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# Dynamics in Networks

IV124

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# Dynamics in Networks

- dynamic processes on a static network

- time-evolving network
  - static network in sliding window
  - temporal network – measures
  - temporal network – events in bursts
  - null models
  - network states
  - change points

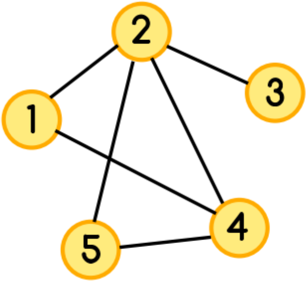
# Time-Evolving Networks

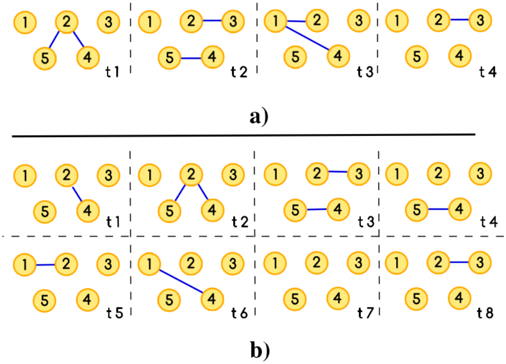
Motivation

- real networks are based on links that are subject to change in time
- static network does not represent information about the sequence of steps and distance in time

communication networks, *face-to-face* interaction, neuronal networks, ecological networks, interaction between species …

# Time-evolving Networks – Motivation[1]




$T = 240$ min, a) $\Delta t = 60$ min, b) $\Delta t = 30$ min

[1] Nicosia V., 2013

 [2]

- network structure = riverbed
- dynamic network = change of riverbed (friendship network)
- temporal network = river flow (network of meetings and communication)

[2] Saramäki, 2014

 [3]

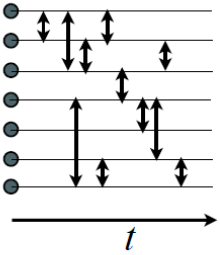
- network structure = aggregation in time
- dynamic network = existing links are active all the time
- temporal network = existing links are switched on and off

[3] Saramäki, 2014

# Temporal, Time-Varying Networks

$\mathscr{G}_{[0,T]} \equiv \mathscr{G} = \{G_1, G_2, \ldots, G_M\}$,
$G_m$ – *network snapshot*

Commonly equidistant snapshots
$t_{m+1} = t_m + \Delta t$, $m = 1, \ldots, M$.




$\mathscr{G}$ fully described by

- adjacency matrix $A(t_m)$
- list of contacts (contact $c = (i, j, t, \delta t)$ between nodes $i, j$, initial contact $0 \leq t \leq T$ and its duration $\delta t$)

# Temporal Scale

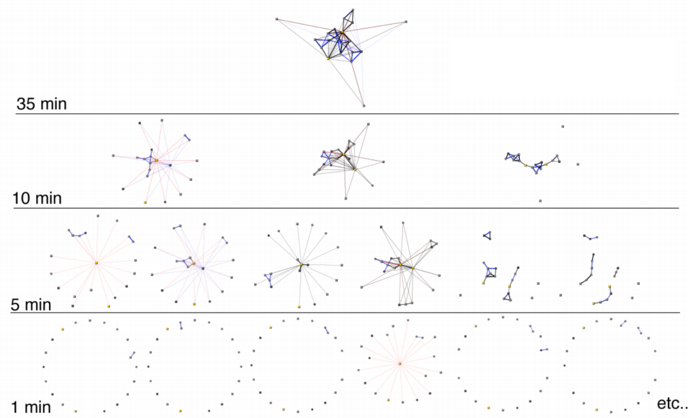
Temporal window of size $\Delta t$

- $\Delta t = T$ ... static network
- $\Delta t \to 0$ ... infinite sequence of instantaneous networks
- Recommended: maximum possible temporal resolution

? Multiscale systems

$\to$ Utilization of knowledge from signal processing, information theory, time series analysis, granularity of the model, time series segmentation, ...

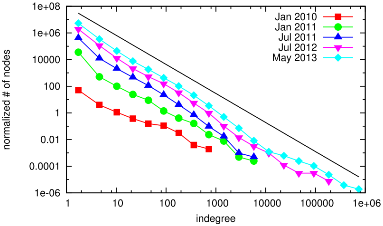
# Temporal Scale – Interactions in a Class[4]

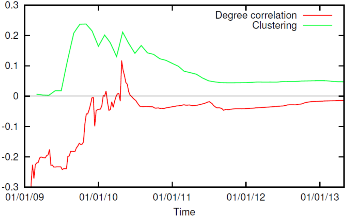



[4] Bender-deMoll S., 2006, Sulo Caceres R., 2013

# Representing the temporal component

- analyzing static networks in sliding window




[5]

- temporal network analysis

[5] Kondor D., 2014

# Topology Evaluation – Connectivity

- temporally strongly connected component of node $i$: In a directed graph, node $i$ is temporally reachable from other nodes of the component in the time interval $[0, T]$, and all nodes of the component are temporally reachable from $i$.
- temporally weakly connected component of node $i$: Node $i$ is temporally reachable from other component nodes and vice versa in the corresponding undirected temporal network.

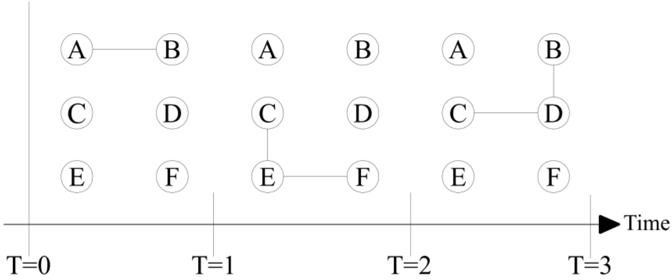
# Metrics – temporal paths I.

$\mathcal{P}_{ij} = \{e_{ik}(t_1), e_{kl}(t_2), \ldots, e_{xj}(t_L) \mid t_1 \leq t_2 \leq \cdots \leq t_L\}$

- topological/temporal path length = number of contacts/time between $i$ and $j$
- temporal distance (latency) $d_{ij}$ = temporal length of the shortest temporal path
- temporal diameter of the network $D = max_{ij} d_{ij}$

- no reciprocity: a path $i \rightarrow j$ doesn't guarantee an existence of path $j \rightarrow i$
- no transitivity: a path $i \rightarrow j$ and a path $j \rightarrow k$ don't guarantee an existence of path $i \rightarrow j \rightarrow k$
- temporal dependence: a path $i \rightarrow j$ in a time $t$ doesn't guarantee the same path in the time $t' > t$

# Metrics – temporal paths II.




[6]

Nodes are often temporally unreachable from each other, i.e., $d_{ij} = \infty$, hence temporal (global) efficiency $\mathscr{E} = \frac{1}{N(N-1)} \sum_{ij} \frac{1}{d_{ij}}$

[6] Tang J., 2009.

# Metrics – clustering coefficient

= ability of events to persist across frames

- $C_i(t_m, t_{m+1})$ topological overlap of the node's neighborhood

- local clustering coefficient

$$C_i = \frac{1}{M-1}\sum_{m=1}^{M-1} C_i(t_m, t_{m+1})$$

- global clustering coefficient

$$C = \frac{1}{N}\sum_{i} C_i$$

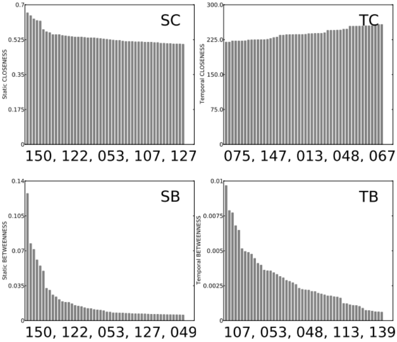
# Metrics – centrality[7]

- betweenness: $C_i^B = \sum_{j \in V} \sum_{k \in V, k \neq j} \frac{\sigma_{jk}(i)}{\sigma_{jk}}$
  Useful to take into account the interval during which information waits at the node before being sent on
- closeness: $C_i^C = \frac{N-1}{\sum_j d_{ij}}$
- *broadcast, receive* centrality:
  - not everything is spread via shortest paths
  - based on static Katz centrality (a version of eigenvector centrality for directed graphs)
  - identification of spreaders and main recipients of information

[7] Nicosia V., 2013, Holme & Saramäki, 2013, Newman, 2010, Grindrod & Parsons, 2011

# Static *vs.* temporal centrality ENRON[8]

static ... corporate role in the organisation
temporal ... information dissemination and the role of information mediators

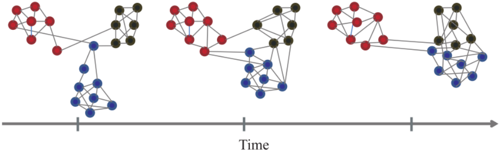



| ID | Name | Role | Notes |
|---|---|---|---|
| 9 | Stephanie Panus | (Unknown) | |
| 13 | Marie Heard | Legal | Senior Legal Specialist |
| 17 | Mike Grigsby | Manager | |
| 48 | Tana Jones | Executive | |
| 53 | John Lavorato | Trader | |
| 54 | Greg Whalley | President | Former Head of Trading |
| 67 | Sara Shackleton | Vice President | Enron Wholesale Services |
| 73 | Jeff Dasovich | Trader | |
| 75 | Gerald Nemec | Director of Trading | |
| 107 | Louise Kitchen | Trader | Head of Online Trading |
| 122 | Sally Beck | Managing Director | |
| 127 | Kenneth Lay | Chairman & CEO | |
| 139 | Mary Hain | Director | |
| 147 | Carol Clair | Trader | |
| 150 | Liz Taylor | Secretary | Assistant to Greg Whalley |

[8] Tang J., 2010. Analysing Information Flows and Key Mediators through Temporal Centrality Metrics

# Metrics – community structure

- rearrangement of cohesive groups
- formation of new groups
- fragmentation of existing ones

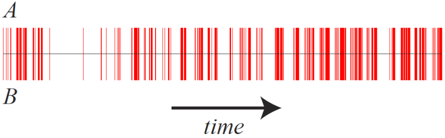


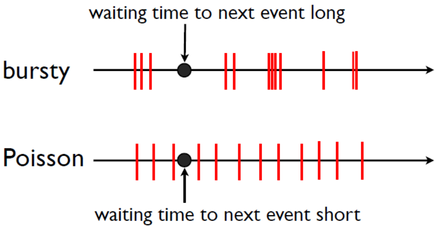

[9]

- maximization of optimization function, parameters of spatial and temporal resolution

[9] Bassett D., 2013

# Burstiness[10]




- temporal inhomogeneity
- events cluster in time

$$B = \frac{\sigma_\tau - m_\tau}{\sigma_\tau + m_\tau}$$

$m_\tau$ …mean time between events
$\sigma_\tau$ …std of times
$B = -1$: periodic
$B = 0$: Poisson
$B = 1$: maximally bursty

An uncorrelated burstiness increases the latency of temporal paths.

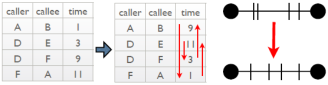
[10] Saramäki, 2014, Goh K.-L., 2008

# Models of Temporal Networks

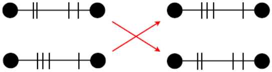
- Randomized null or reference models
  - used to interpret significance, to understand the effects of diverse temporal and structural characteristics
  - A randomize a network in one way; the rest is kept as is
  - B normalized metrics
  - C z-scores of unnormalized metrics against normalized counterpart
  - there is no 'THE ONE' null model (compared to static networks with the configuration model)
- Generative, mechanistic and predictive models
  - generative model to capture structure
  - mechanistic models that explain the evolution of large-scale structures (temporal extensions to WS small-world, BA scale-free)
  - predictive models to forecast a graph behavior in the near future

# Reference (Null) Models I.

- randomly permuted times (DCW): disturbs all temporal correlations, keeps static topology and numbers of contacts between node-pairs




- random swaps of whole sequences (DCB): disturbs correlations between neighboring events while preserving a sequence character and weights



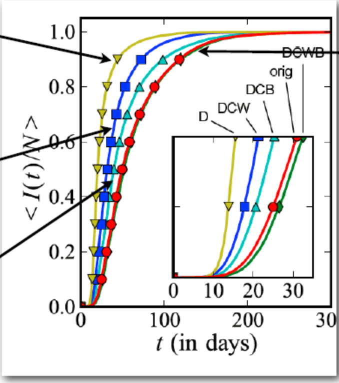
## *Small but slow world: how network topology and burstiness slow down spreading*[11]

**Diffusion in networks over time: real system and null models**

**Removing most correlations**

**Removing burstiness**

**Removing bottlenecks, burstiness kept**



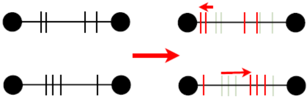

**Original sequence of phone calls**

**Bursty dynamics significantly slows down diffusion!**

[11] Karsai et al., 2011; D: configuration model – null model from a static network;
DCWB: as DCB but shuffles only sequences with the same number of events

# Reference (Null) Models II.

- randomly shifted times in a sequence: disturbs correlations between neighboring events, leaves nodes sequences



- random times in a sequence: disturbs other correlations in a sequence and between sequences

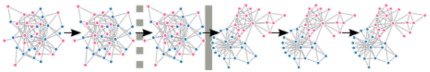
# Temporal Networks Summary

- describes network topology and properties with respect to time
- defined as a set of network slices tracking the flow of time
- can use a fast temporal scale
- ... which is comparable with the temporal scale of dynamic processes on a network
- defines time-respecting paths
- real-world systems exhibit small-world characteristics and bursty timing of events

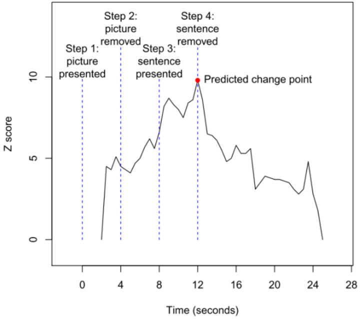
# Change-Point Detection

## identification of important moments

- similarity between two snapshots
  - task-based experiments
  - interictal – ictal phases

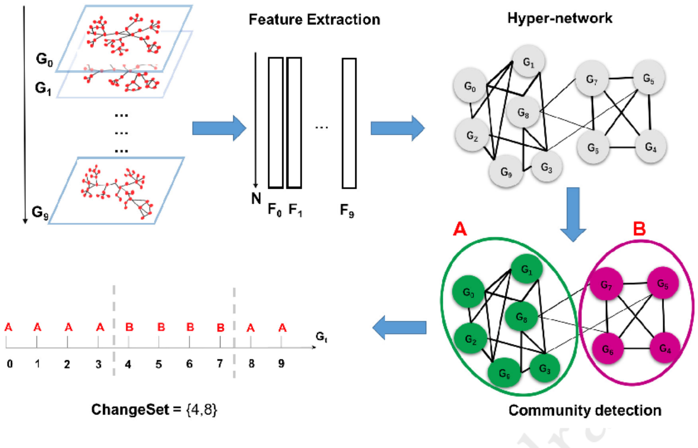


Peel, 2014



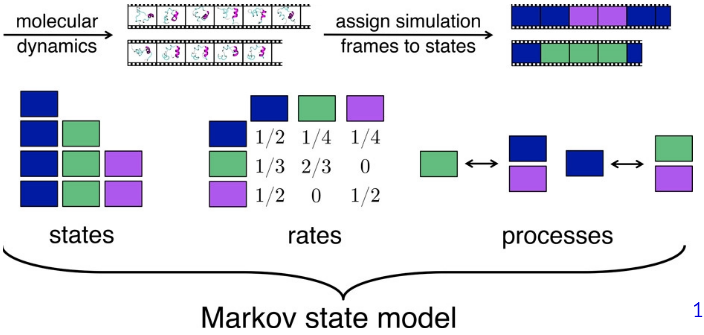

Barnett, 2014

# Change-Point Detection II.[12]




[12] Zhu, 2018

# Network States



[13]

- identification of stable states of a network
- dwell-time, the fraction of total time spent in each state, transition matrix

[13] Husic, 2018

# Sliding Window Approaches for Correlation Networks

- sliding window approach (SW)
  - Pearson’s correlation
- tapered sliding window approach (TSW)
  - weighted Pearson’s correlation
  - weights distributed according to Gaussian distribution centered at $t$
- dynamic conditional correlations (DCC)
  - Engle 2000[14], Lindquist 2014
  - model-based multivariate method from GARCH family
  - estimates conditional variances and correlations
  - uses past values

[14] https://escholarship.org/uc/item/56j4143f

# Summary

An aggregated static network leads to

- overestimating the number of paths and walks
- underestimating the effective distances
- BUT is essential for topological (rather than temporal) analysis.

Studying network dynamics allows us to

- capture network topology evolving in time
- identify network states
- detect exact change points
- asses temporal network properties and identify key nodes in processes on a network
- reveal real-life behavior of complex systems.

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