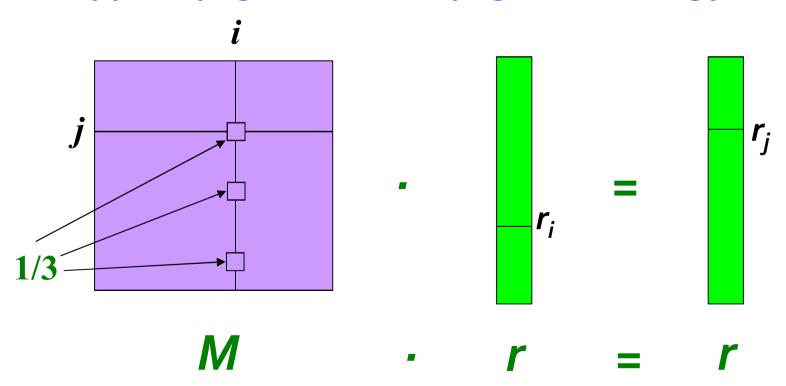
PageRank: Matrix Formulation

- Stochastic adjacency matrix
 - Let page has out-links
 - If , then else
 - is a column stochastic matrix
 - Columns sum to 1
- Rank vector : vector with an entry per page
 - is the importance score of page
- The flow equations can be written

$$r_j = \sum_{i \to j} \frac{r_i}{d_i}$$

Example

- Remember the flow equation: $r_j = \sum_{i \to j} \frac{r_i}{d_i}$ Flow equation in the matrix form
 - Suppose page i links to 3 pages, including j

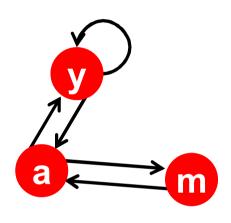


Eigenvector Formulation

- The flow equations can be written
- So the rank vector r is an eigenvector of the stochastic web matrix M
 - In fact, its first or principal eigenvector, with corresponding eigenvalue 1
 - Largest eigenvalue of *M* is 1 since *M* is column stochastic (with non-negative entries)
 - We know r is unit length and each column of M sums to one, so
- We can now efficiently solve for r!
 The method is called Power iteration

NOTE: *x* is an eigenvector with the corresponding eigenvalue λ if:

Example: Flow Equations & M



$$r_y = r_y/2 + r_a/2$$

$$r_a = r_y/2 + r_m$$

$$r_m = r_a/2$$

$$r = M \cdot r$$

$$\begin{bmatrix} y \\ a \\ m \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} y \\ a \\ m \end{bmatrix}$$

Power Iteration Method

- Given a web graph with n nodes, where the nodes are pages and edges are hyperlinks
- Power iteration: a simple iterative scheme
 - Suppose there are N web pages
 - Initialize: $\mathbf{r}^{(0)} = [1/N,....,1/N]^T$
 - Iterate: $\mathbf{r}^{(t+1)} = \mathbf{M} \cdot \mathbf{r}^{(t)}$
 - Stop when $|\mathbf{r}^{(t+1)} \mathbf{r}^{(t)}|_1 < \varepsilon$

 $|\mathbf{x}|_1 = \sum_{1 \le i \le N} |\mathbf{x}_i|$ is the **L**₁ norm Can use any other vector norm, e.g., Euclidean

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathbf{d}_i}$$

d_i out-degree of node i

PageRank: How to solve?

Power Iteration:

Set

/N

1:

- **2**:
- Goto 1
- Example:

$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = 1/3$$

$$1/3$$

$$1/3$$

$$1/3$$

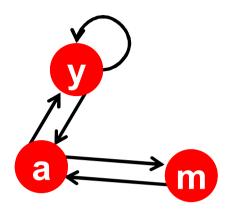
$$1/3$$

$$1/3$$

$$1/3$$

$$1/3$$

$$1/3$$



	У	a	m
у	1/2	1/2	0
a	1/2	0	1
m	0	1/2	0

$$r_y = r_y/2 + r_a/2$$

$$r_a = r_y/2 + r_m$$

$$r_m = r_a/2$$

PageRank: How to solve?

Power Iteration:

Set

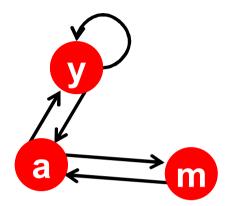
/N

1:

- **2**:
- Goto 1
- Example:

1/3	1/3	5/12	9/24	
1/3	3/6	1/3	11/24	
1/3	1/6	3/12	1/6	

Iteration 0, 1, 2, ...



	У	a	m
у	1/2	1/2	0
a	1/2	0	1
m	0	1/2	0

$$r_y = r_y/2 + r_a/2$$

$$r_a = r_y/2 + r_m$$

$$r_m = r_a/2$$

6/15

6/15

3/15

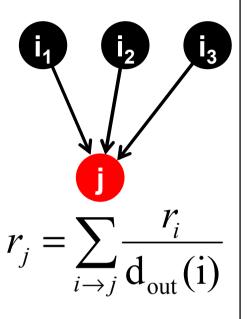
Random Walk Interpretation

Imagine a random web surfer:

- At any time , surfer is on some page
- At time , the surfer follows an out-link from uniformly at random
- Ends up on some page linked from
- Process repeats indefinitely

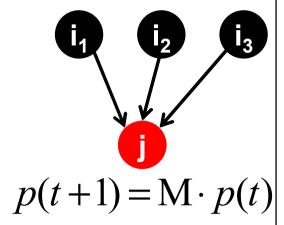
Let:

- ... vector whose th coordinate is the prob. that the surfer is at page at time
- So, is a probability distribution over pages



The Stationary Distribution

- Where is the surfer at time t+1?
 - Follows a link uniformly at random()



Suppose the random walk reaches a state()

then is stationary distribution of a random walk

- Our original rank vector satisfies
 - So, is a stationary distribution for the random walk

Existence and Uniqueness

 A central result from the theory of random walks (a.k.a. Markov processes):

For graphs that satisfy **certain conditions**, the **stationary distribution is unique** and eventually will be reached no matter what the initial probability distribution at time **t** = **0**

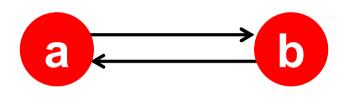
PageRank: The Google Formulation

PageRank: Three Questions

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathbf{d}_i}$$
 or equivalently $r = Mr$

- Does this converge?
- Does it converge to what we want?
- Are results reasonable?

Does this converge?



$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{d_i}$$

Example:

Does it converge to what we want?

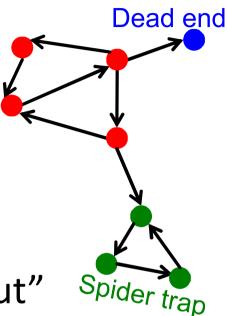
$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{d_i}$$

Example:

PageRank: Problems

2 problems:

- (1) Some pages are dead ends (have no out-links)
 - Random walk has "nowhere" to go to
 - Such pages cause importance to "leak out"



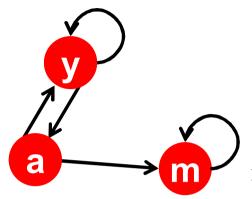
- (2) Spider traps:
 - (all out-links are within the group)
 - Random walked gets "stuck" in a trap
 - And eventually spider traps absorb all importance

Problem: Spider Traps

Power Iteration:

Set

And iterate



	У	a	m
у	1/2	1/2	0
a	1/2	0	0
m	0	1/2	1

m is a spider trap

$$r_y = r_y/2 + r_a/2$$

$$r_a = r_y/2$$

$$r_m = r_a/2 + r_m$$

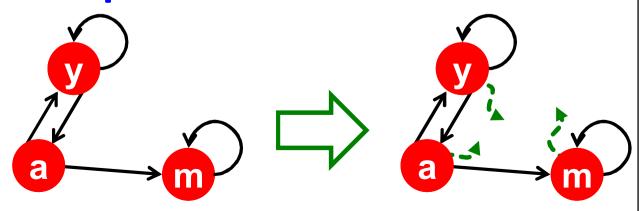
Example:

Iteration 0, 1, 2, ...

All the PageRank score gets "trapped" in node m.

Solution: Teleports!

- The Google solution for spider traps: At each time step, the random surfer has two options
 - With prob. β , follow a link at random
 - With prob. **1-** β , jump to some random page
 - Common values for β are in the range 0.8 to 0.9
- Surfer will teleport out of spider trap within a few time steps



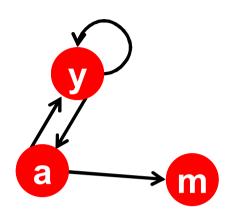
Problem: Dead Ends

Power Iteration:

Set

__

And iterate



	У	a	m
у	1/2	1/2	0
a	1/2	0	0
m	0	1/2	0

$$r_y = r_y/2 + r_a/2$$

$$r_a = r_y/2$$

$$r_m = r_a/2$$

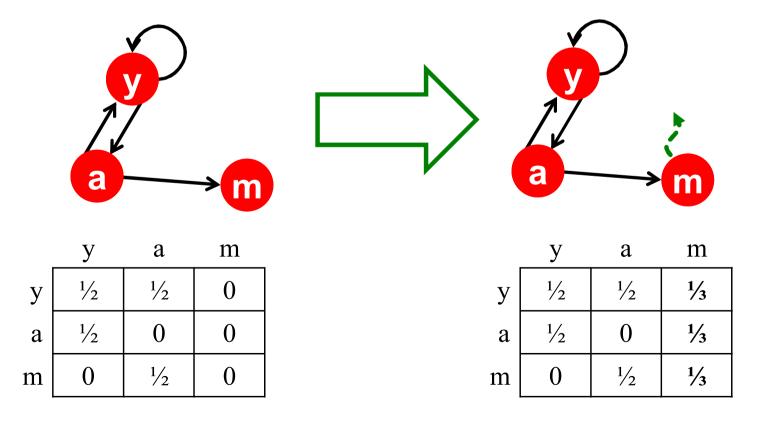
Example:

Iteration 0, 1, 2, ...

Here the PageRank "leaks" out since the matrix is not stochastic.

Solution: Always Teleport!

- Teleports: Follow random teleport links with probability 1.0 from dead-ends
 - Adjust matrix accordingly



Why Teleports Solve the Problem?

Why are dead-ends and spider traps a problem and why do teleports solve the problem?

- Spider-traps are not a problem, but with traps
 PageRank scores are not what we want
 - Solution: Never get stuck in a spider trap by teleporting out of it in a finite number of steps
- Dead-ends are a problem
 - The matrix is not column stochastic so our initial assumptions are not met
 - Solution: Make matrix column stochastic by always teleporting when there is nowhere else to go

Solution: Random Teleports

- Google's solution that does it all:
 At each step, random surfer has two options:
 - With probability β , follow a link at random
 - With probability $1-\beta$, jump to some random page
- PageRank equation [Brin-Page, 98]

d_i ... out-degree of node i

This formulation assumes that has no dead ends. We can either preprocess matrix to remove all dead ends or explicitly follow random teleport links with probability 1.0 from dead-ends.

The Google Matrix

PageRank equation [Brin-Page, '98]

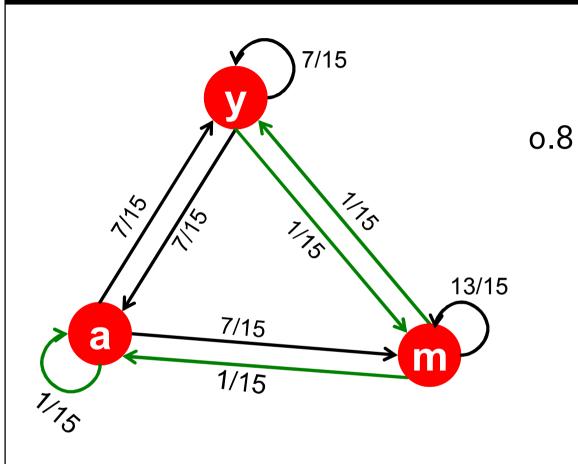
The Google Matrix A:

 $[1/N]_{NxN}...N$ by N matrix where all entries are 1/N

$$() \left[- \right]$$

- We have a recursive problem:
 And the Power method still works!
- What is β ?
 - In practice $\beta = 0.8, 0.9$ (make 5 steps on avg., jump)

Random Teleports ($\beta = 0.8$)



M

1/2 1/2 0 1/2 0 0 0 1/2 1 [1/N]_{N×N}

+ 0.2 | 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3

y 7/15 7/15 1/15 a 7/15 1/15 1/15 m 1/15 7/15 13/15

A

y
$$1/3$$
 0.33 0.24 0.26 7/33
a = 1/3 0.20 0.20 0.18 ... 5/33
m 1/3 0.46 0.52 0.56 21/33

How do we actually compute the PageRank?

Computing Page Rank

- Key step is matrix-vector multiplication
 - $r^{\text{new}} = A \cdot r^{\text{old}}$
- Easy if we have enough main memory to hold A, r^{old}, r^{new}
- Say N = 1 billion pages
 - We need 4 bytes for each entry (say)
 - 2 billion entries for vectors, approx 8GB
 - Matrix A has N² entries
 - 10¹⁸ is a large number!

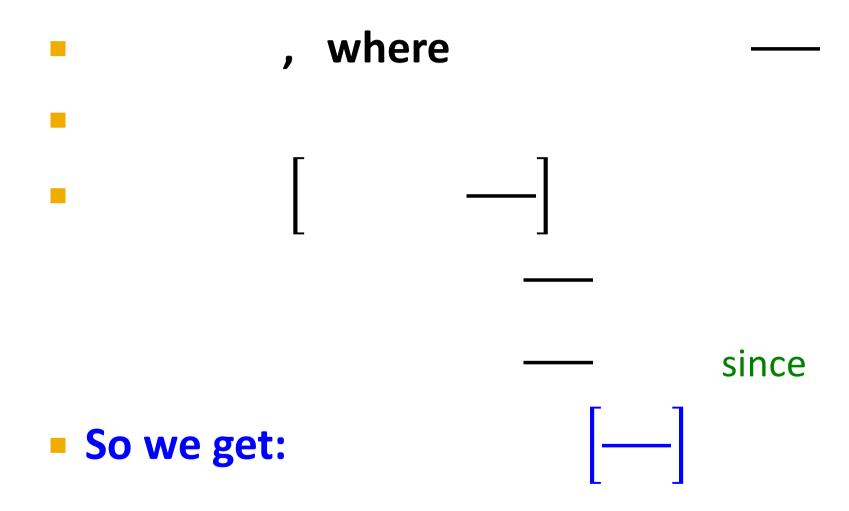
$$\mathbf{A} = \beta \cdot \mathbf{M} + (\mathbf{1} - \beta) \left[\mathbf{1} / \mathbf{N} \right]_{\mathbf{N} \times \mathbf{N}}$$

$$\mathbf{A} = 0.8 \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 1 \end{bmatrix} + 0.2 \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

Matrix Formulation

- Suppose there are N pages
- Consider page i, with d_i out-links
- We have $M_{jj} = 1/|d_i|$ when $i \rightarrow j$ and $M_{jj} = 0$ otherwise
- The random teleport is equivalent to:
 - Adding a **teleport link** from i to every other page and setting transition probability to $(1-\beta)/N$
 - Reducing the probability of following each out-link from $1/|d_i|$ to $\beta/|d_i|$
 - Equivalent: Tax each page a fraction $(1-\beta)$ of its score and redistribute evenly

Rearranging the Equation

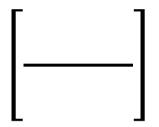


Note: Here we assumed **M** has no dead-ends

 $[x]_N$... a vector of length N with all entries x

Sparse Matrix Formulation

We just rearranged the PageRank equation



- where $[(1-\beta)/N]_N$ is a vector with all **N** entries $(1-\beta)/N$
- M is a sparse matrix! (with no dead-ends)
 - 10 links per node, approx 10N entries
- So in each iteration, we need to:
 - Compute $r^{\text{new}} = \beta M \cdot r^{\text{old}}$
 - Add a constant value (1- β)/N to each entry in r^{new}
 - Note if M contains dead-ends then and we also have to renormalize r^{new} so that it sums to 1

PageRank: The Complete Algorithm

- Input: Graph and parameter
 - Directed graph (can have spider traps and dead ends)
 - Parameter
- Output: PageRank vector
 - Set: —
 - repeat until convergence:

if in-degree of is **0**

Now re-insert the leaked PageRank:

— where:

If the graph has no dead-ends then the amount of leaked PageRank is **1-β**. But since we have dead-ends the amount of leaked PageRank may be larger. We have to explicitly account for it by computing **S**.

Sparse Matrix Encoding

- Encode sparse matrix using only nonzero entries
 - Space proportional roughly to number of links
 - Say 10N, or 4*10*1 billion = 40GB

COLLECO

Still won't fit in memory, but will fit on disk

node	degree	destination nodes		
0	3	1, 5, 7		
1	5	17, 64, 113, 117, 245		
2	2	13, 23		

Some Problems with PageRank

- Measures generic popularity of a page
 - Will ignore/miss topic-specific authorities
 - Solution: Topic-Specific PageRank (next)
- Uses a single measure of importance
 - Other models of importance
 - Solution: Hubs-and-Authorities
- Susceptible to Link spam
 - Artificial link topographies created in order to boost page rank
 - Solution: TrustRank

Topic-Specific PageRank

Topic-Specific PageRank

- Instead of generic popularity, can we measure popularity within a topic?
- Goal: Evaluate Web pages not just according to their popularity, but by how close they are to a particular topic, e.g. "sports" or "history"
- Allows search queries to be answered based on interests of the user
 - Example: Query "Trojan" wants different pages depending on whether you are interested in sports, history and computer security

Topic-Specific PageRank

- Random walker has a small probability of teleporting at any step
- Teleport can go to:
 - Standard PageRank: Any page with equal probability
 - To avoid dead-end and spider-trap problems
 - Topic Specific PageRank: A topic-specific set of "relevant" pages (teleport set)
- Idea: Bias the random walk
 - When walker teleports, she pick a page from a set S
 - S contains only pages that are relevant to the topic
 - E.g., Open Directory (DMOZ) pages for a given topic/query
 - For each teleport set S, we get a different vector r_S

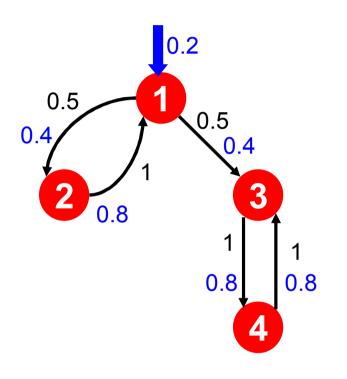
Matrix Formulation

To make this work all we need is to update the teleportation part of the PageRank formulation:

if
 otherwise

- A is stochastic!
- We weighted all pages in the teleport set 5 equally
 - Could also assign different weights to pages!
- Compute as for regular PageRank:
 - Multiply by M, then add a vector
 - Maintains sparseness

Example: Topic-Specific PageRank



Suppose $S = \{1\}, \beta = 0.8$

Node	Iteration				
	0	1	2	stable	
1	0.25	0.4	0.28	0.294	
2	0.25	0.1	0.16	0.118	
3	0.25	0.3	0.32	0.327	
4	0.25	0.2	0.24	0.261	

 $S=\{1\}$, $\beta=0.90$: r=[0.17, 0.07, 0.40, 0.36] $S=\{1\}$, $\beta=0.8$: r=[0.29, 0.11, 0.32, 0.26] $S=\{1\}$, $\beta=0.70$: r=[0.39, 0.14, 0.27, 0.19] $S=\{1,2,3,4\}$, $\beta=0.8$: r=[0.13, 0.10, 0.39, 0.36] $S=\{1,2,3\}$, $\beta=0.8$: r=[0.17, 0.13, 0.38, 0.30] $S=\{1,2\}$, $\beta=0.8$: r=[0.26, 0.20, 0.29, 0.23] $S=\{1\}$, $\beta=0.8$: r=[0.29, 0.11, 0.32, 0.26]

TrustRank: Combating the Web Spam

What is Web Spam?

Spamming:

 Any deliberate action to boost a web page's position in search engine results, incommensurate with page's real value

Spam:

- Web pages that are the result of spamming
- This is a very broad definition
 - SEO industry might disagree!
 - SEO = search engine optimization
- Approximately 10-15% of web pages are spam

Web Search

Early search engines:

- Crawl the Web
- Index pages by the words they contained
- Respond to search queries (lists of words) with the pages containing those words
- Early page ranking:
 - Attempt to order pages matching a search query by "importance"
 - First search engines considered:
 - (1) Number of times query words appeared
 - (2) Prominence of word position, e.g. title, header

First Spammers

 As people began to use search engines to find things on the Web, those with commercial interests tried to exploit search engines to bring people to their own site – whether they wanted to be there or not

Example:

- Shirt-seller might pretend to be about "movies"
- Techniques for achieving high relevance/importance for a web page

First Spammers: Term Spam

- How do you make your page appear to be about movies?
 - (1) Add the word movie 1,000 times to your page
 - Set text color to the background color, so only search engines would see it
 - (2) Or, run the query "movie" on your target search engine
 - See what page came first in the listings
 - Copy it into your page, make it "invisible"
- These and similar techniques are term spam

Google's Solution to Term Spam

- Believe what people say about you, rather than what you say about yourself
 - Use words in the anchor text (words that appear underlined to represent the link) and its surrounding text
- PageRank as a tool to measure the "importance" of Web pages

Why It Works?

Our hypothetical shirt-seller looses

- Saying he is about movies doesn't help, because others don't say he is about movies
- His page isn't very important, so it won't be ranked high for shirts or movies

Example:

- Shirt-seller creates 1,000 pages, each links to his with "movie" in the anchor text
- These pages have no links in, so they get little PageRank
- So the shirt-seller can't beat truly important movie pages, like IMDB

Google vs. Spammers: Round 2!

- Once Google became the dominant search engine, spammers began to work out ways to fool Google
- Spam farms were developed to concentrate
 PageRank on a single page
- Link spam:
 - Creating link structures that boost PageRank of a particular page



Link Spamming

- Three kinds of web pages from a spammer's point of view
 - Inaccessible pages
 - Accessible pages
 - e.g., blog comments pages
 - spammer can post links to his pages
 - Owned pages
 - Completely controlled by spammer
 - May span multiple domain names

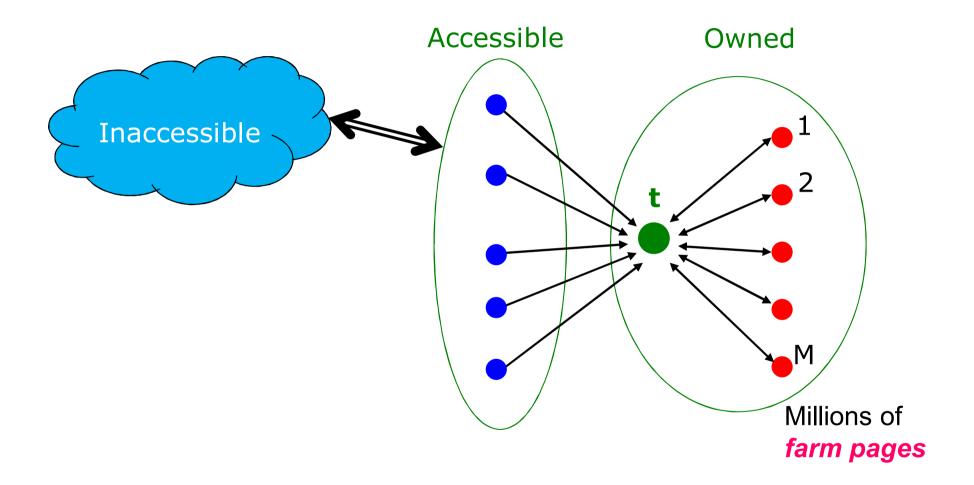
Link Farms

- Spammer's goal:
 - Maximize the PageRank of target page t

Technique:

- Get as many links from accessible pages as possible to target page t
- Construct "link farm" to get PageRank multiplier effect

Link Farms



One of the most common and effective organizations for a link farm

TrustRank: Combating the Web Spam

Combating Spam

- Combating term spam
 - Analyze text using statistical methods
 - Similar to email spam filtering
 - Also useful: Detecting approximate duplicate pages
- Combating link spam
 - Detection and blacklisting of structures that look like spam farms
 - Leads to another war hiding and detecting spam farms
 - TrustRank = topic-specific PageRank with a teleport set of trusted pages
 - Example: .edu domains, similar domains for non-US schools

TrustRank: Idea

- Basic principle: Approximate isolation
 - It is rare for a "good" page to point to a "bad" (spam) page
- Sample a set of seed pages from the web
- Have an oracle (human) to identify the good pages and the spam pages in the seed set
 - Expensive task, so we must make seed set as small as possible

Why is it a good idea?

Trust attenuation:

 The degree of trust conferred by a trusted page decreases with the distance in the graph

Trust splitting:

- The larger the number of out-links from a page, the less scrutiny the page author gives each outlink
- Trust is split across out-links

Hubs and Authorities

- HITS (Hypertext-Induced Topic Selection)
 - Is a measure of importance of pages or documents, similar to PageRank
 - Proposed at around same time as PageRank ('98)
- Goal: Say we want to find good newspapers
 - Don't just find newspapers. Find "experts" people who link in a coordinated way to good newspapers
- Idea: Links as votes
 - Page is more important if it has more links
 - In-coming links? Out-going links?

PageRank and HITS

- PageRank and HITS are two solutions to the same problem:
 - What is the value of an in-link from u to v?
 - In the PageRank model, the value of the link depends on the links into u
 - In the HITS model, it depends on the value of the other links out of u
- The destinies of PageRank and HITS post-1998 were very different