

# Language Modeling (and the Noisy Channel)

PA154 Language Modeling (2.1)

**Pavel Rychlý**

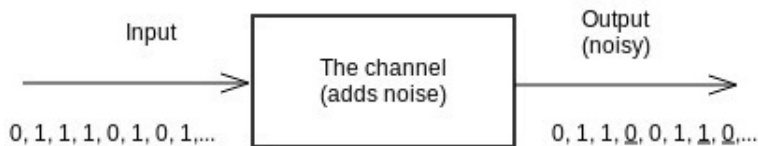
pary@fi.muni.cz

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**Source:** Introduction to Natural Language Processing (600.465)  
Jan Hajič, CS Dept., Johns Hopkins Univ.  
[www.cs.jhu.edu/~hajic](http://www.cs.jhu.edu/~hajic)

# The Noisy Channel

- Prototypical case



- Model: probability of error (noise):

- Example:  $p(0|1) = .3$   $p(1|1) = .7$   $p(1|0) = .4$   $p(0|0) = .6$

- The task:

known: the noisy output; want to know; the input (decoding)

# Noisy Channel Applications

- OCR
    - straightforward: text  $\rightarrow$  print (adds noise), scan  $\rightarrow$  image
  - Handwriting recognition
    - text  $\rightarrow$  neurons, muscles ("noise"), scan/digitize  $\rightarrow$  image
  - Speech recognition (dictation, commands, etc.)
    - text  $\rightarrow$  conversion to acoustic signal ("noise")  $\rightarrow$  acoustic waves
  - Machine Translation
    - text in target language  $\rightarrow$  translation ("noise")  $\rightarrow$  source language
  - Also: Part of Speech Tagging
    - sequence of tags  $\rightarrow$  selection of word forms  $\rightarrow$  text
-

# The Golden Rule of OCR, ASR, HR, MT,...

- Recall:

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)} \quad (\text{Bayes formula})$$

$$A_{best} = \underset{A}{\operatorname{argmax}} p(B|A)p(A) \quad (\text{The Golden Rule})$$

- $p(B|A)$ : the acoustic/image/translation/lexical model
  - application-specific name
  - will explore later
- $p(A)$ : language model

# The Perfect Language Model

- Sequence of word forms (forget about tagging for the moment)
- Notation:  $A \sim W = (w_1, w_2, w_3, \dots, w_d)$
- The big (modeling) question:

$$p(W) = ?$$

- Well, we know (Bayes/chain rule)  $\rightarrow$ ):

$$p(W) = p(w_1, w_2, w_3, \dots, w_d) = \\ p(w_1) \times p(w_2|w_1) \times p(w_3|w_1, w_2) \times \dots \times p(w_d|w_1, w_2, \dots, w_{d-1})$$

- Not practical (even short  $W \rightarrow$  too many parameters)

# Markov Chain

- Unlimited memory (cf. previous foil):
  - for  $w_i$  we know all its predecessors  $w_1, w_2, w_3, \dots, w_{i-1}$
- Limited memory:
  - we disregard "too old" predecessors
  - remember only  $k$  previous words:  $w_{i-k}, w_{i-k+1}, \dots, w_{i-1}$
  - called " $k^{\text{th}}$  order Markov approximation"
- + stationary character (no change over time):

$$p(W) \cong \prod_{i=1..d} p(w_i | w_{i-k}, w_{i-k+1}, \dots, w_{i-1}), d = |W|$$

# n-gram Language Models

- $(n - 1)^{th}$  order Markov approximation  $\rightarrow$  n-gram LM:

$$p(W) =_{df} \prod_{i=1..d} p(\overset{\text{prediction}}{w_i} \mid \overset{\text{history}}{w_{i-n+1}, w_{i-n+2}, \dots, w_{i-1}})$$

- In particular (assume vocabulary  $|V| = 60k$ ):

0-gram LM: uniform model,

$$p(w) = 1/|V|,$$

1 parameter

1-gram LM: unigram model,

$$p(w),$$

$6 \times 10^4$  parameters

2-gram LM: bigram model,

$$p(w_i | w_{i-1}),$$

$3.6 \times 10^9$  parameters

3-gram LM: trigram model,

$$p(w_i | w_{i-2}, w_{i-1}),$$

$2.16 \times 10^{14}$  parameters

# LM: Observations

- How large  $n$ ?
  - nothing in enough (theoretically)
  - but anyway: as much as possible ( $\rightarrow$  close to "perfect" model)
  - empirically: 3
    - parameter estimation? (reliability, data availability, storage space, ...)
    - 4 is too much:  $|V|=60k \rightarrow 1.296 \times 10^{19}$  parameters
    - but: 6–7 would be (almost) ideal (having enough data): in fact, one can recover original from 7-grams!
- Reliability  $\sim(1/\text{Detail})$  ( $\rightarrow$  need compromise)
- For now, keep word forms (no "linguistic" processing)



# The Length Issue

- $\forall n; \sum_{w \in \Omega^n} p(w) = 1 \Rightarrow \sum_{n=1.. \infty} \sum_{w \in \Omega^n} p(w) \gg 1 (\rightarrow \infty)$
- We want to model all sequences of words
  - for "fixed" length tasks: no problem – n fixed, sum is 1
    - tagging, OCR/handwriting (if words identified ahead of time)
  - for "variable" length tasks: have to account for
    - discount shorter sentences
- General model: for each sequence of words of length n, define  $p'(w) = \lambda_n p(w)$  such that  $\sum_{n=1.. \infty} \lambda_n = 1 \Rightarrow$ 
$$\sum_{n=1.. \infty} \sum_{w \in \Omega^n} p'(w) = 1$$

e.g. estimate  $\lambda_n$  from data; or use normal or other distribution

# Parameter Estimation

- Parameter: numerical value needed to compute  $p(w|h)$
- From data (how else?)
- Data preparation:
  - get rid of formatting etc. ("text cleaning")
  - define words (separate but include punctuation, call it "word")
  - define sentence boundaries (insert "words" `<s>` and `</s>`)
  - letter case: keep, discard, or be smart:
    - name recognition
    - number type identification (these are huge problems per se!)
    - numbers: keep, replace by `<num>`, or be smart (form ~ punctuation)

# Maximum Likelihood Estimate

- MLE: Relative Frequency...
  - ...best predicts the data at hand (the "training data")
- Trigrams from training Data T:
  - count sequences of three words in T:  $c_3(w_{i-2}, w_{i-1}, w_i)$
  - (NB: notation: just saying that three words follow each other)
  - count sequences of two words in T:  $c_2(w_{i-1}, w_i)$ 
    - either use  $c_2(y, z) = \sum_w c_3(y, z, w)$
    - or count differently at the beginning (& end) of the data!

$$p(w_i | w_{i-2}, w_{i-1}) =_{est.} \frac{c_3(w_{i-2}, w_{i-1}, w_i)}{c_2(w_{i-2}, w_{i-1})} \quad !$$

# Character Language Model

- Use individual characters instead of words:

$$p(W) =_{df} \prod_{i=1..d} p(c_i | c_{i-n+1}, c_{i-n+2}, \dots, c_i)$$

- Same formulas etc.
- Might consider 4-grams, 5-grams or even more
- Good only for language comparison)
- Transform cross-entropy between letter- and word-based models:

$$H_S(p_c) = H_S(p_w) / \text{avg. \# of characters/word in } S$$

# LM: an Example

## ■ Training data:

<s> <s> He can buy the can of soda.

### – Unigram:

$$p_1(\text{He}) = p_1(\text{buy}) = p_1(\text{the}) = p_1(\text{of}) = p_1(\text{soda}) = p_1(\text{.}) = .125$$
$$p_1(\text{can}) = .25$$

### – Bigram:

$$p_2(\text{He}|\text{<s>}) = 1, p_2(\text{can}|\text{He}) = 1, p_2(\text{buy}|\text{can}) = .5, p_2(\text{of}|\text{can}) = .5,$$
$$p_2(\text{the}|\text{buy}) = 1, \dots$$

### – Trigram:

$$p_3(\text{He}|\text{<s>, <s>}) = 1, p_3(\text{can}|\text{<s>, He}) = 1, p_3(\text{buy}|\text{He, can}) = 1,$$
$$p_3(\text{of}|\text{the, can}) = 1, \dots, p_3(\text{.}|\text{of, soda}) = 1.$$

### – Entropy:

$$H(p_1) = 2.75, H(p_2) = .25, H(p_3) = 0 \leftarrow \text{Great?!}$$

## LM: an Example (The Problem)

- Cross-entropy:
- $S = \langle s \rangle \langle s \rangle$  It was the greatest buy of all.
- Even  $H_S(p_1)$  fails ( $= H_S(p_2) = H_S(p_3) = \infty$ ), because:
  - all unigrams but  $p_1(\text{the})$ ,  $p_1(\text{buy})$ ,  $p_1(\text{of})$  and  $p_1(\cdot)$  are 0.
  - all bigram probabilities are 0.
  - all trigram probabilities are 0.
- We want: to make all (theoretically possible\*) probabilities non-zero.

\*in fact, all: remeber our graph from day1?