Language Modeling (and the Noisy Channel)

PA154 Language Modeling (3.1)

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Source: Introduction to Natural Language Processing (600.465)
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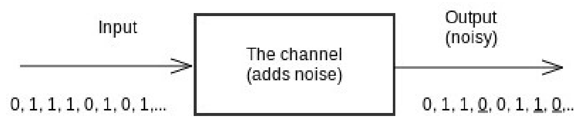
The Noisy Channel

- Prototypical case



- Model: probability of error (noise):
- Example: p(0|1) = .3 p(1|1) = .7 p(1|0) = .4 p(0|0) = .6
- The task:
  known: the noisy output; want to know; the input (*decoding*)

Noisy Channel Applications

- OCR
  – straightforward: text → print (adds noise), scan → image
- Handwriting recognition
  – text → neurons, muscles ("noise"), scan/digitize → image
- Speech recognition (dictation, commands, etc.)
  – text → conversion to acoustic signal ("noise") → acoustic waves
- Machine Translation
  – text in target language → translation ("noise") → source language
- Also: Part of Speech Tagging
  – sequence of tags → selection of word forms → text

The Golden Rule of OCR, ASR, HR, MT,...

- Recall:
  $$p(A|B) = \frac{p(B|A)p(A)}{p(B)} \quad \text{(Bayes formula)}$$
  $$A_{best} = argmax_A p(B|A)p(A) \quad \text{(The Golden Rule)}$$
- p(B|A): the acoustic/image/translation/lexical model
  – application-specific name
  – will explore later
- p(A): *language model*

The Perfect Language Model

- Sequence of word forms (forget about tagging for the moment)
- Notation: A ~ W = $(w_1, w_2, w_3, ..., w_d)$
- The big (modeling) question:
  $$p(W) = ?$$
- Well, we know (Bayes/chain rule) →):
  $$p(W) = p(w_1, w_2, w_3, ..., w_d) =$$
  $$p(w_1) \times p(w_2|w_1) \times p(w_3|w_1, w_2) \times ... \times p(w_d|w_1, w_2, ...w_{d-1})$$
- Not practical (even short W → too many parameters)

Markov Chain

- Unlimited memory (cf. previous foil):
  – for w_i we know all its predecessors $w_1, w_2, w_3, ..., w_{i-1}$
- Limited memory:
  – we disregard "too old" predecessors
  – remember only k previous words: $w_{i-k}, w_{i-k+1}, ..., w_{i-1}$
  – called "k^{th} order Markov approximation"
- \+ stationary character (no change over time):
  $$p(W) \cong \prod_{i=1..d} p(w_i|w_{i-k}, w_{i-k+1}, ..., w_{i-1}), d = |W|$$

n-gram Language Models

- $(n-1)^{th}$ order Markov approximation → n-gram LM:

prediction — history

$$p(W) =_{df} \prod_{i=1..d} p(w_i \mid w_{i-n+1}, w_{i-n+2}, ..., w_{i-1})$$

- In particular (assume vocabulary |V| = 60k):

0-gram LM: uniform model,	p(w) = 1/|V|,	1 parameter
1-gram LM: unigram model,	p(w),	6×10^4 parameters
2-gram LM: bigram model,	$p(w_i \mid w_{i-1})$,	3.6×10^9 parameters
3-gram LM: trigram model,	$p(w_i \mid w_{i-2}, w_{i-1})$,	2.16×10^{14} parameters

LM: Observations

- How large *n*?
  - – nothing in enough (theoretically)
  - – but anyway: as much as possible (→ close to "perfect" model)
  - – empirically: *3*
    - parameter estimation? (reliability, data availability, storage space, ...)
    - 4 is too much: |V|=60k → 1.296×10^{19} parameters
    - but: 6–7 would be (almost) ideal (having enough data): in fact, one can recover original from 7-grams!
- Reliability ~(1/Detail) (→ need compromise)
- For now, keep word forms (no "linguistic" processing)

The Length Issue

- $\forall n; \Sigma_{w\in\Omega^n} p(w) = 1 \Rightarrow \Sigma_{n=1..\infty}\Sigma_{w\in\Omega^n} p(w) \gg 1 (\rightarrow \infty)$
- We want to model all sequences of words
  - – for "fixed" length tasks: no problem – n fixed, sum is 1
    - tagging, OCR/handwriting (if words identified ahead of time)
  - – for "variable" length tasks: have to account for
    - discount shorter sentences
- General model: for each sequence of words of length n, define p'(w) = $\lambda_n p(w)$ such that $\Sigma_{n-1..\infty}\lambda_n = 1 \Rightarrow$

$$\Sigma_{n=1..\infty}\Sigma_{w\in\Omega^n} p'(w) = 1$$

e.g. estimate λ_n from data; or use normal or other distribution

Parameter Estimation

- Parameter: numerical value needed to compute p(w|h)
- From data (how else?)
- Data preparation:
  - get rid of formating etc. ("text cleaning")
  - define words (separate but include punctuation, call it "word")
  - define sentence boundaries (insert "words" <s> and </s>)
  - letter case: keep, discard, or be smart:
    - – name recognition
    - – number type identification (these are huge problems per se!)
    - – numbers: keep, replace by <num>, or be smart (form ~ punctuation)

Maximum Likelihood Estimate

- MLE: Relative Frequency...
  - – ...best predicts the data at hand (the "training data")
- Trigrams from training Data T:
  - – count sequences of three words in T: $c_3(w_{i-2}, w_{i-1}, w_i)$
  - – (NB: notation: just saying that three words follow each other)
  - – count sequences of two words in T: $c_2(w_{i-1}, w_i)$
    - either use $c_2(y,z) = \Sigma_w c_3(y,z,w)$
    - or count differently at the beginning (& end) of the data!

$$p(w_i \mid w_{i-2}, w_{i-1}) =_{est.} \frac{c_3(w_{i-2}, w_{i-1}, w_i)}{c_2(w_{i-2}, w_{i-1})} \quad !$$

Character Language Model

- Use individual characters instead of words:

$$p(W) =_{df} \Pi_{i=1..d} p(c_i \mid c_{i-n+1}, c_{i-n+2}, ..., c_i)$$

- Same formulas etc.
- Might consider 4-grams, 5-grams or even more
- Good only for language comparison)
- Transform cross-entropy between letter- and word-based models:

$$H_S(p_c) = H_S(p_w)/avg.\ \text{\# of characters/word in S}$$

LM: an Example

- Training data:

  <s> <s> He can buy the can of soda.

  – Unigram:
  p_1(He) = p_1(buy) = p_1(the) = p_1(of) p_1(soda) = p_1(.) = .125
  p_1(can) = .25

  – Bigram:
  p_2(He|<s>) = 1, p_2(can|He) = 1, p_2(buy|can) = .5, p_2(of|can) = .5,
  p_2(the|buy) = 1,...

  – Trigram:
  p_3(He|<s>, <s>) = 1, p_3(can|<s>,He) = 1, p_3(buy|He,can) = 1,
  p_3(of|the,can) = 1, ..., p_3(.|of,soda) = 1.

  – Entropy:
  H(p_1) = 2.75, H(p_2) = .25, H(p_3) = 0 ← Great?!

LM: an Example (The Problem)

- Cross-entropy:
- S = <s><s> It was the greatest buy of all.
- Even $H_S(p_1)$ fails (= $H_S(p_2) = H_S(p_3) = \infty$), because:
  - all unigrams but p_1(the), p_1(buy), p_1(of) and p_1(.) are 0.
  - all bigram probabilities are 0.
  - all trigram probabilities are 0.
- We want: to make all (theoretically possible*) probabilities non-zero.

*in fact, all: remeber our graph from day1?