HMM Parameter Estimation: the Baum-Welch algorithm

PA154 Language Modeling (5.1)

Pavel Rychlý

pary@fi.muni.cz

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Source: Introduction to Natural Language Processing (600.465)
Jan Hajič, CS Dept., Johns Hopkins Univ.
www.cs.jhu.edu/˜hajic

HMM: The Tasks

- HMM(the general case):
  - five-tuple (S, S_0, Y, P_S, P_Y), where:
    - $S = \{s_1, s_2, \ldots, s_T\}$ is the set of states, S_0 is the initial state,
    - $Y = \{y_1, y_2, \ldots, y_y\}$ is the output alphabet,
    - $P_S(s_j|s_i)$ is the set of prob. distributions of transitions,
    - $P_Y(y_k|s_i, s_j)$ is the set of output (emission) probability distributions.
- Given an HMM & an output sequence $Y = \{y_1, y_2, \ldots, y_k\}$:
  - (Task 1) compute the probability of Y;
  - (Task 2) compute the most likely sequence of states which has generated Y
  - (Task 3) Estimating the parameters (transition/output distributions)

A variant of Expectation–Maximization

- Idea($\sim$EM, for another variant see LM smoothing (lecture 2.2)):
  - Start with (possibly random) estimates of P_S and P_Y.
  - Compute (fractional) "counts" of state transitions/emissions taken, from P_S and P_Y, given data Y
  - Adjust the estimates of P_S and P_Y from these "counts" (using MLE, i.e. relative frequency as the estimate).
- Remarks:
  - many more parameters than the simple four-way smoothing
  - no proofs here; see Jelinek Chapter 9

Setting

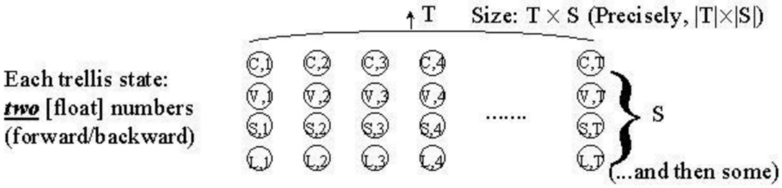
- HMM (without P_S, P_Y)(S, S_0, Y), and data $T = \{y_i \in Y\}_{i=1...|T|}$
  - will use $T \sim |T|$
- HMM structure is given: (S, S_0)
- P_S: Typically, one wants to allow "fully connected" graph
  - (i.e. no transitions forbidden $\sim$ no transitions set to hard 0)
  - why? $\rightarrow$ we better leave it on the learning phase, based on the data!
  - sometimes possible to remove some transitions ahead of time
- P_Y : should be restricted (if not, we will not get anywhere!)
  - restricted $\sim$ hard 0 probabilities of $p(y|s, s')$
  - "Dictionary": states $\leftrightarrow$ words, "m:n" mapping on $S \times Y$ (in general)

Initialization

- For computing the initial expected "counts"
- Important part
  - EM guaranteed to find a *local* maximum only (albeit a good one in most cases)
- P_Y initialization more important
  - fortunately, often easy to determine
    - together with dictionary $\leftrightarrow$ vocabulary mapping, get counts, then MLE
- P_S initialization less important
  - e.g. uniform distribution for each $p(.|s)$

Data structures

- Will need storage for:
  - The predetermined structure of the HMM (unless fully connected $\rightarrow$ need not to keep it!)
  - The parameters to be estimated (P_S, P_Y)
  - The expected counts (same size as (P_S, P_Y))
  - The training data $T = \{y_i \in Y\}_{i=1...T}$
  - The trellis (if f.c.):



The Algorithm Part I

1. Initialize P_S, P_Y
2. Compute "forward" probabilities:
   - follow the procedure for trellis (summing), compute $\alpha(s, i)$ everywhere
   - use the current values of $P_S, P_Y(p(s'|s), p(y|s, s'))$: $\alpha(s', i) = \sum_{s \to s'} \alpha(s, i-1) \times p(s'|s) \times p(y_i|s, s')$
   - NB: do not throw away the previous stage!
3. Compute "backward" probabilities
   - start at all nodes of the last stage, proceed backwards, $\beta(s, i)$
   - i.e., probability of the "tail" of data from stage i to the end of data $\beta(s', i) = \sum_{s' \leftarrow s} \beta(s, i+1) \times p(s|s') \times p(y_{i+1}|s', s)$
   - also, keep the $\beta(s, i)$ at all trellis states

The Algorithm Part II

4. Collect counts:
   - for each output/transition pair compute

$$c(y,s,s') = \sum_{i=0..k-1,\, y=y_{i+1}} \alpha(s,i)\ \underbrace{p(s'|s)\ p(y_{i+1}|s,s')}_{\text{this transition prob} \times \text{output prob}}\ \beta(s',i+1)$$

one pass through data, only stop at (output) y

prefix prob.

tail prob

$c(s,s') = \sum_{y \in Y} c(y,s,s')$ (assuming all observed y_i in Y)
$c(s) = \sum_{s' \in S} c(s,s')$

5. Reestimate: $p'(s'|s) = c(s,s')/c(s) \quad p'(y|s,s') = c(y,s,s')/c(s,s')$
6. Repeat 2-5 until desired convergence limit is reached

Baum-Welch: Tips & Tricks

- Normalization badly needed
  - long training data $\rightarrow$ extremely small probabilities
- Normalize α, β using the same norm.factor:

  $$N(i) = \sum_{s \in S} \alpha(s, i)$$

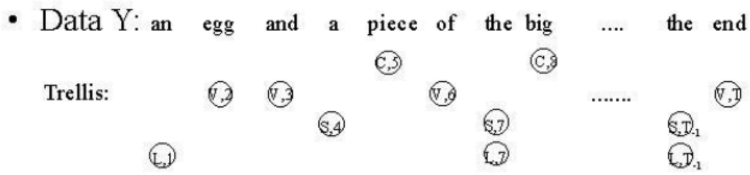
  as follows:
  - compute $\alpha(s, i)$ as usual (Step 2 of the algorithm), computing the sum $N(i)$ at the given stage i as you go.
  - at the end of each stage, recompute all αs (for each state s):

    $$\alpha^*(s, i) = \alpha(s, i)/N(i)$$
  - use the same $N(i)$ for βs at the end of each backward (Step 3) stage:

    $$\beta^*(s, i) = \beta(s, i)/N(i)$$

Example

- Task: pronunciation of ”the”
- Solution: build HMM, fully connected, 4 states:
  - S - short article, L - long article, C,V - word starting w/consonant, vowel
  - thus, only ”the” is ambiguous (a, an, the - not members of C,V)
- Output form states only ($p(w|s,s') = p(w|s')$)

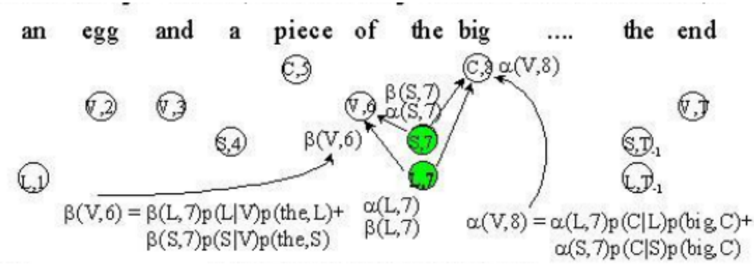


Example: Initialization

- Output probabilities:
  - $p_{init}(w|c) = c(c,w)/c(c)$; where $c(S, the) = c(L, the) = c(the)/2$ (other than that, everything is deterministic)
- Transition probabilities:
  - $p_{init}(c'|c) = 1/4(uniform)$
- Don't forget:
  - about the space needed
  - initialize $\alpha(X, 0) = 1$ (X : the never-occuring front buffer st.)
  - initialize $\beta(s, T) = 1$ for all s (except for $s = X$)

Fill in alpha, beta

- Left to right, alpha: $\alpha(s', i) = \sum_{s \to s'} \alpha(s, i-1) \times p(s'|s) \times p(w_i|s')$, where s' is the output from states
- Remember normalization (N(i)).
- Similary, beta (on the way back from the end).

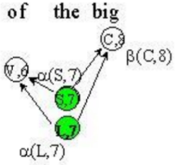


Counts & Reestimation

- One pass through data
- At each position i, go through all pairs (s_i, s_{i+1})
- Increment appropriate counters by frac. counts (Step 4):
  - $\text{inc}(y_{i+1}, s_i, s_{i+1}) = a(s_i, i)p(s_{i+1}|s_i)p(y_{i+1}|s_{i+1})b(s_{i+1,i+1})$
  - $c(y, s_i, s_{i+1}) += \text{inc}$ (for y at pos i+1)
  - $c(s_i, s_{i+1}) += \text{inc}$ (always)
  - $c(s_i) += \text{inc}$ (always)

    inc(big,L,C)=$\alpha(L,7)p(C|L)p(\text{big},C)\beta(V,8)$

    inc(big,S,C)=$\alpha(S,7)p(C|S)p(\text{big},C)\beta(V,8)$
- Reestimate $p(s'|s), p(y|s)$
  - and hope for increase in $p(C|S)$ and $p(V|L)$ ...!!



HMM: Final Remarks

- Parameter ”tying”
  - keep certain parameters same ($\sim$ just one ”counter” for all of them)
  - any combination in principle possible
  - ex.: smoothing (just one set of lambdas)
- Real Numbers Output
  - Y of infinite size (R, R^n)
    - parametric (typically: few) distribution needed (e.g., ”Gaussian”)
- ”Empty” transitions: do not generate output
  - $\sim$ vertical areas in trellis; do not use in ”counting”