Exercise 1 Let f be a binary function symbol, g, h unary, and c a constant symbol. Find the most general unifier for the following pairs of terms.

(i) $f(g(x), y)$ and $f(x, h(y))$

(ii) $f(h(x), x)$ and $f(x, h(y))$

(iii) $f(x, f(x, g(y)))$ and $f(y, f(h(c), x))$

(iv) $f(f(x, c), g(f(y, x)))$ and $f(x, g(x))$

Exercise 2 Suppose we are given a predicate flight(*From*, *To*, *Time*, *Price*) containing information about direct flights including the starting airport, the destination, the flight time, and the price of a ticket. Write a Prolog program computing a predicate travel(*From*, *To*, *Stops*, *Time*, *Price*) indicating all possibilities to travel from one city to another using one or several flights.

Exercise 3 Write a Prolog predicate fib(N, X) computing the Fibonacci sequence. Evaluate fib$(3, X)$ and fib$(N, 5)$.

Exercise 4 Write Prolog definitions of the following predicates.

length(*List*, N)	N is the length of *List* .
append(X, Y, Z)	Z is the concatenation of the lists X and Y .
reverse(X, Y)	Y is the reverse of the list X .
map(X, Y)	maps a list $X = [X_1, \dots, X_n]$ to $Y = [f(X_1), \dots, f(X_n)]$.
fold_left(X, Y, Z)	maps $Y = [Y_1, \dots, Y_n]$ to $Z = f(\cdots f(f(X, Y_1), Y_2)\cdots, Y_n)$.
fold_right(X, Y, Z)	maps $Y = [Y_1, \dots, Y_n]$ to $Z = f(Y_1, f(Y_2, \dots, f(Y_n, X)\dots)))$.

The Prolog notation for lists is as follows:

$$[] \quad [X, Y, Z] \quad [X|Y] \quad [X, Y|Z].$$

Exercise 5 Write a naive sort function

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naive_sort(X,Y) :- permute(X,Y), sorted(Y).
```

by implementing the relations

sorted(X)	checks that the list X is sorted.
insert(X, Y, Z)	if the list Z is obtained from Y by inserting X at an arbitrary position.
permute(X, Y)	if the list Y is a permutation of X .

Implement merge sort using the relations

merge(X, Y, Z)	merges two sorted lists X and Y into Z .
split(X, Y, Z)	splits the list X into two lists Y and Z .

Exercise 6 We consider directed graphs of the form $\langle V, E\rangle$. Express the following relation in relational algebra.

(a) x and y are not connected by an edge.

(b) The edge $\langle x, y\rangle$ is part of a triangle.

(c) x has at least two neighbours.

(d) Every neighbour of x is also a neighbour of y.

Exercise 7 Evaluate the following Datalog program on the tree $\langle V, E, P\rangle$ to the right.

$$
\begin{aligned}
U &\leftarrow S(x,y) \wedge W(x) \wedge W(y) \\
W(x) &\leftarrow P(x) \\
W(x) &\leftarrow E(x,y) \wedge W(y) \\
S(x,y) &\leftarrow E(z,x) \wedge E(z,y) \wedge x \neq y \\
R(x,y) &\leftarrow P(x) \wedge x = y \\
R(x,y) &\leftarrow E(x,z) \wedge R(z,y) \\
R(x,y) &\leftarrow R(x,z) \wedge E(z,y)
\end{aligned}
$$