**Exercise 1** Prove that the set of even numbers is not first-order definable in  $(\mathbb{Z}, \leq)$ .

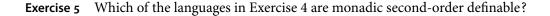
**Exercise 2** We consider structures with the empty signature. Use a back-and-forth argument to show that

 $\mathfrak{A} \equiv_m \mathfrak{B}$  iff |A| = |B| or  $|A|, |B| \ge m$ .

**Exercise 3** Prove that the set of all complete finite binary trees whose number of vertices is divisible by 3 is not first-order definable.

**Exercise 4** Which of the following languages (over  $\{a, b\}$ ) are first-order definable?

- (a)  $a^*(bb)^*$
- (b) (*ab*)\*
- (c)  $\{a^{n^2} \mid n \in \mathbb{N}\}$
- (d)  $\{a^n b^m a^n \mid m, n \in \mathbb{N}\}$
- (e) The set of all palindromes.



**Exercise 6** Which of the following graph classes are first-order definable? (We assume that all graphs are finite and undirected.)

- (a) graphs of degree at most 3
- (b) trees
- (c) paths
- (d) graphs of diameter at most 3
- (e) graphs of even diameter
- (f) graphs with a Hamiltonian cycle