IA159 Formal Verification Methods

Verification via Automata, Symbolic Execution, and Interpolation

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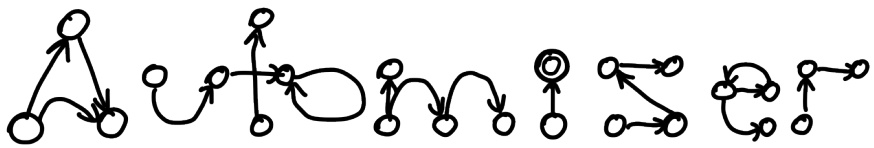
Masaryk University

Focus and sources

Focus

- basic principle
- interpolation
- example

ULTIMATE



Sources

- M. Heizmann, J. Hoenicke, A. Podelski: Software Model Checking for People Who Love Automata, CAV 2013.
- M. Heizmann, J. Hoenicke, A. Podelski: Termination Analysis by Learning Terminating Programs, CAV 2014.

Special thanks to M. Heizmann for providing me his slides.

Programs as languages

New view on programs

A program defines a language over program statements.

- alphabet = the set of program statements
- finite automaton = control flow graph
- accepting states = error locations

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while x<20 {
   if i!=0 then z++
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}
assert(z<=5)
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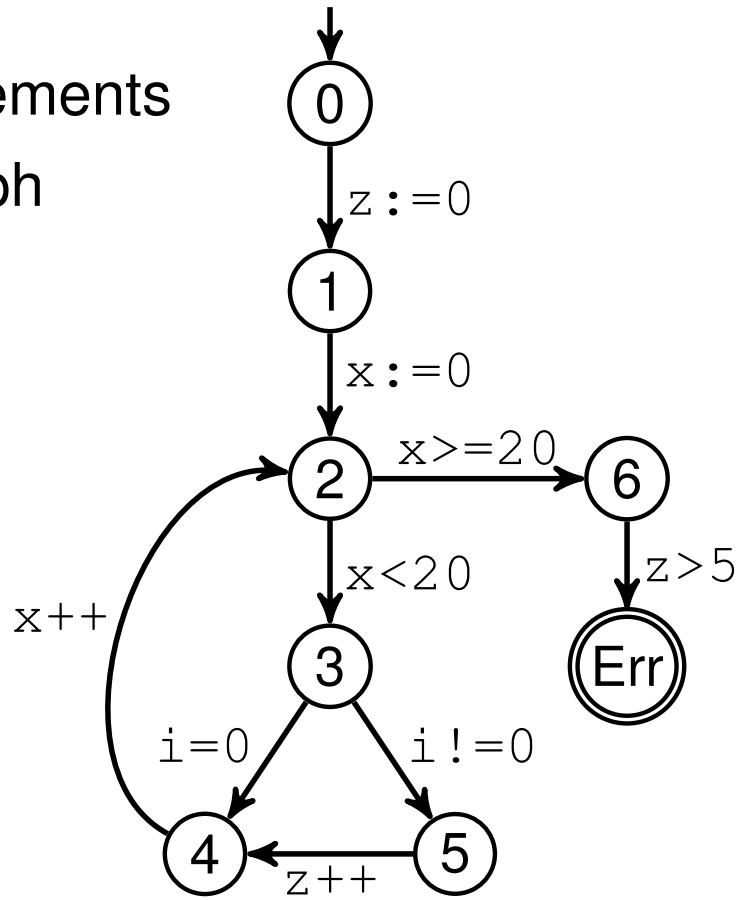


The goal

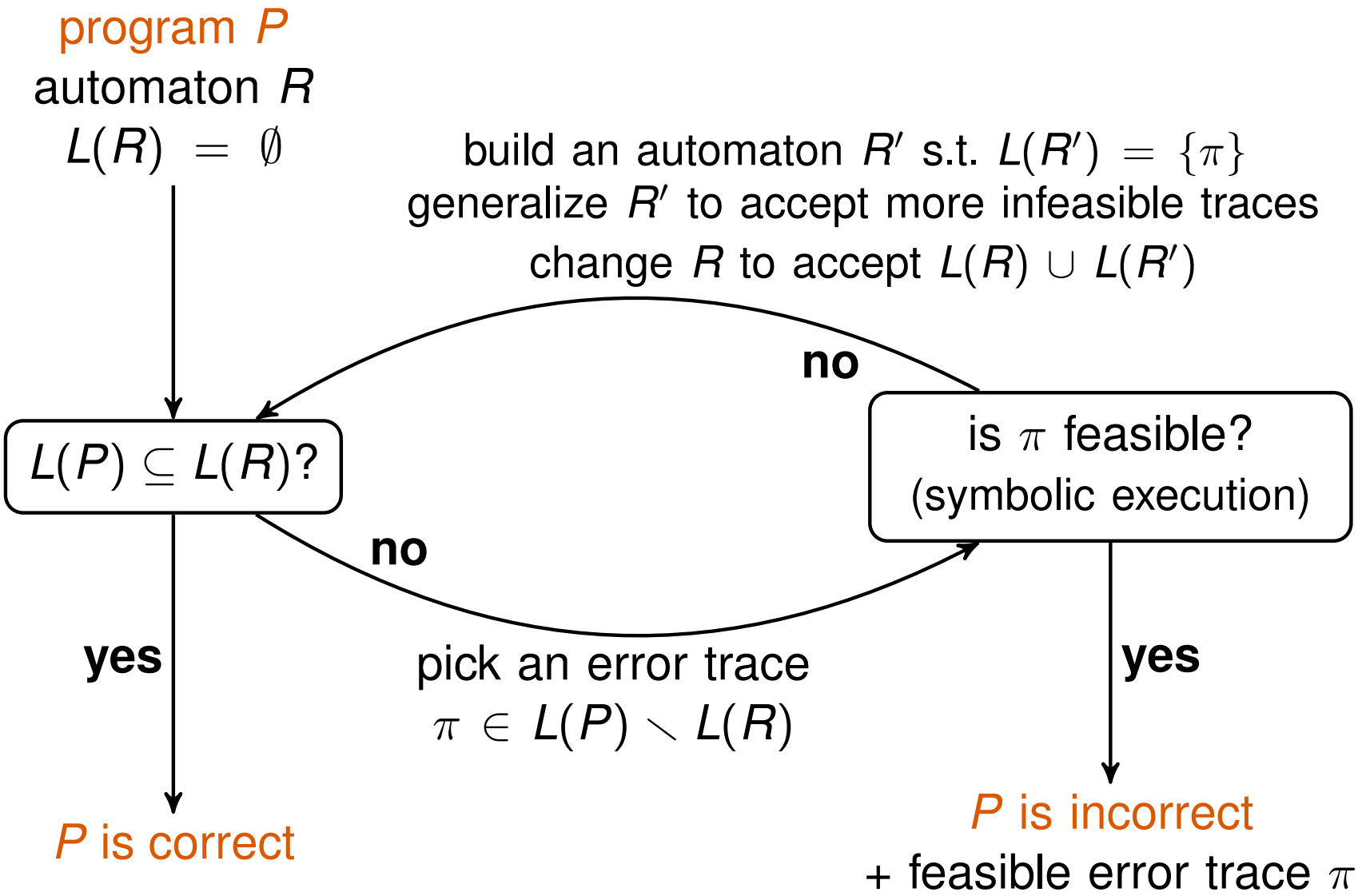
- such an automaton accepts error traces
- not all error traces are feasible

The goal

To decide whether there exists a feasible error trace accepted by the automaton.

The program is correct iff all error traces accepted by the automaton are infeasible.

The algorithm



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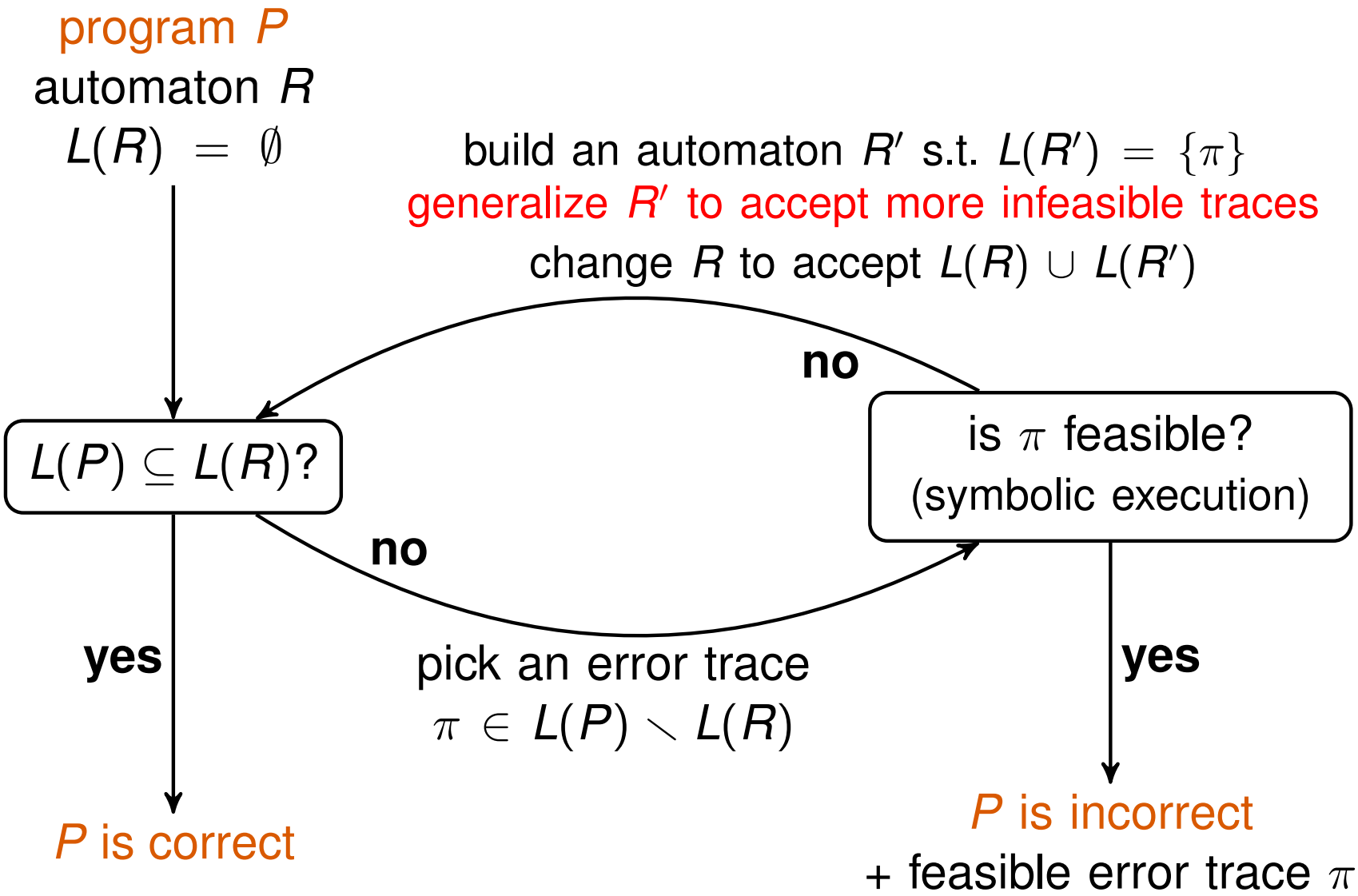

Craig interpolation

Theorem (William Craig, 1957)

Let φ, ψ be two first-order formulae such that $\varphi \implies \psi$. Then there exists a first order-formula θ called interpolant such that

- *all non-logical symbols in θ occur in both φ and ψ,*
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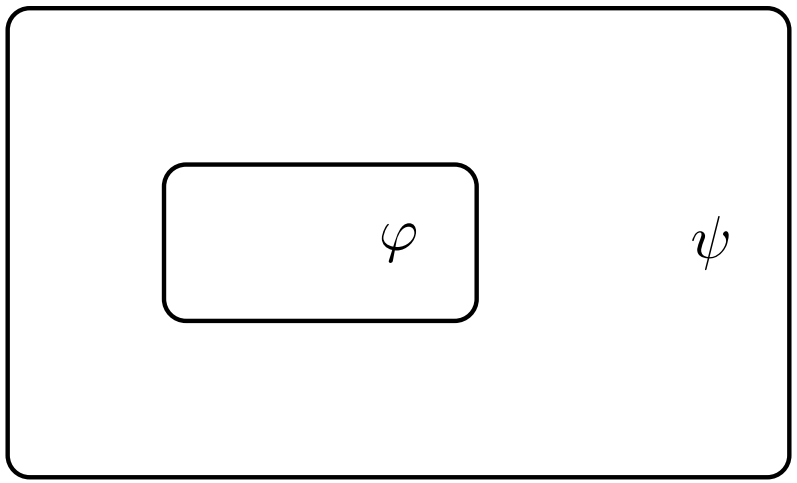

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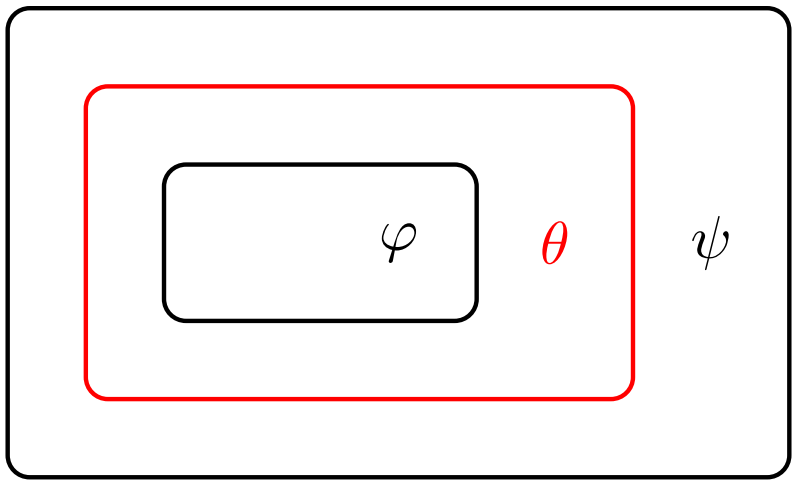

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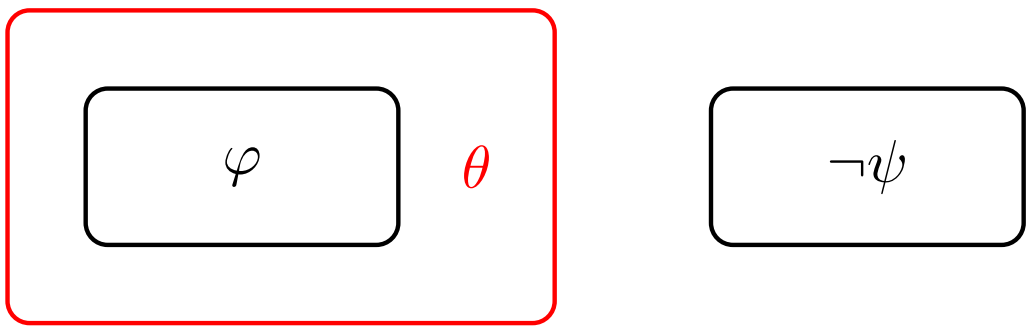

Example: program

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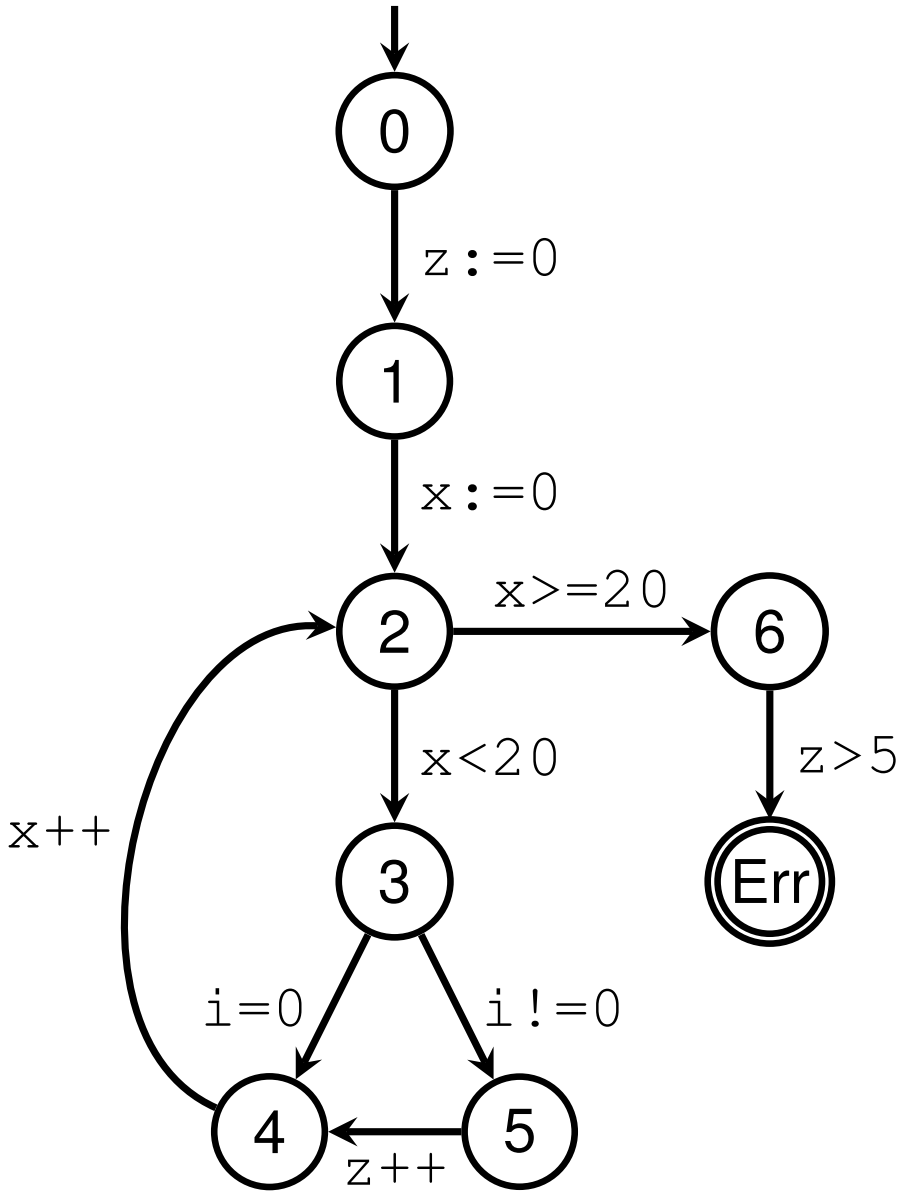

Example: error path

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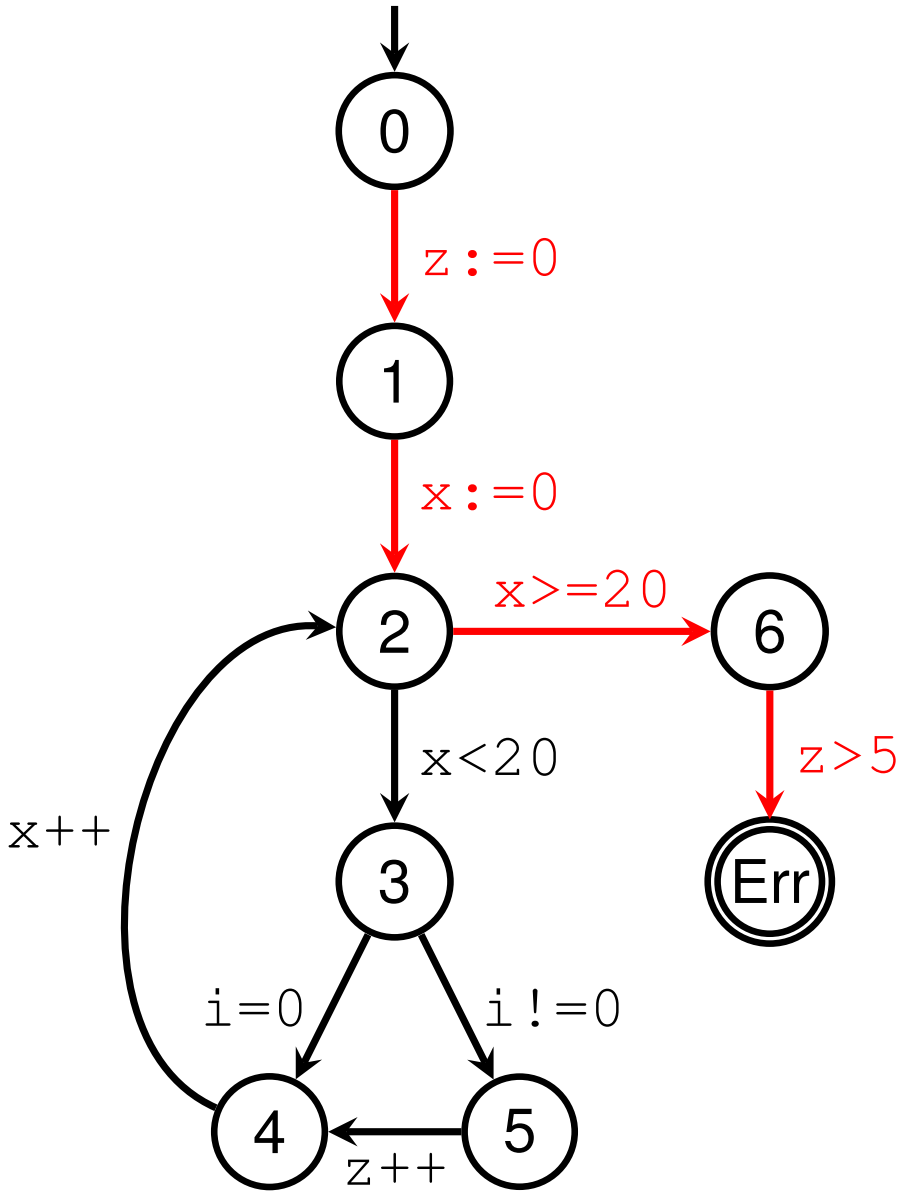

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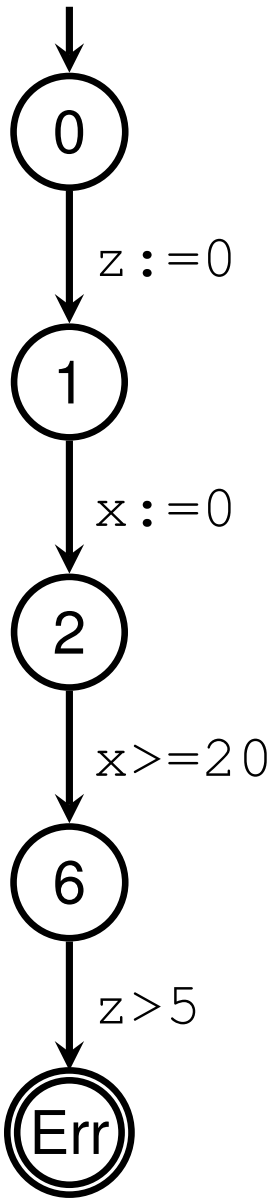
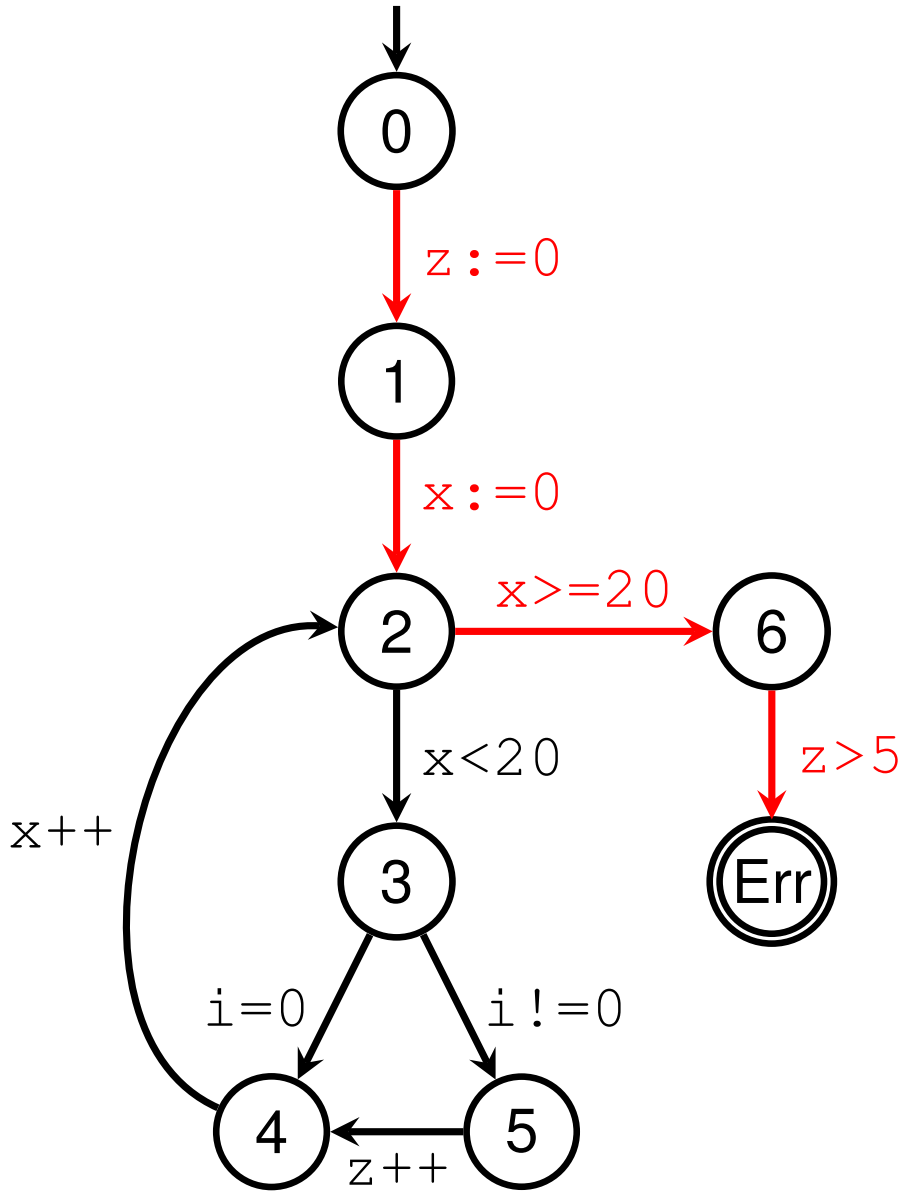
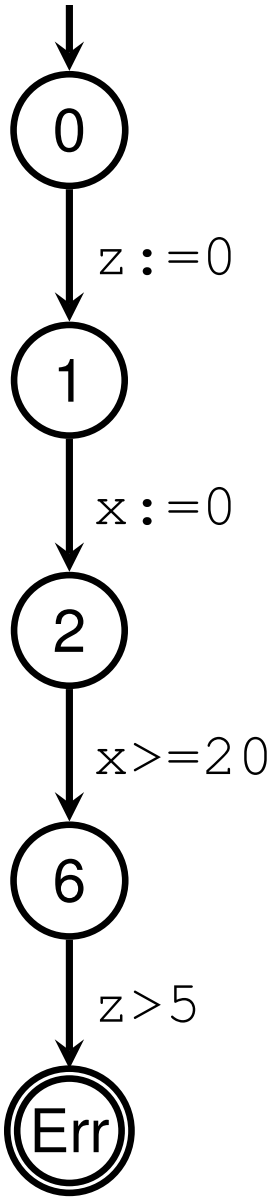




Example: feasability analysis



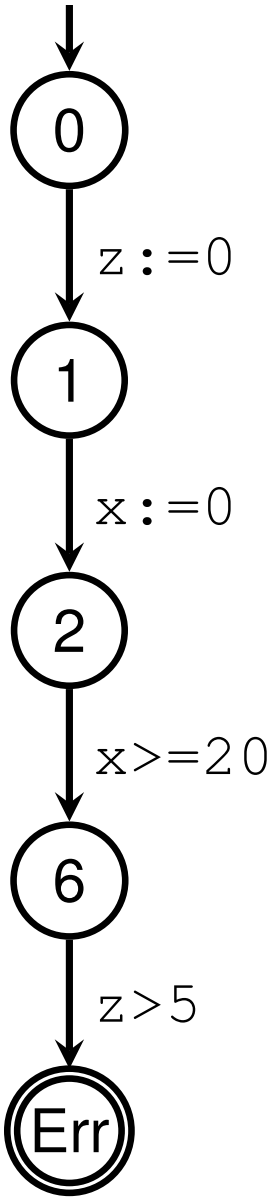
symbolic execution produces

$z = 0 \ \wedge \ x = 0 \ \wedge \ x \geq 20 \ \wedge \ z > 5$

$\equiv$ false

$\Longrightarrow$ the error trace is infeasible

Example: generalization of infeasibility arguments



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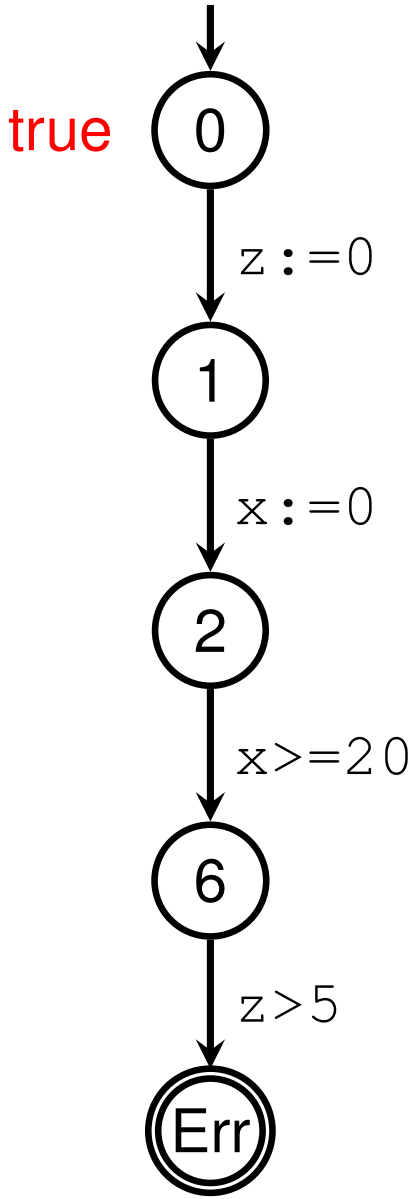

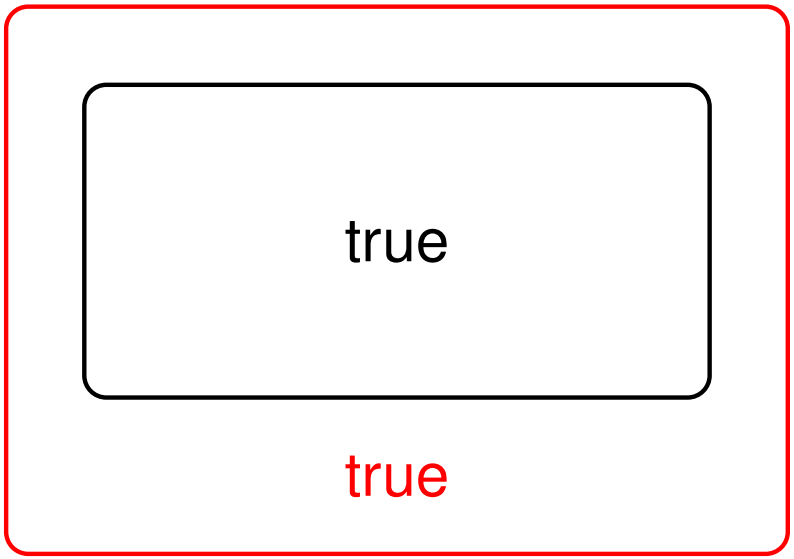

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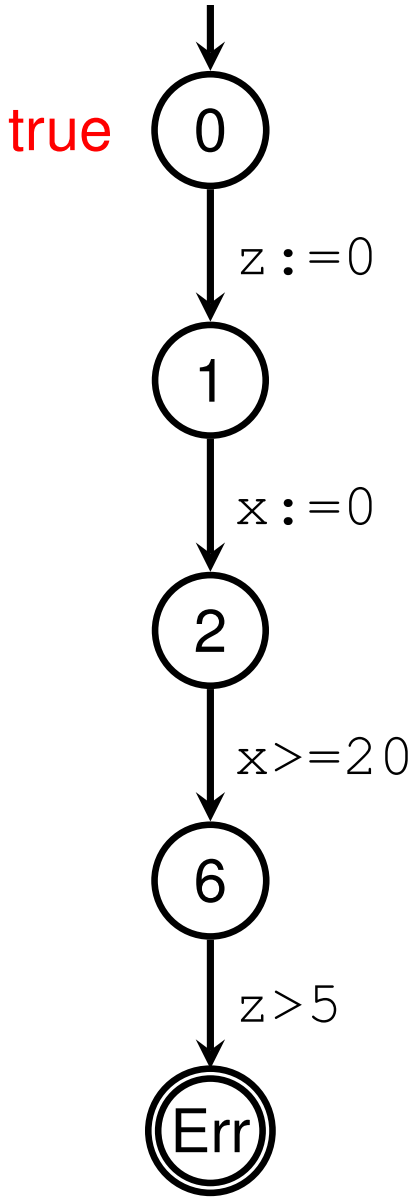

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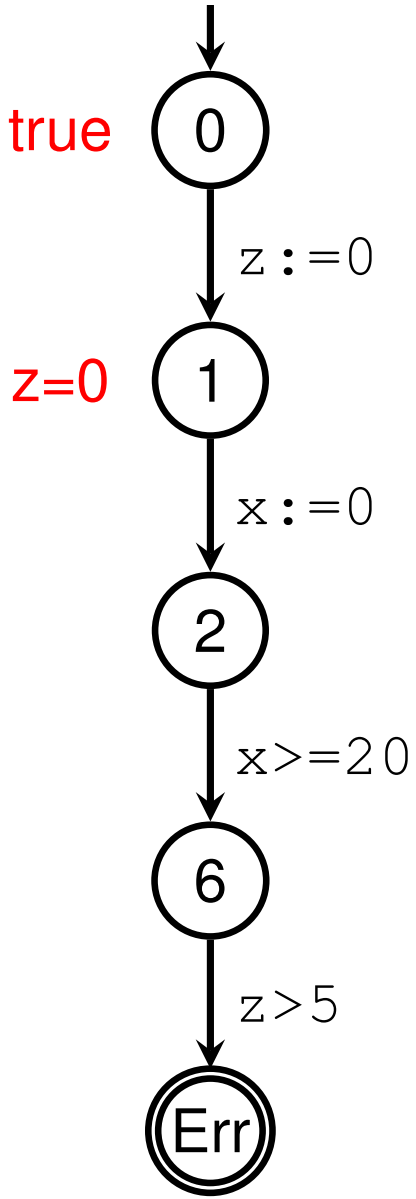

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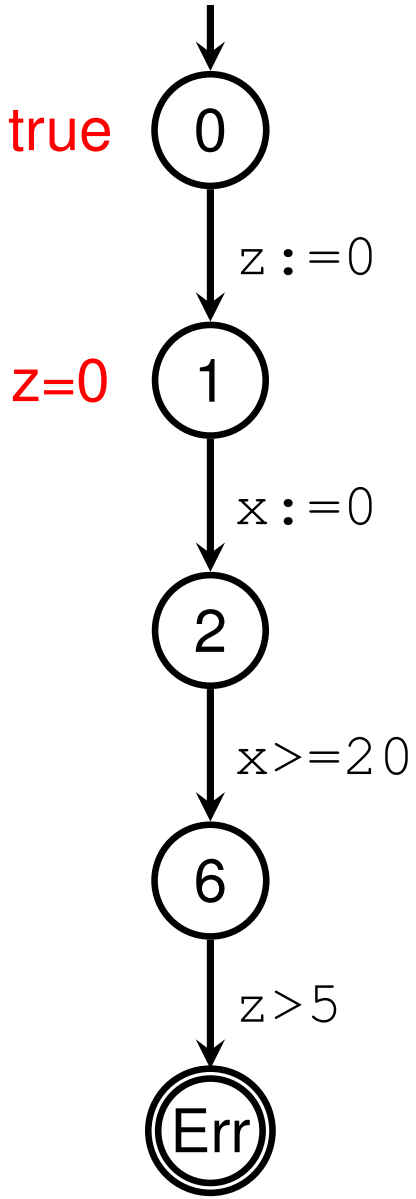

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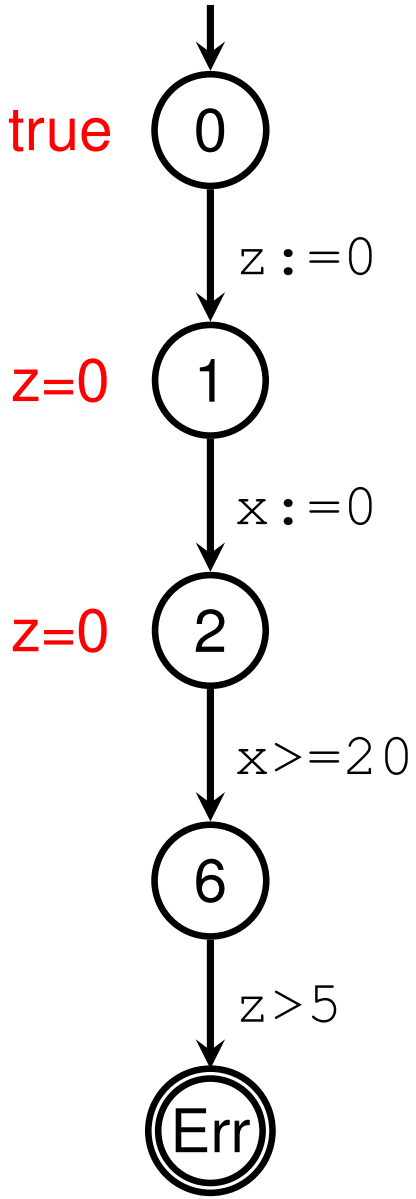

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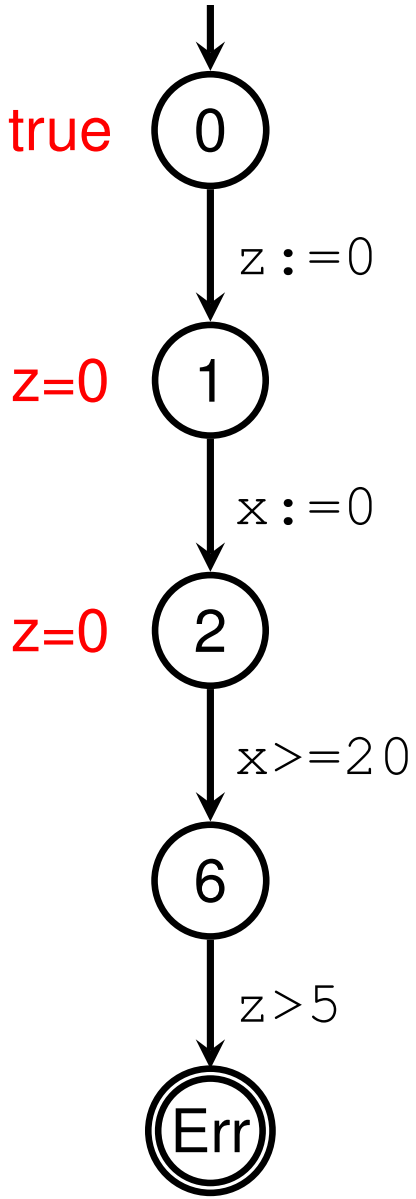

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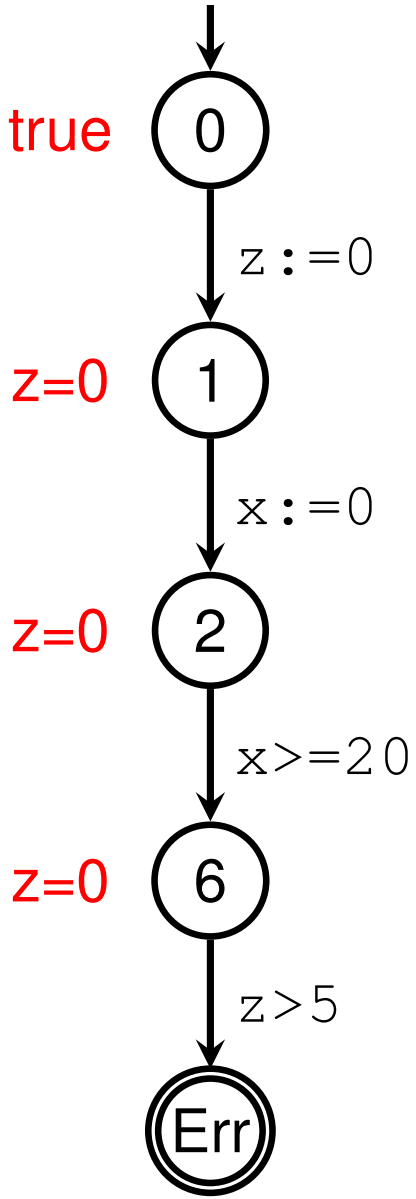

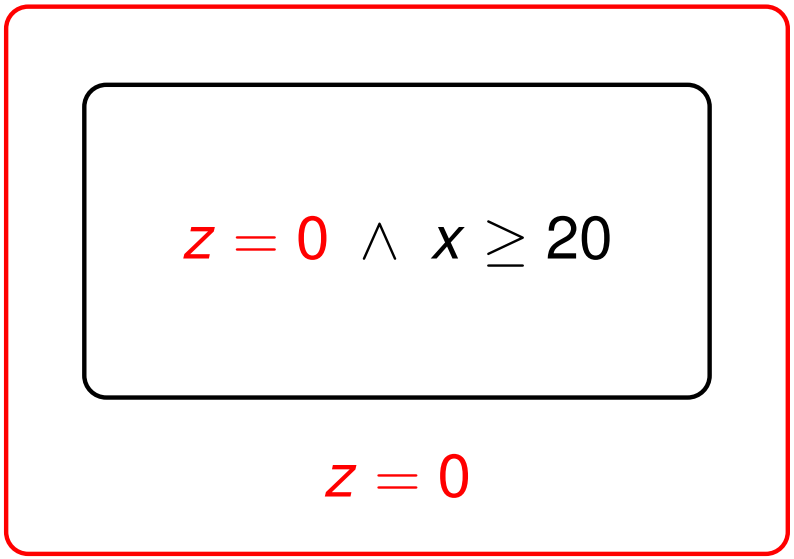

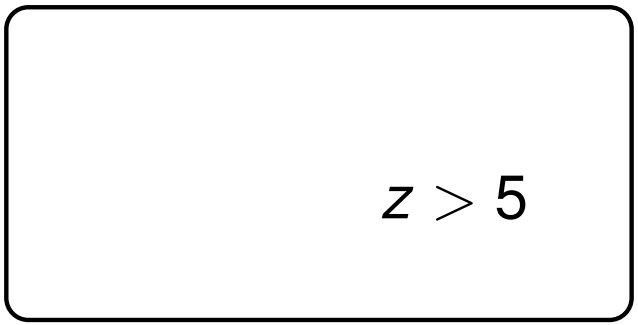

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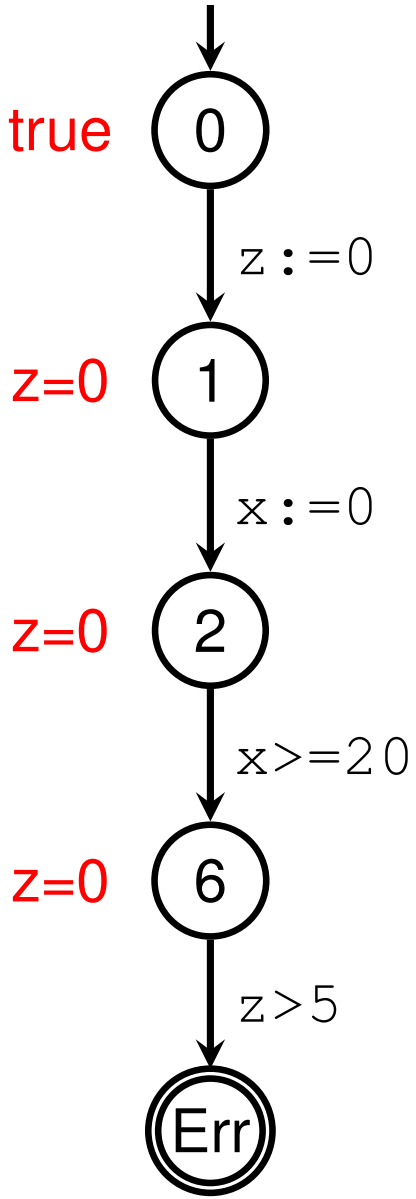

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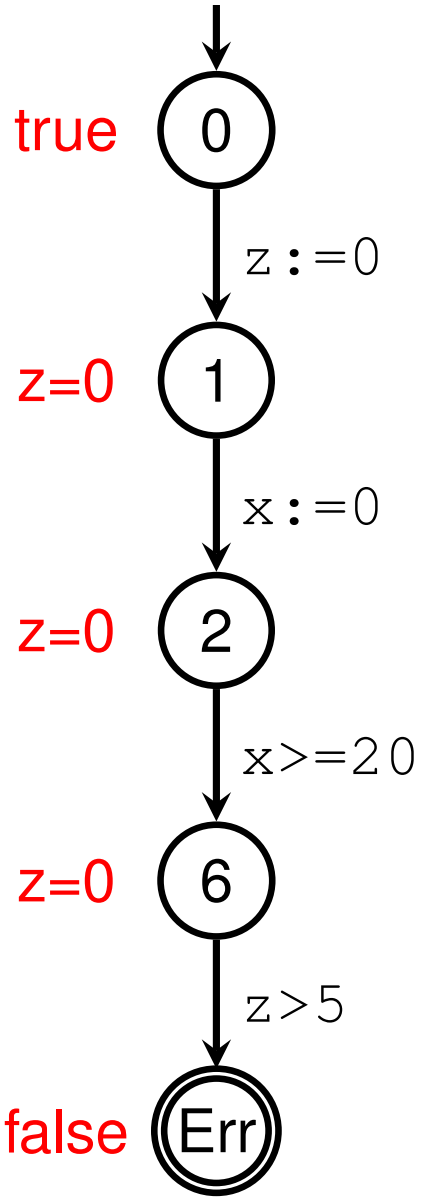

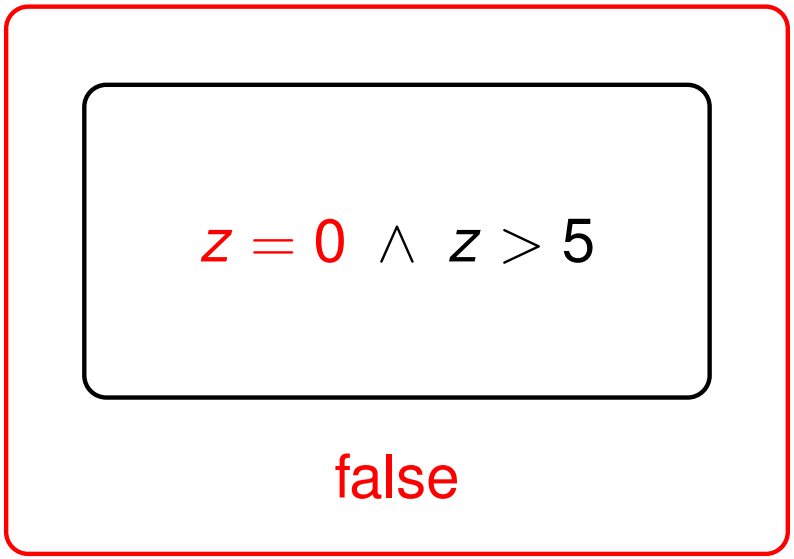

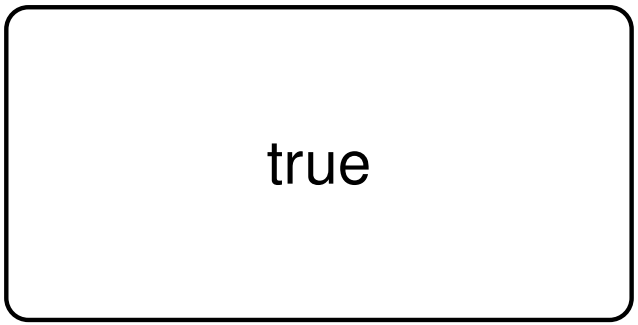

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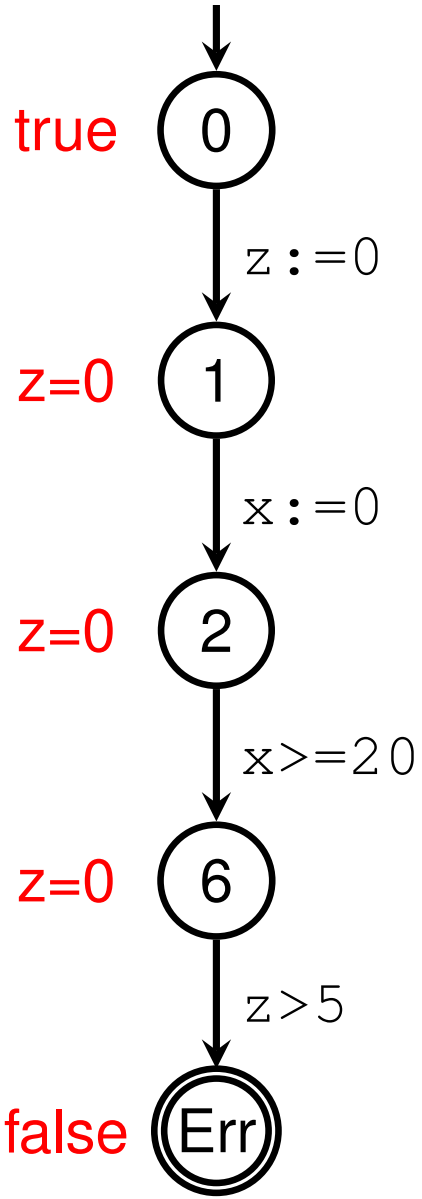

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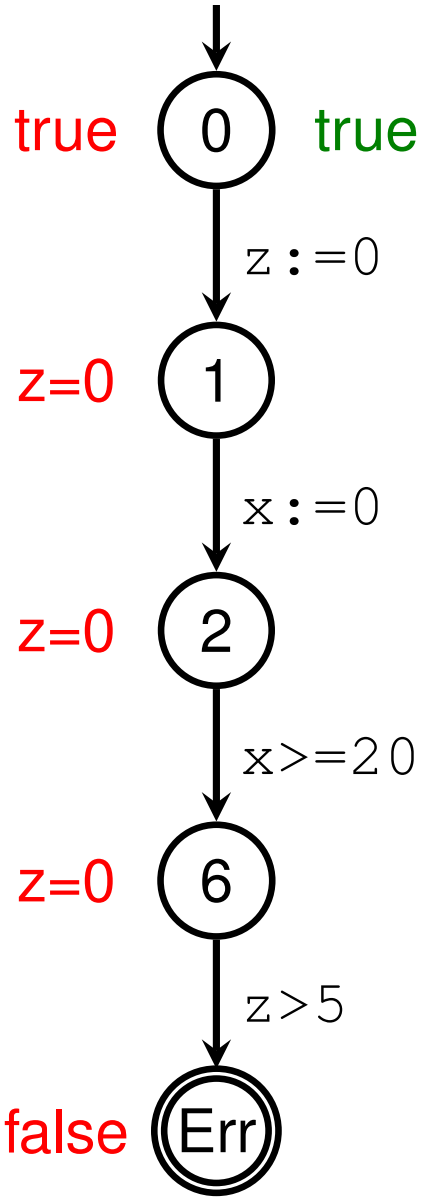

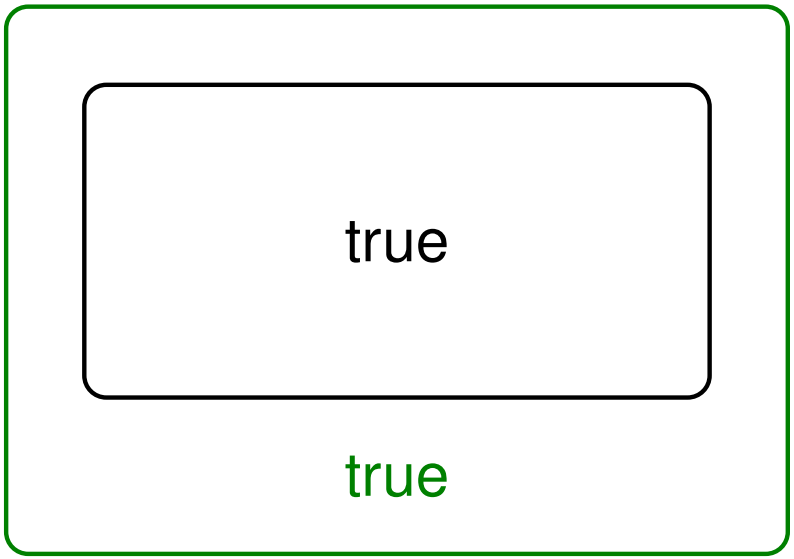

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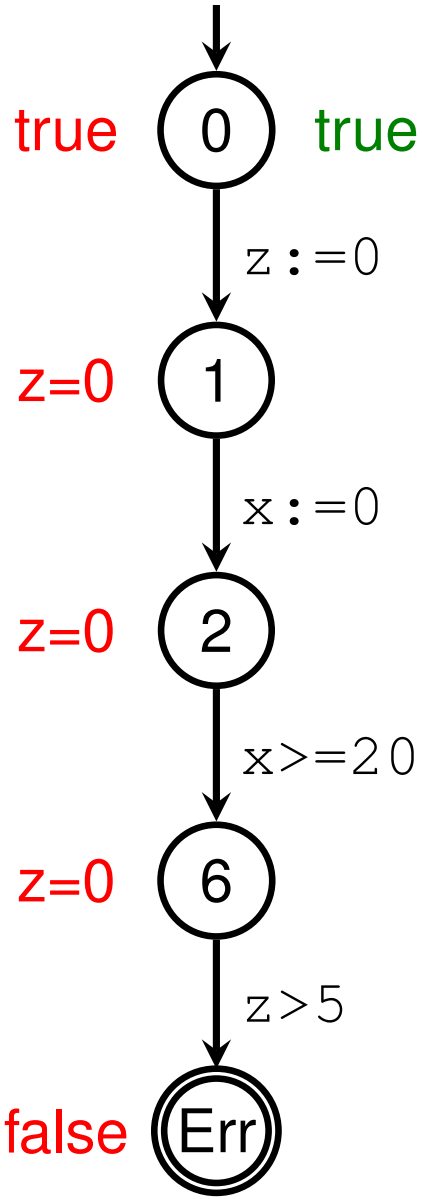

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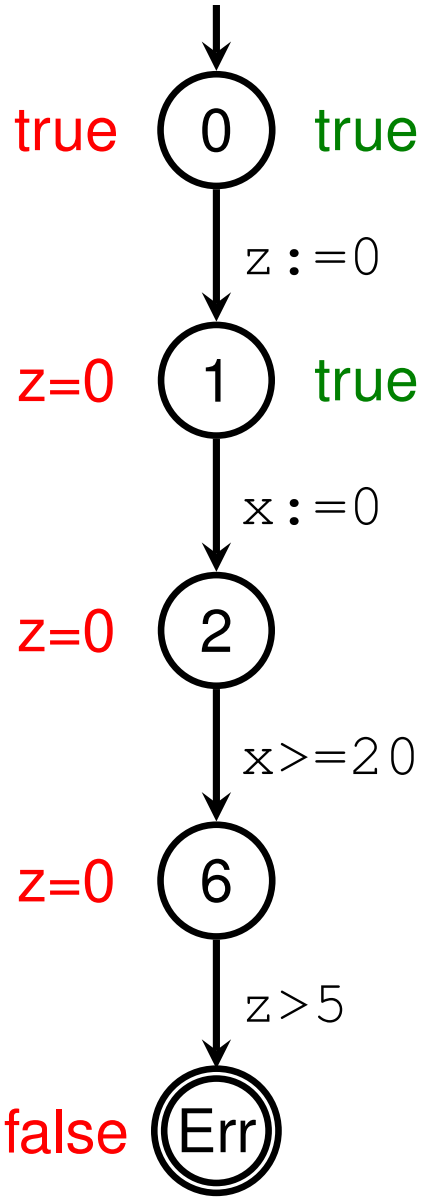

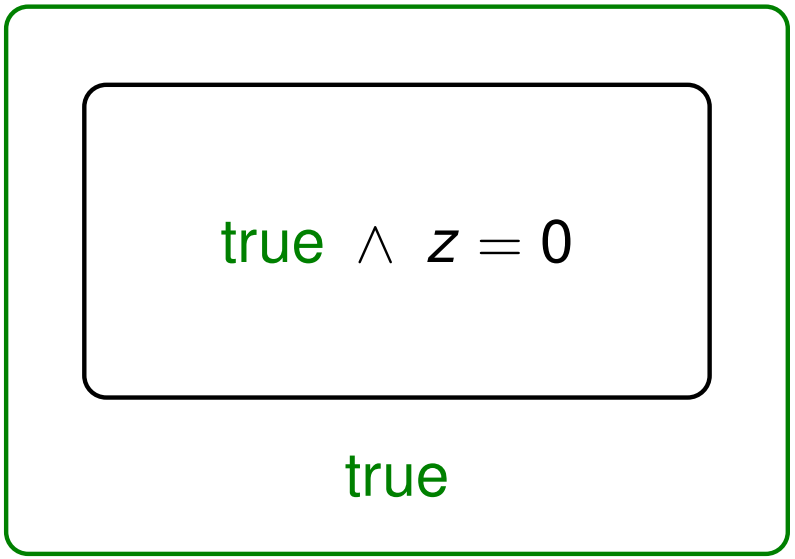

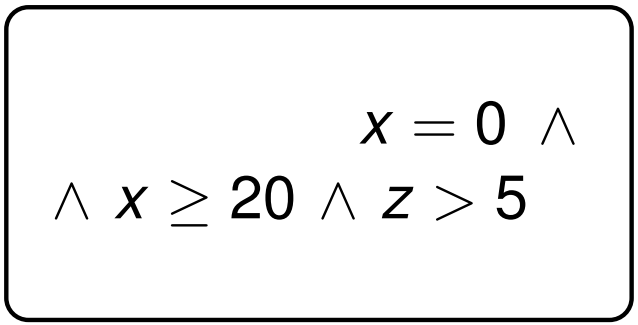

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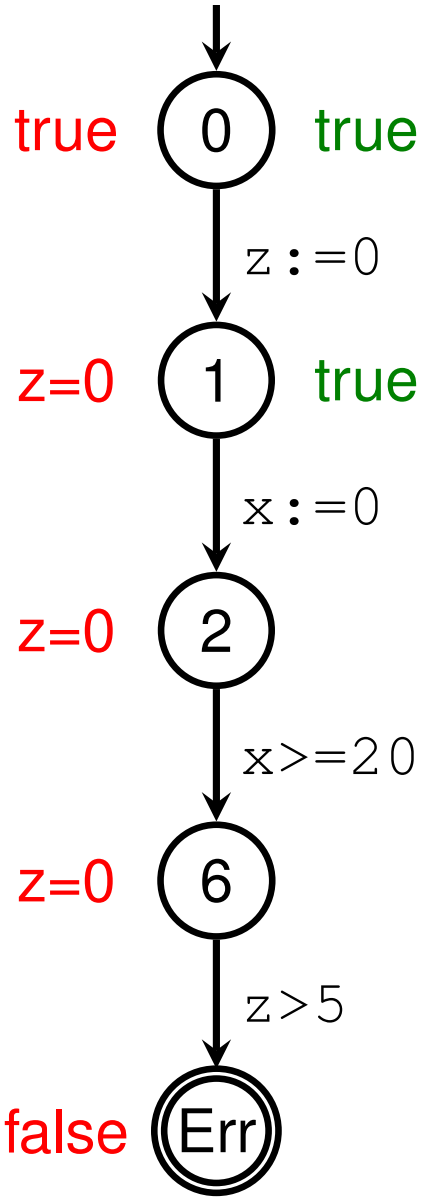

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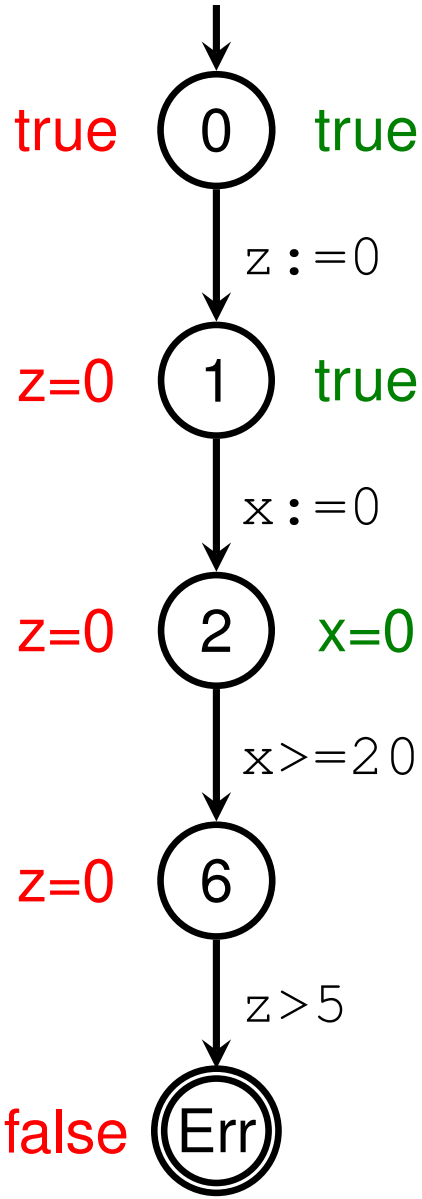

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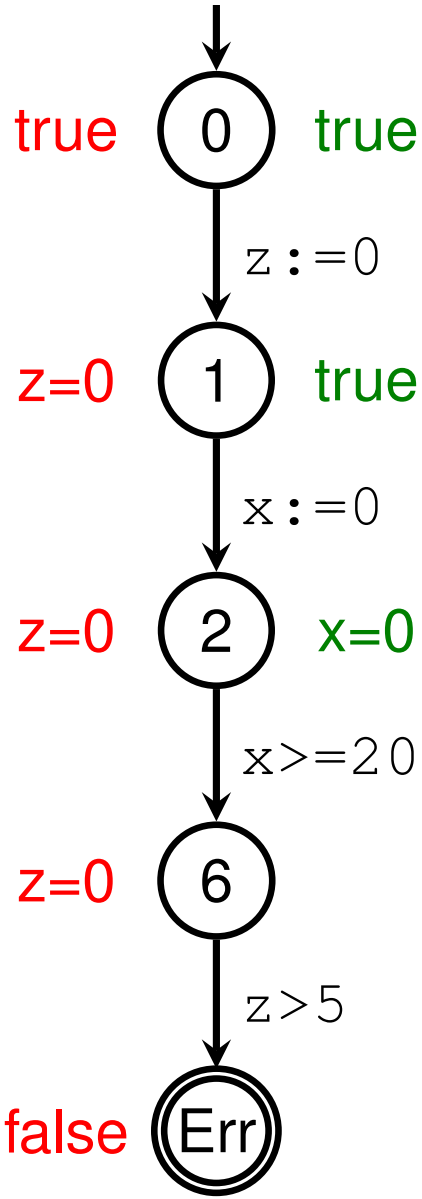

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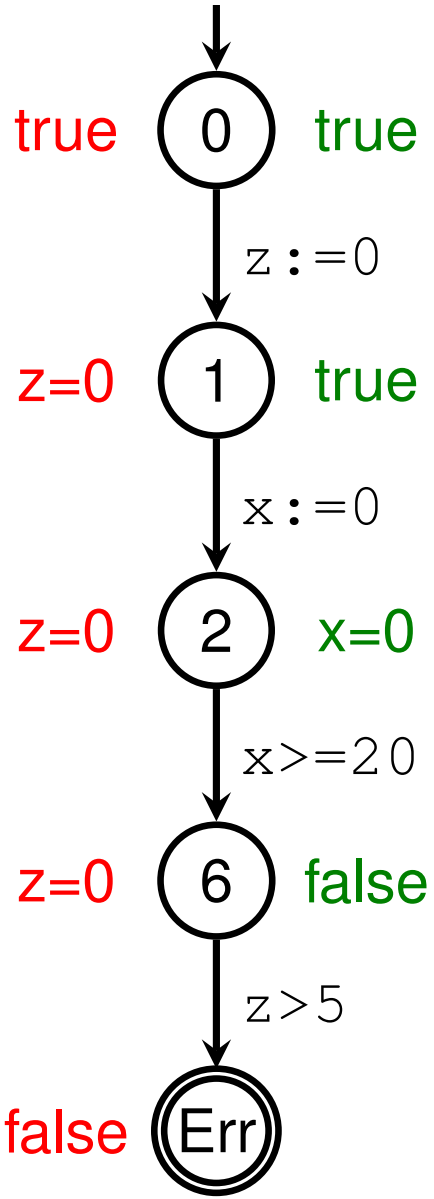

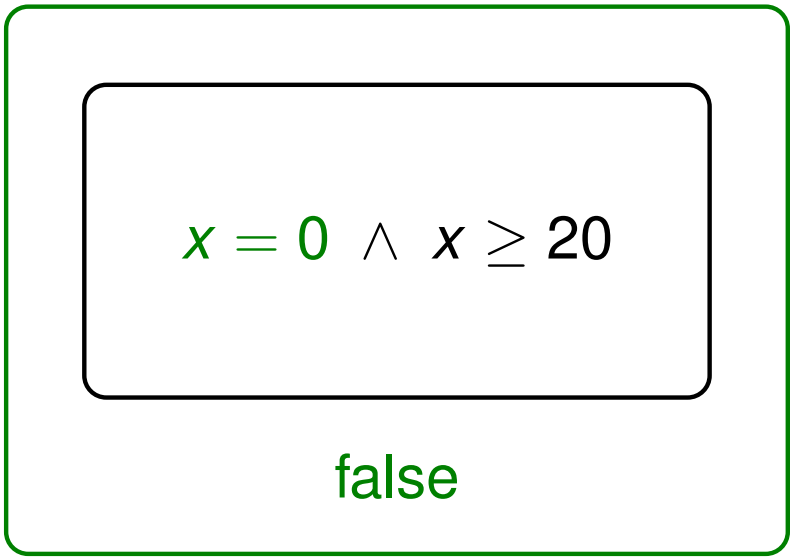

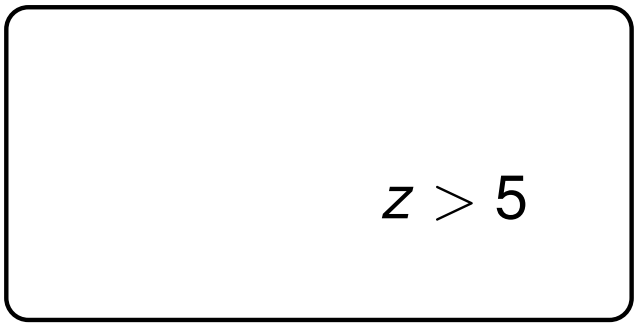

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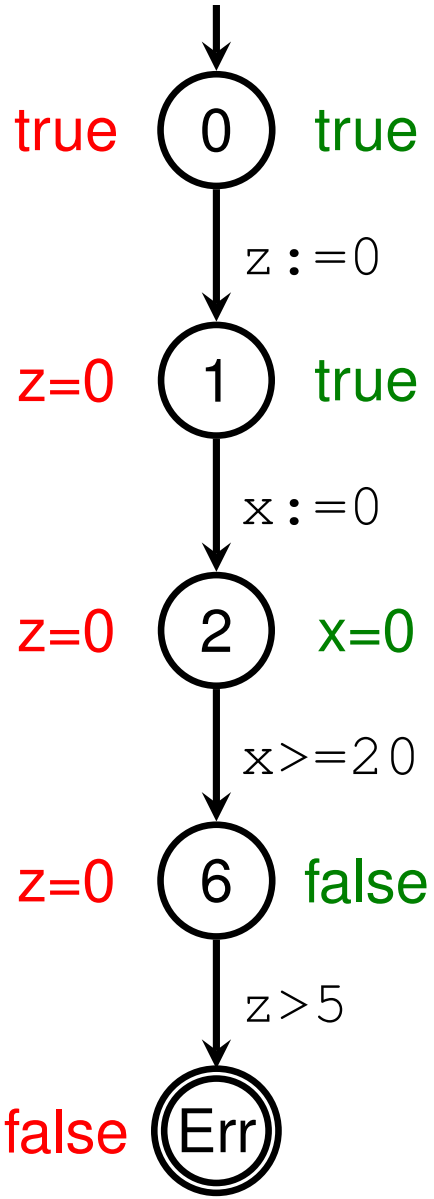

false $\wedge$ $z > 5$

true

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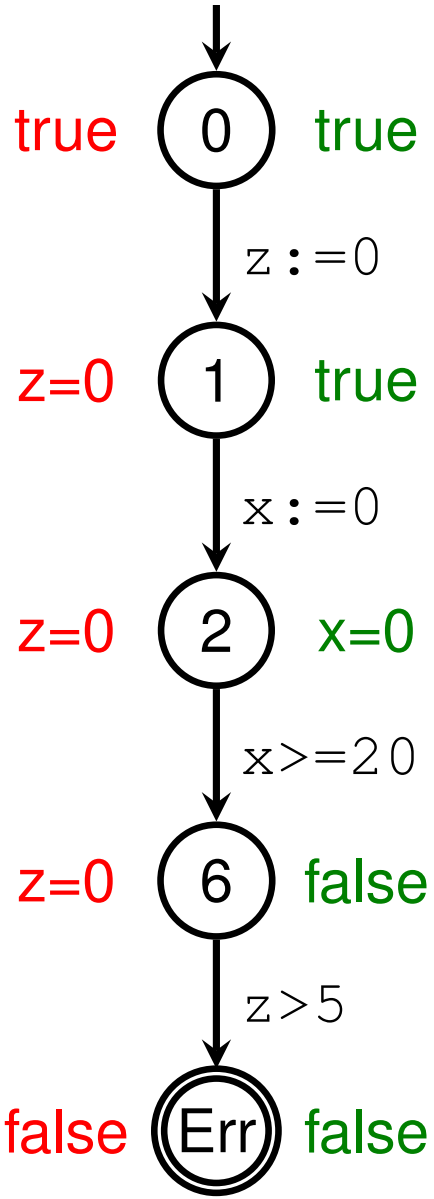

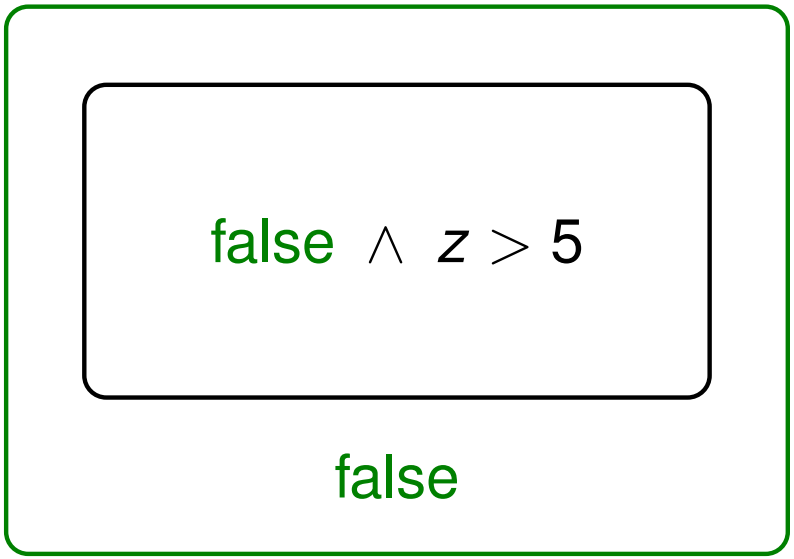

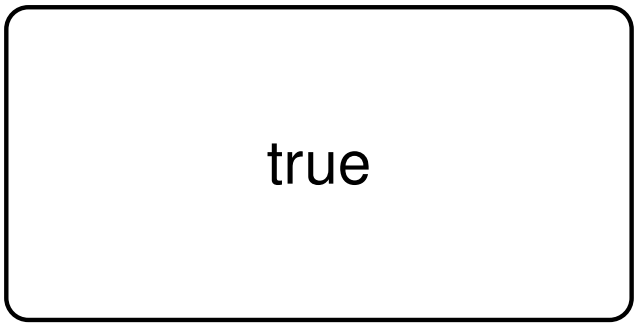

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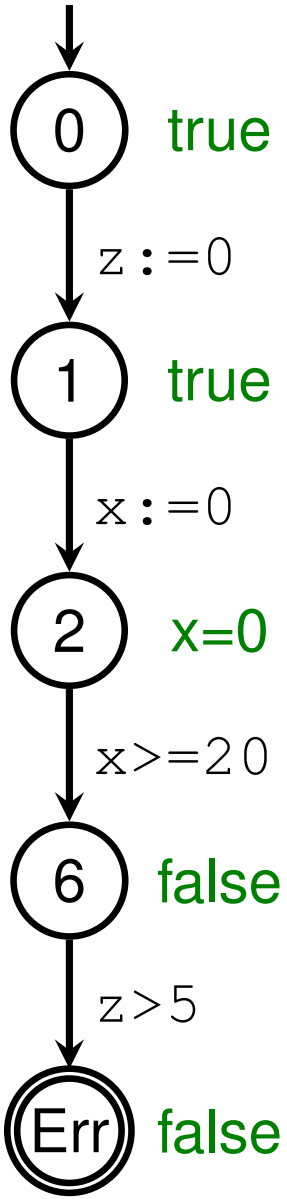

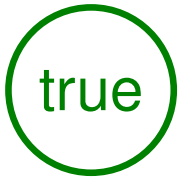

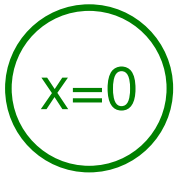

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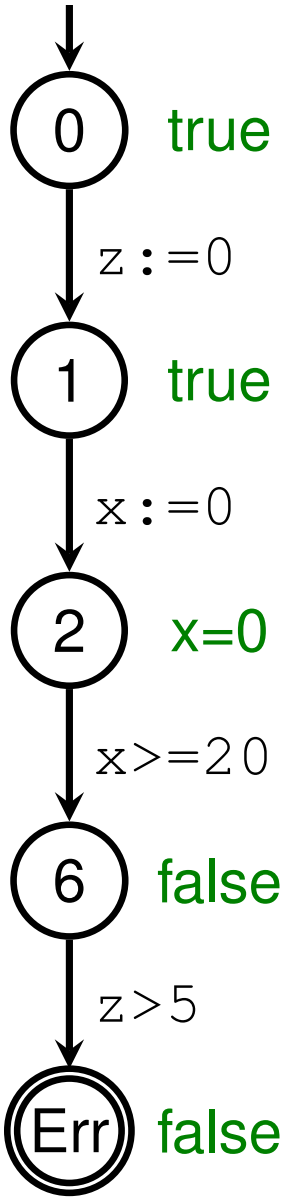

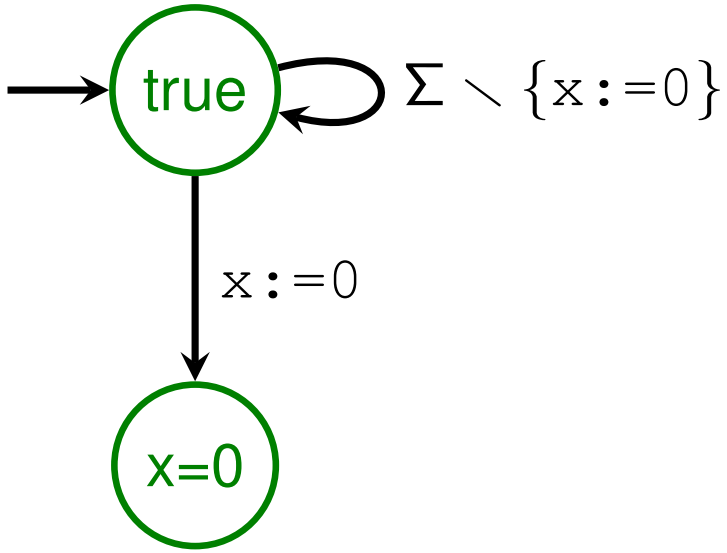

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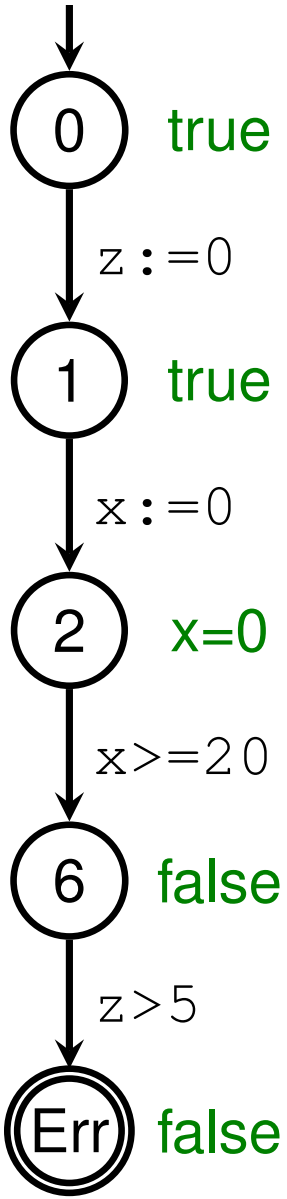
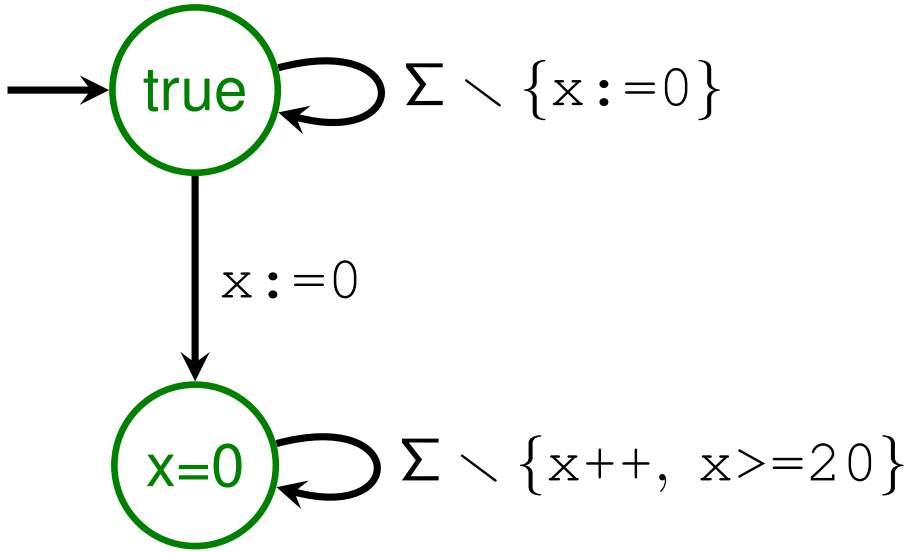
Example: generalization of infeasibility arguments





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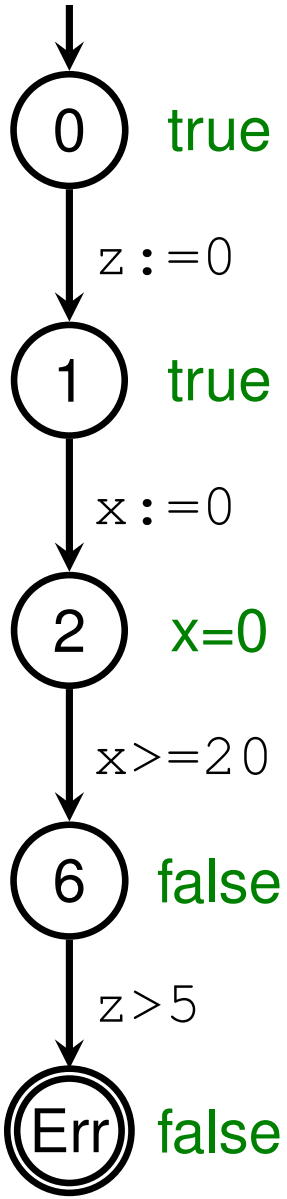

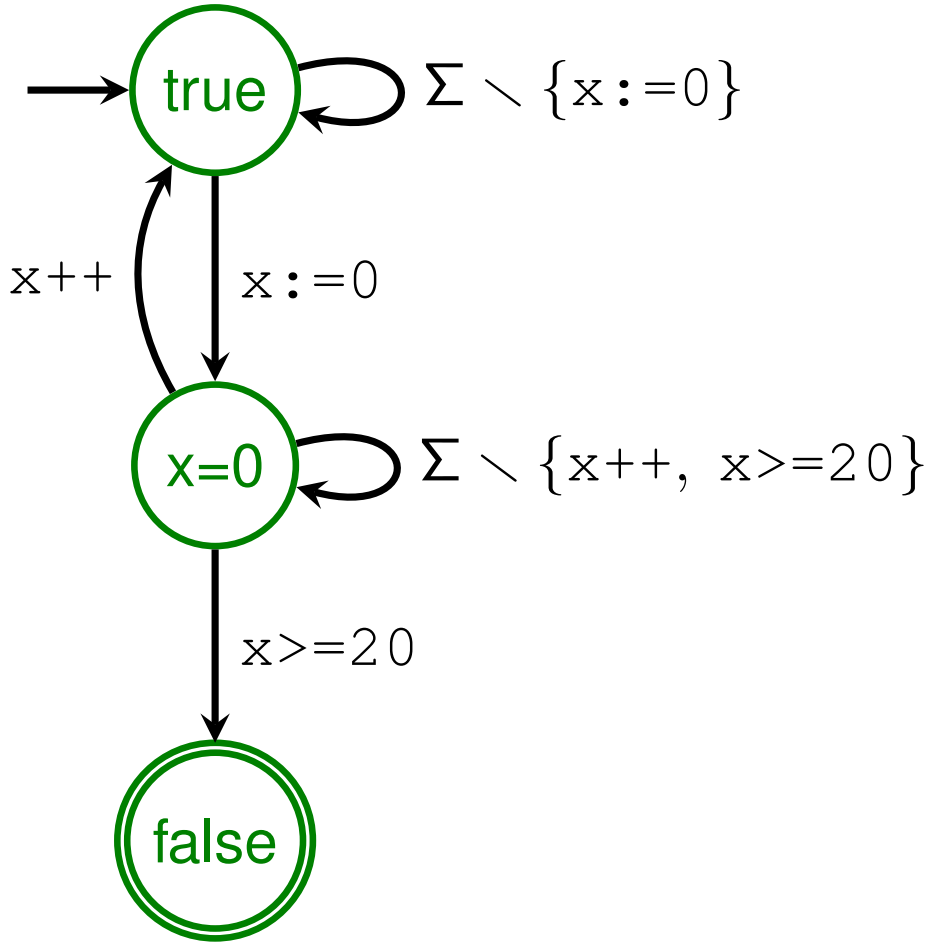

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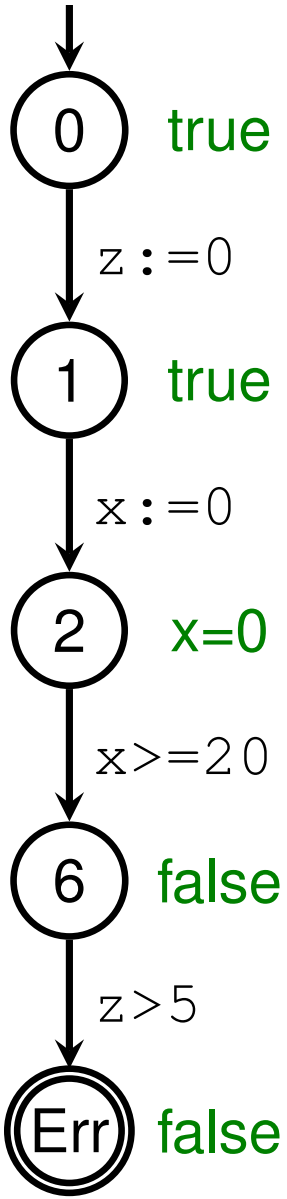

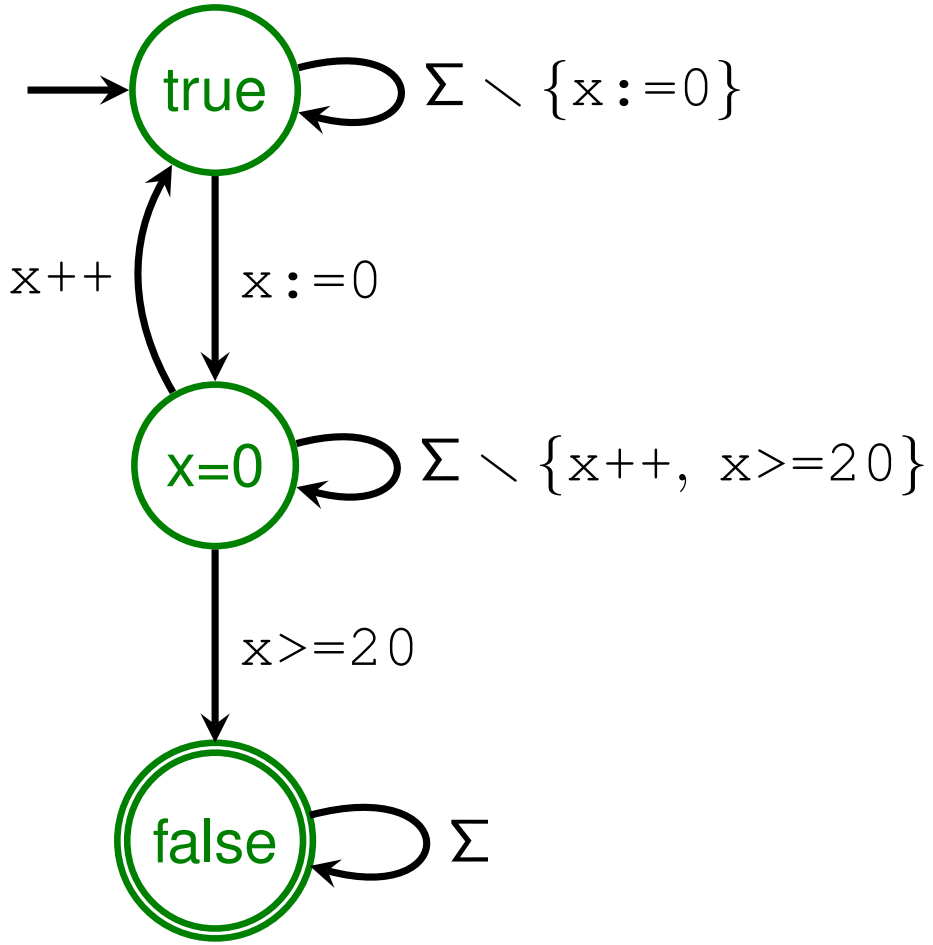

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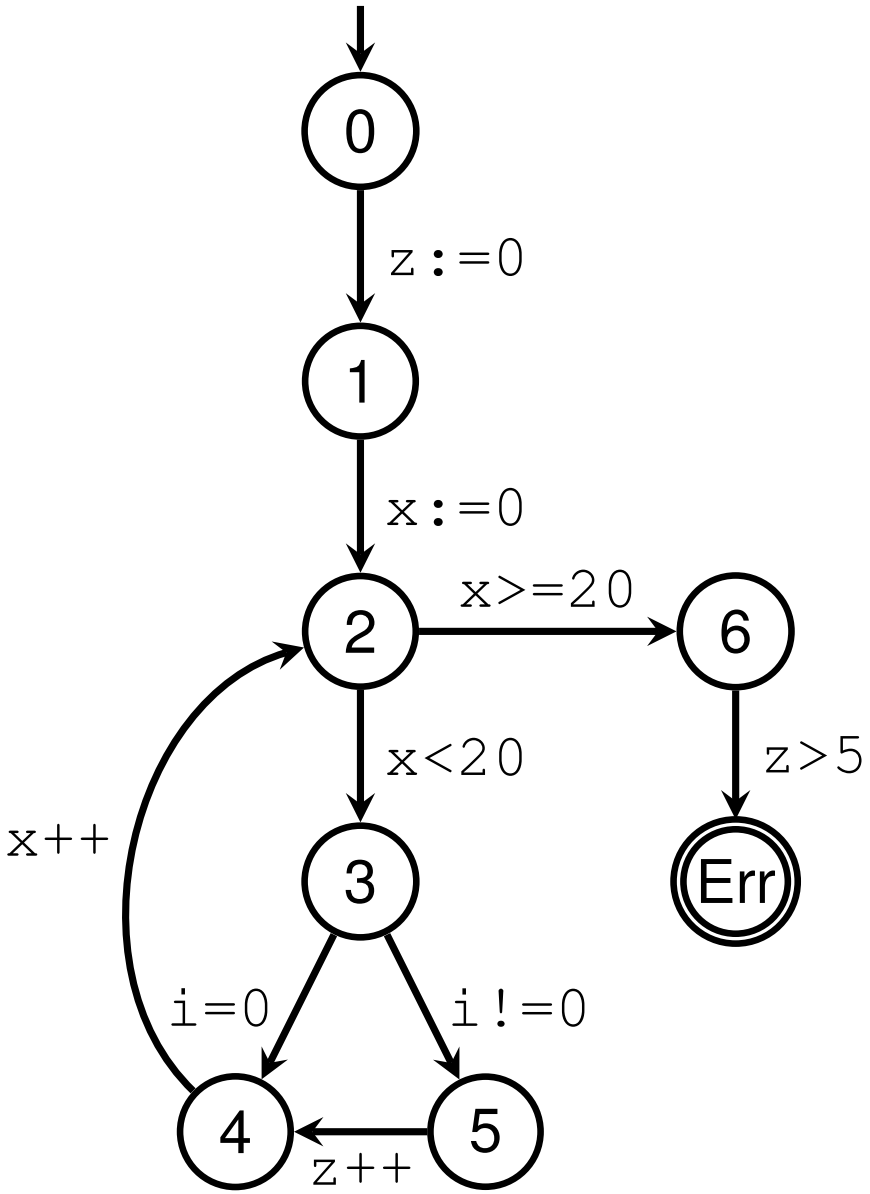
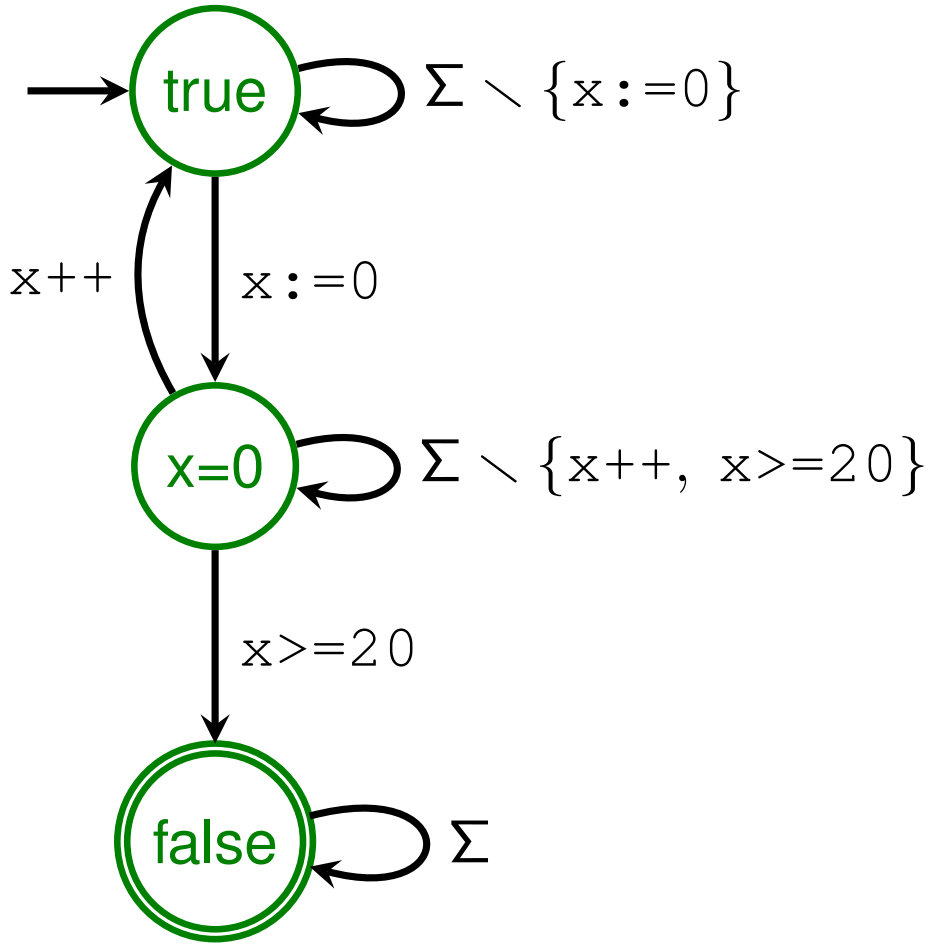
Example: $L(P) \subseteq L(R)$?





$L(P) \subseteq L(R)$ iff the language of the synchronous product of P and complemented R is empty.

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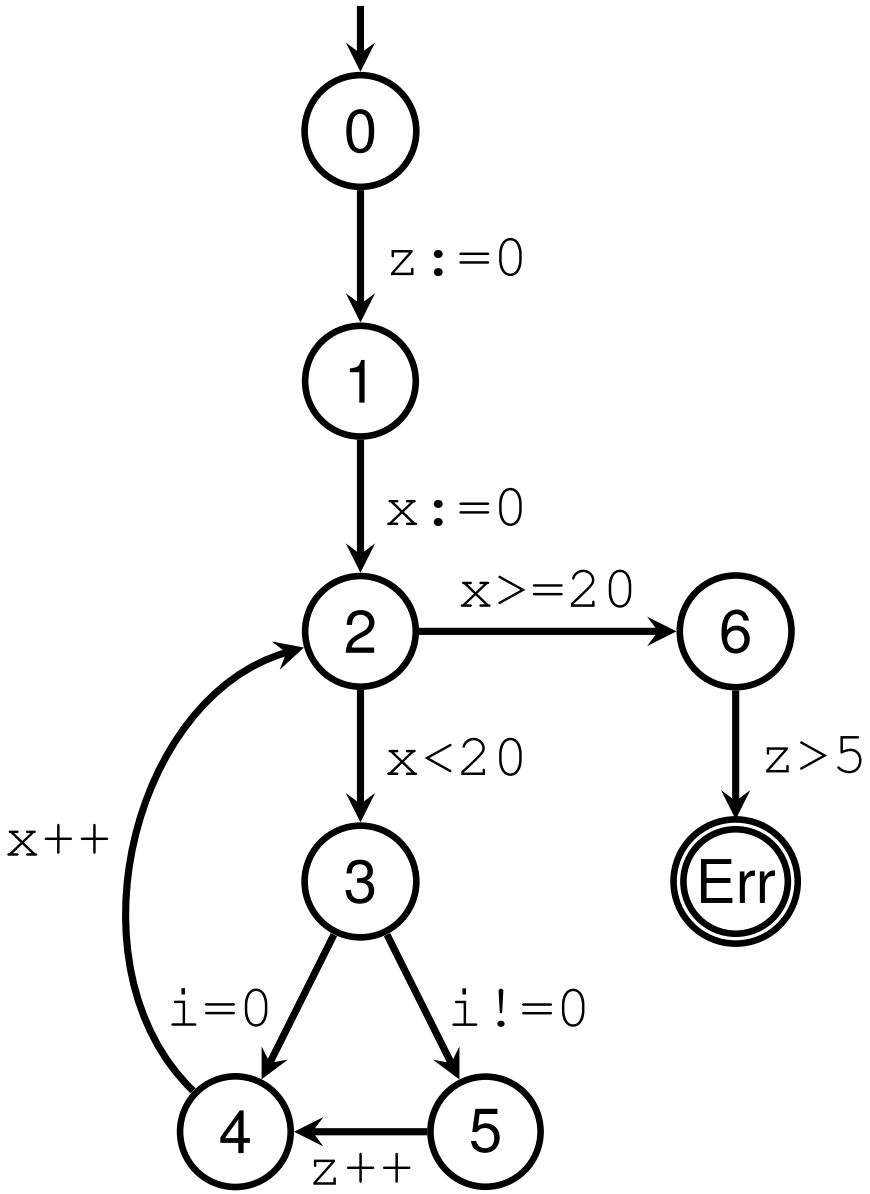

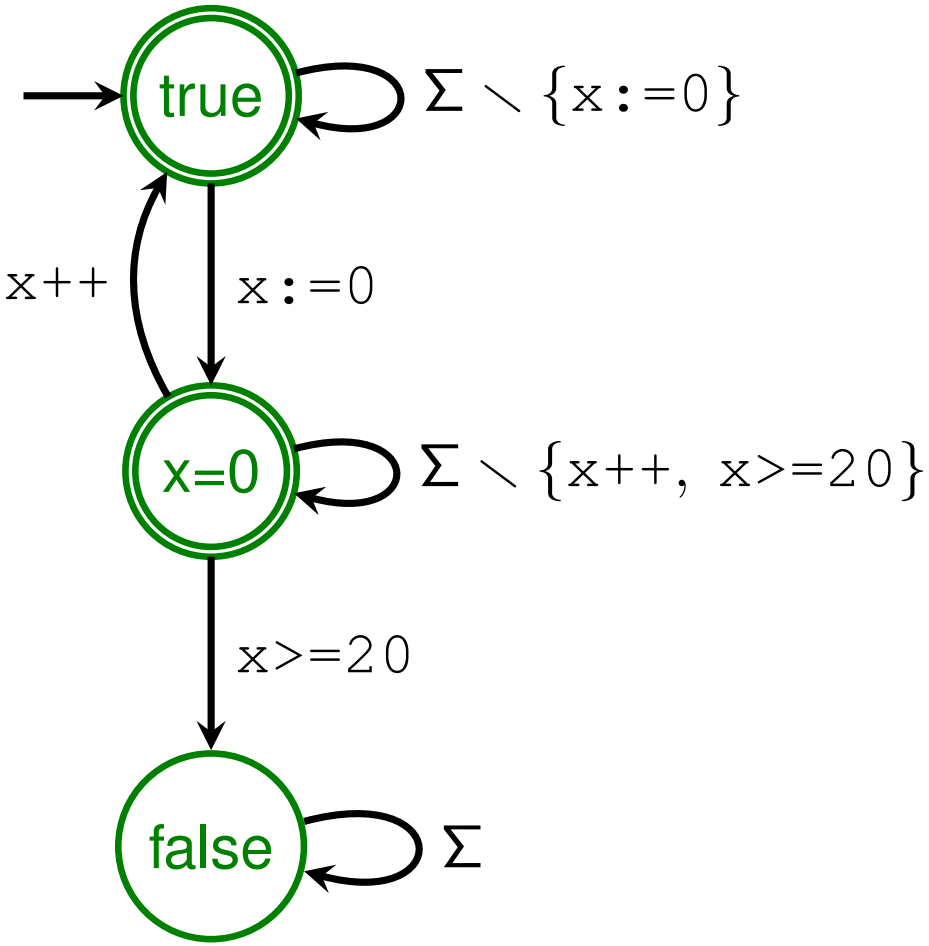

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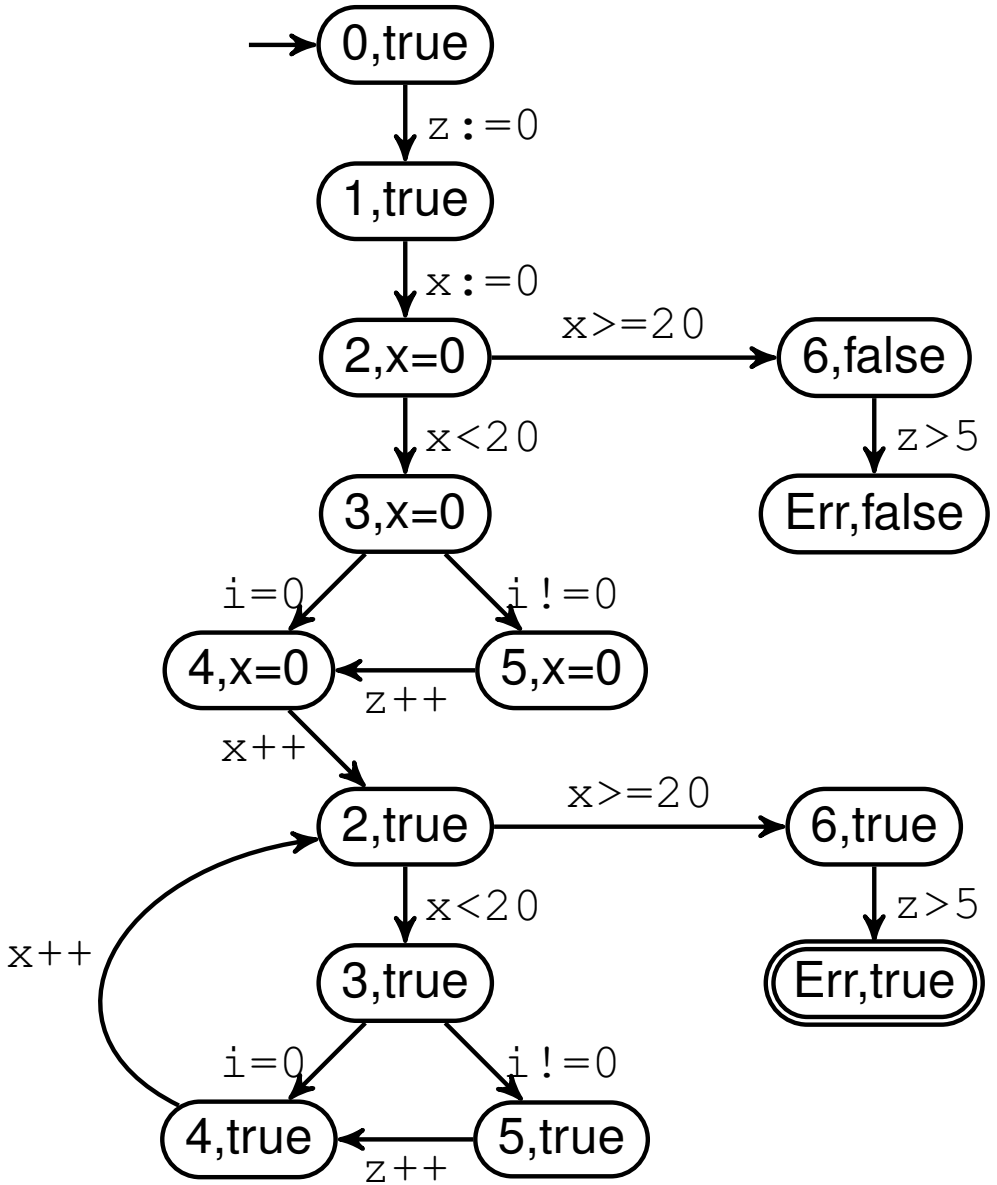
Example: the product



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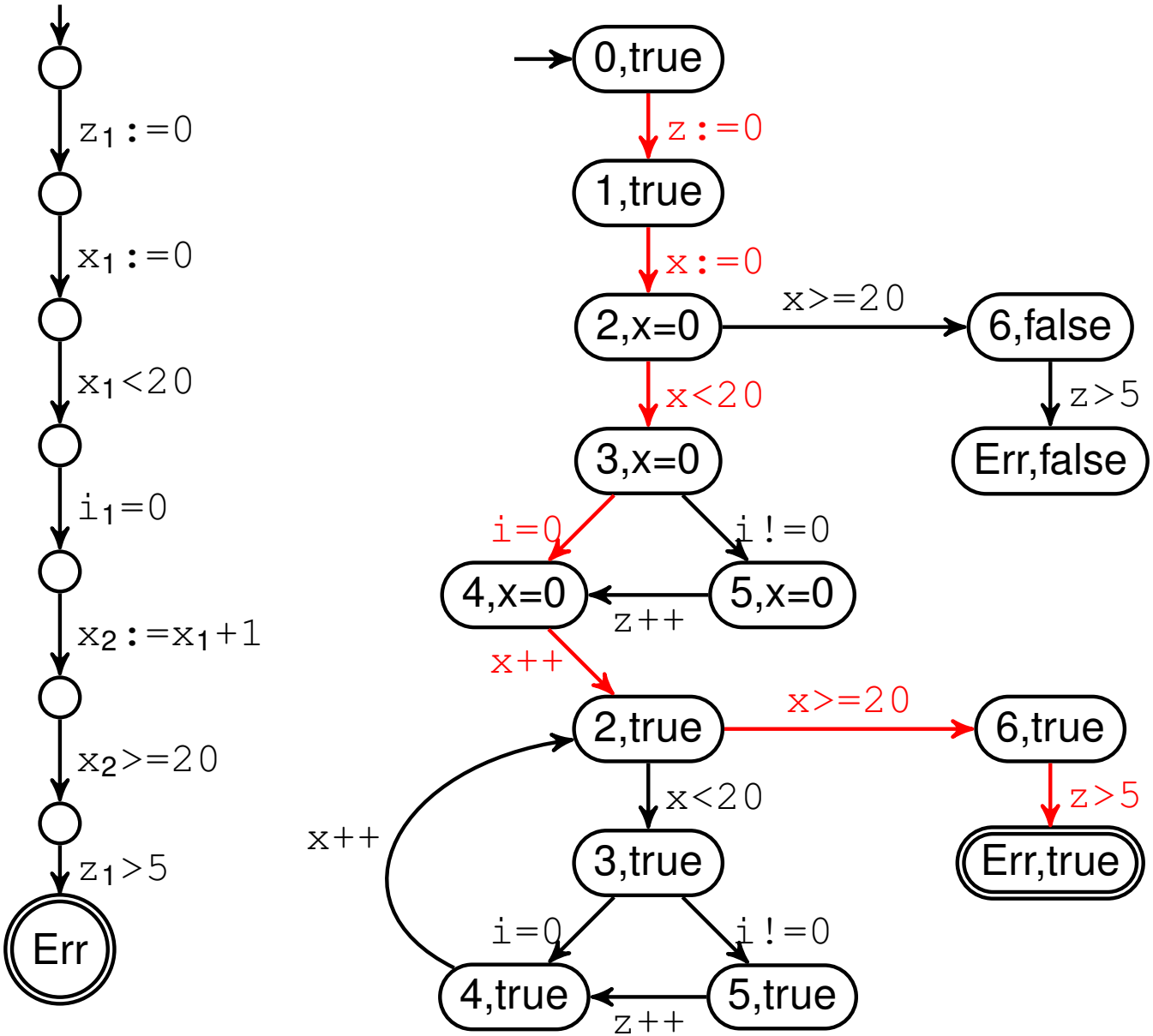

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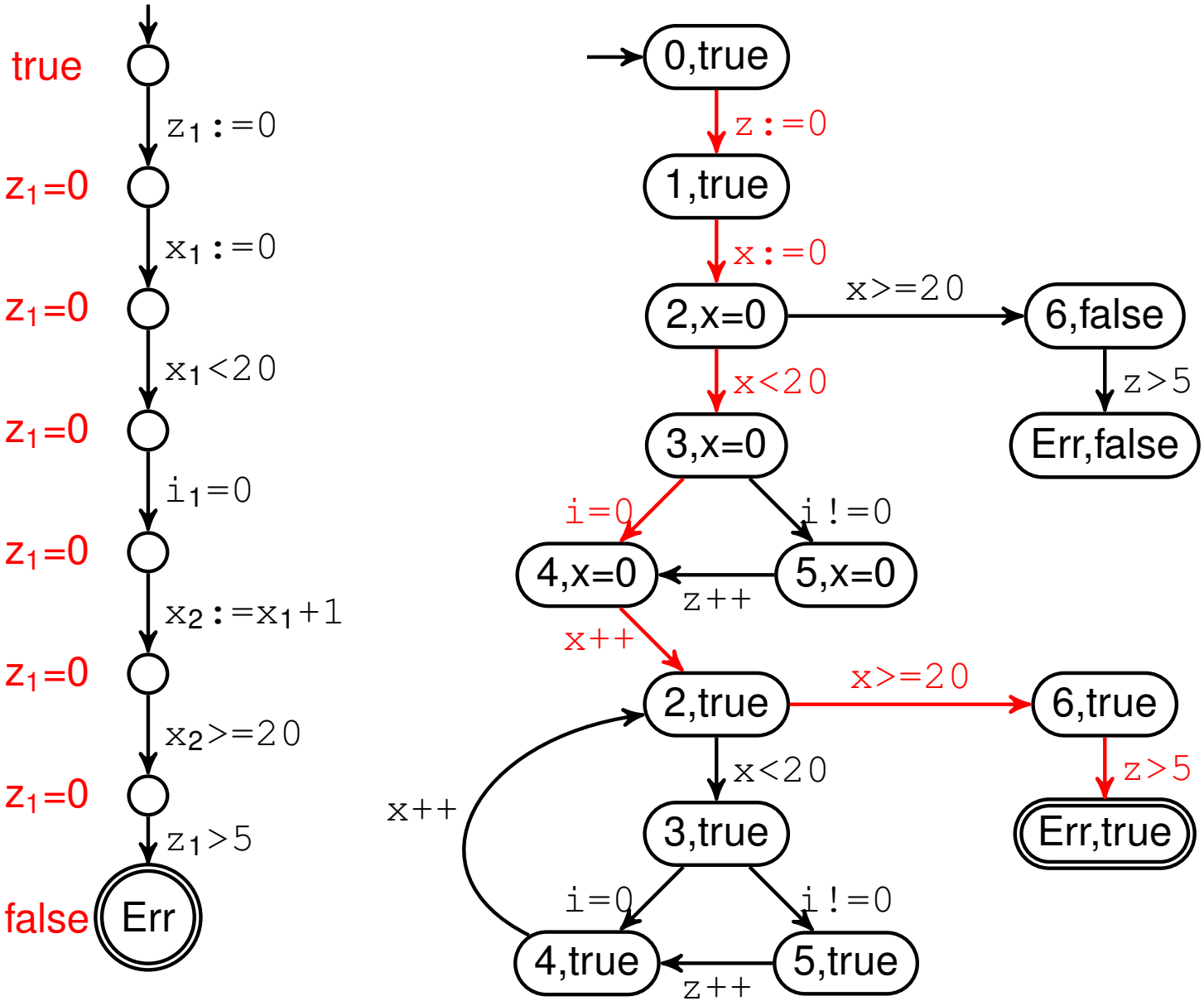

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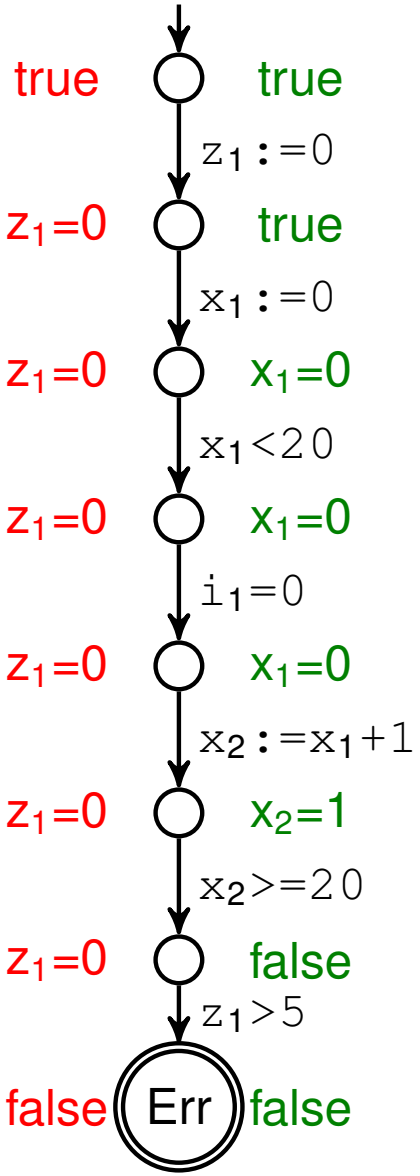

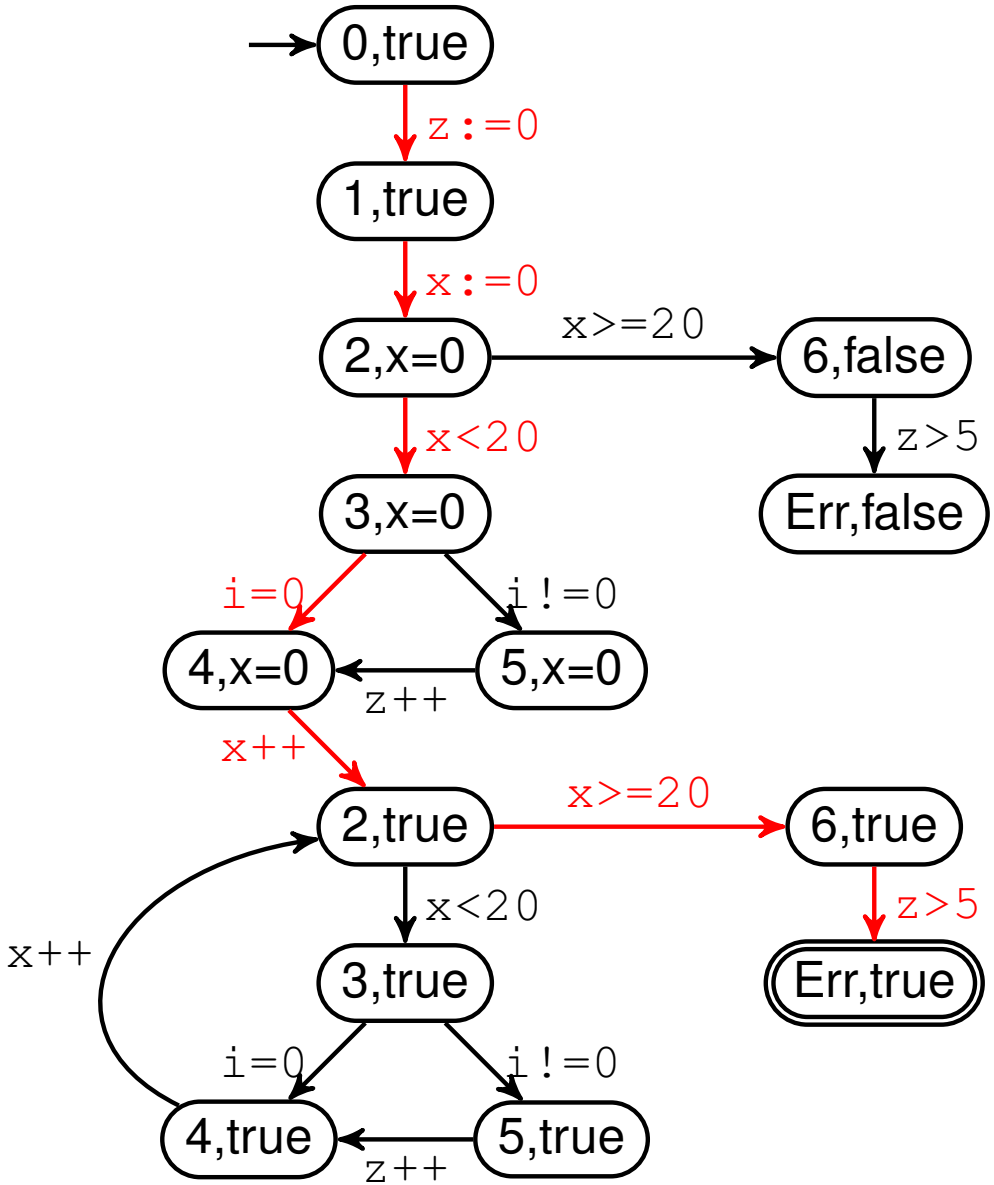

Notes

- interpolation algorithms exist only for some logics/theories
- to handle function calls and unbounded recursion, nested word automata and nested interpolants are needed
- implemented in Ultimate Automizer with numerous optimizations, e.g. on-the-fly complementation and emptiness check of the product
- extensions
  - for termination analysis: Ultimate Büchi Automizer
  - for LTL properties: Ultimate LTL Automizer

Try it out online:

`https://monteverdi.informatik.uni-freiburg.de/tomcat/Website/`

The End

Thank you for your attention!

- Oral exam (subscribe via IS!)
- 30 min preparation + 30 min exam
- Questions = topics
  - model checking PDA
  - translation of LTL to Büchi automata
  - ...