PV211: Introduction to Information Retrieval
https://www.fi.muni.cz/~sojka/PV211

IIR 5: Index compression
Handout version

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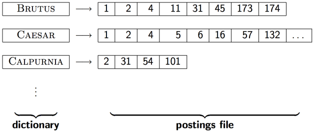
Overview

Roadmap

- Today: index compression, and vector space model
- Next week: the whole picture of complete search system, scoring and ranking

Take-away today

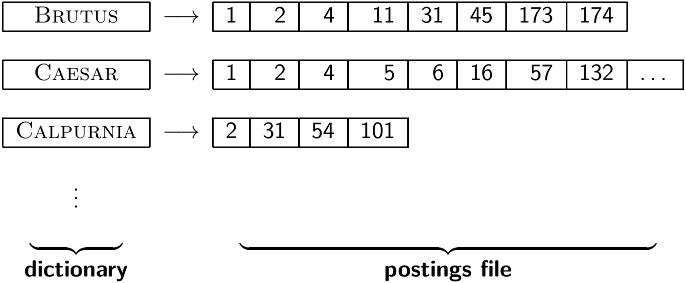
For each term t, we store a list of all documents that contain t.



- Motivation for compression in information retrieval systems
- How can we compress the dictionary component of the inverted index?
- How can we compress the postings component of the inverted index?
- Term statistics: how are terms distributed in document collections?

Inverted index

For each term t, we store a list of all documents that contain t.



Today:

- How much space do we need for the dictionary?
- How much space do we need for the postings file?
- How can we compress them?

Why compression? (in general)

- Use less disk space (saves money).
- Keep more stuff in memory (increases speed).
- Increase speed of transferring data from disk to memory (again, increases speed).

  [read compressed data and decompress in memory]
  is faster than
  [read uncompressed data]
- Premise: Decompression algorithms are fast.
- This is true of the decompression algorithms we will use.

Why compression in information retrieval?

- First, we will consider space for dictionary:
  - Main motivation for dictionary compression: make it small enough to keep in main memory.
- Then for the postings file
  - Motivation: reduce disk space needed, decrease time needed to read from disk.
  - Note: Large search engines keep significant part of postings in memory.
- We will devise various compression schemes for dictionary and postings.

Lossy vs. lossless compression

- Lossy compression: Discard some information
- Several of the preprocessing steps we frequently use can be viewed as lossy compression:
  - downcasing, stop words, porter, number elimination
- Lossless compression: All information is preserved.
  - What we mostly do in index compression

Model collection: The Reuters collection

symbol	statistic	value
N	documents	800,000
L	avg. # word tokens per document	200
M	word types	400,000
	avg. # bytes per word token (incl. spaces/punct.)	6
	avg. # bytes per word token (without spaces/punct.)	4.5
	avg. # bytes per word type	7.5
T	non-positional postings	100,000,000

Effect of preprocessing for Reuters

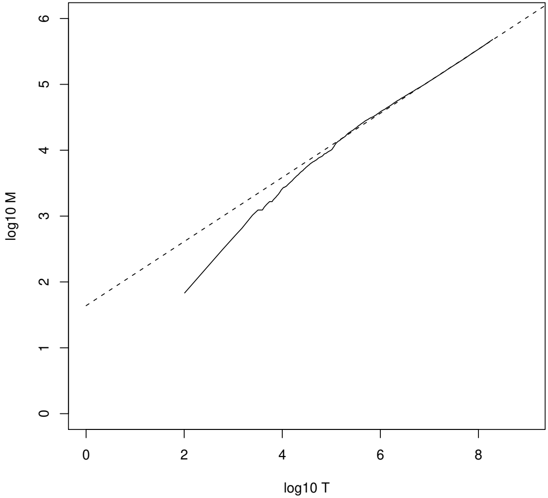
	word types (terms)			non-positional postings			positional postings (word tokens)		
size of	dictionary			non-positional index			positional index		
	size	Δ	cml	size	Δ	cml	size	Δ	cml
unfiltered	484,494			109,971,179			197,879,290		
no numbers	473,723	-2	-2	100,680,242	-8	-8	179,158,204	-9	-9
case folding	391,523	-17	-19	96,969,056	-3	-12	179,158,204	-0	-9
30 stopw's	391,493	-0	-19	83,390,443	-14	-24	121,857,825	-31	-38
150 stopw's	391,373	-0	-19	67,001,847	-30	-39	94,516,599	-47	-52
stemming	322,383	-17	-33	63,812,300	-4	-42	94,516,599	-0	-52

Explain differences between numbers non-positional vs positional: -3 vs 0, -14 vs -31, -30 vs -47, -4 vs 0

How big is the term vocabulary?

- That is, how many distinct words are there?
- Can we assume there is an upper bound?
- Not really: At least $70^{20} \approx 10^{37}$ different words of length 20.
- The vocabulary will keep growing with collection size.
- Heaps' law: $M = kT^b$
- M is the size of the vocabulary, T is the number of tokens in the collection.
- Typical values for the parameters k and b are: $30 \leq k \leq 100$ and $b \approx 0.5$.
- Heaps' law is linear in log-log space.
  - It is the simplest possible relationship between collection size and vocabulary size in log-log space.
  - Empirical law

Heaps' law for Reuters



Vocabulary size M as a function of collection size T (number of tokens) for Reuters-RCV1. For these data, the dashed line $\log_{10} M = 0.49 * \log_{10} T + 1.64$ is the best least squares fit. Thus, $M = 10^{1.64} T^{0.49}$ and $k = 10^{1.64} \approx 44$ and $b = 0.49$.

Empirical fit for Reuters

- Good, as we just saw in the graph.
- Example: for the first 1,000,020 tokens Heaps' law predicts 38,323 terms:

$$44 \times 1{,}000{,}020^{0.49} \approx 38{,}323$$

- The actual number is 38,365 terms, very close to the prediction.
- Empirical observation: fit is good in general.

Exercise

1. What is the effect of including spelling errors vs. automatically correcting spelling errors on Heaps' law?
2. Compute vocabulary size M
   - Looking at a collection of web pages, you find that there are 3,000 different terms in the first 10,000 tokens and 30,000 different terms in the first 1,000,000 tokens.
   - Assume a search engine indexes a total of 20,000,000,000 (2×10^{10}) pages, containing 200 tokens on average
   - What is the size of the vocabulary of the indexed collection as predicted by Heaps' law?

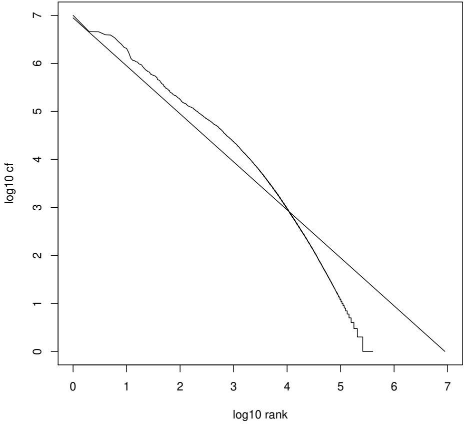
Zipf's law

- Now we have characterized the growth of the vocabulary in collections.
- We also want to know how many frequent vs. infrequent terms we should expect in a collection.
- In natural language, there are a few very frequent terms and very many very rare terms.
- Zipf's law: The i^{th} most frequent term has frequency cf_i proportional to $1/i$.
- $\mathrm{cf}_i \propto \frac{1}{i}$
- cf_i is collection frequency: the number of occurrences of the term t_i in the collection.

Zipf's law

- Zipf's law: The i^{th} most frequent term has frequency proportional to $1/i$.
- $\mathrm{cf}_i \propto \frac{1}{i}$
- cf is collection frequency: the number of occurrences of the term in the collection.
- So if the most frequent term (*the*) occurs cf_1 times, then the second most frequent term (*of*) has half as many occurrences $\mathrm{cf}_2 = \frac{1}{2}\mathrm{cf}_1 \ldots$
- ... and the third most frequent term (*and*) has a third as many occurrences $\mathrm{cf}_3 = \frac{1}{3}\mathrm{cf}_1$, etc.
- Equivalent: $\mathrm{cf}_i = ci^k$ and $\log \mathrm{cf}_i = \log c + k \log i$ (for $k = -1$)
- Example of a power law

Zipf's law for Reuters



Fit is not great. What is important is the key insight: Few frequent terms, many rare terms.

Dictionary compression

- The dictionary is small compared to the postings file.
- But we want to keep it in memory.
- Also: competition with other applications, cell phones, onboard computers, fast startup time
- So compressing the dictionary is important.

Recall: Dictionary as array of fixed-width entries

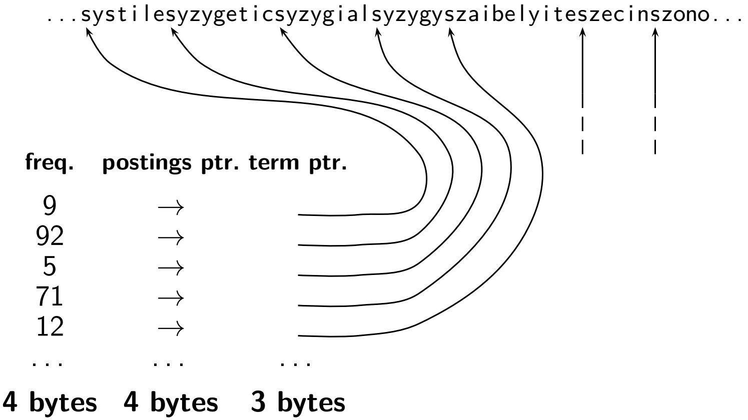
	term	document frequency	pointer to postings list
	a	656,265	$\longrightarrow$
	aachen	65	$\longrightarrow$
	...	...	...
	zulu	221	$\longrightarrow$
space needed:	20 bytes	4 bytes	4 bytes

Space for Reuters: (20+4+4)*400,000 = 11.2 MB

Fixed-width entries are bad.

- Most of the bytes in the term column are wasted.
  - We allot 20 bytes for terms of length 1.
- We cannot handle HYDROCHLOROFLUOROCARBONS and SUPERCALIFRAGILISTICEXPIALIDOCIOUS
- Average length of a term in English: 8 characters (or a little bit less)
- How can we use on average 8 characters per term?

Dictionary as a string



Space for dictionary as a string

- 4 bytes per term for frequency
- 4 bytes per term for pointer to postings list
- 8 bytes (on average) for term in string
- 3 bytes per pointer into string (need $\log_2 8 \cdot 400{,}000 < 24$ bits to resolve $8 \cdot 400{,}000$ positions)
- Space: $400{,}000 \times (4 + 4 + 3 + 8) = 7.6$ MB (compared to 11.2 MB for fixed-width array)

Dictionary as a string with blocking

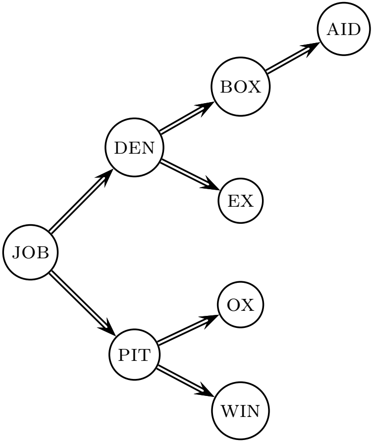
. . . **7** s y s t i l e **9** s y z y g e t i c **8** s y z y g i a l **6** s y z y g y **11** s z a i b e l y i t e **6** s z e c i n . . .

freq.	postings ptr.	term ptr.
9	→	
92	→	
5	→	
71	→	
12	→	
. . .	. . .	. . .

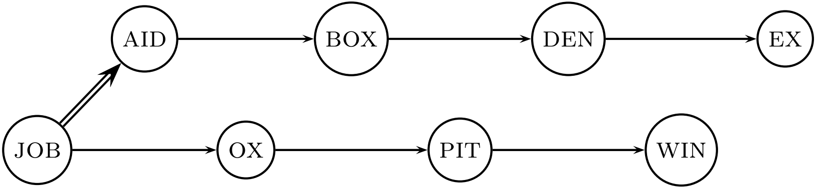
Space for dictionary as a string with blocking

- Example block size $k = 4$
- Where we used 4×3 bytes for term pointers without blocking . . .
- . . . we now use 3 bytes for one pointer plus 4 bytes for indicating the length of each term.
- We save $12 - (3 + 4) = 5$ bytes per block.
- Total savings: $400{,}000/4 * 5 = 0.5$ MB
- This reduces the size of the dictionary from 7.6 MB to 7.1 MB.

Lookup of a term without blocking



Lookup of a term with blocking: (slightly) slower



Front coding

One block in blocked compression ($k = 4$) ...

8 a u t o m a t a **8** a u t o m a t e **9** a u t o m a t i c **10** a u t o m a t i o n

$\Downarrow$

...further compressed with front coding.

8 a u t o m a t $*$ a **1** $\diamond$ e **2** $\diamond$ i c **3** $\diamond$ i o n

Dictionary compression for Reuters: Summary

data structure	size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
$\sim$, with blocking, $k = 4$	7.1
$\sim$, with blocking & front coding	5.9

Exercise

- Which prefixes should be used for front coding? What are the tradeoffs?
- Input: list of terms (= the term vocabulary)
- Output: list of prefixes that will be used in front coding

Postings compression

- The postings file is much larger than the dictionary, factor of at least 10.
- Key desideratum: store each posting compactly
- A posting for our purposes is a docID.
- For Reuters (800,000 documents), we would use 32 bits per docID when using 4-byte integers.
- Alternatively, we can use $\log_2 800{,}000 \approx 19.6 < 20$ bits per docID.
- Our goal: use a lot less than 20 bits per docID.

Key idea: Store gaps instead of docIDs

- Each postings list is ordered in increasing order of docID.
- Example postings list: COMPUTER: 283154, 283159, 283202, . . .
- It suffices to store gaps: $283159 - 283154 = 5$, $283202 - 283159 = 43$
- Example postings list using gaps: COMPUTER: 283154, 5, 43, . . .
- Gaps for frequent terms are small.
- Thus: We can encode small gaps with fewer than 20 bits.

Gap encoding

	encoding	postings list									
THE	docIDs	...		283042		283043		283044		283045	...
	gaps				1		1		1		...
COMPUTER	docIDs	...		283047		283154		283159		283202	...
	gaps				107		5		43		...
ARACHNOCENTRIC	docIDs	252000		500100							
	gaps	252000	248100								

Variable length encoding

- Aim:
  - For ARACHNOCENTRIC and other rare terms, we will use about 20 bits per gap (= posting).
  - For THE and other very frequent terms, we will use only a few bits per gap (= posting).
- In order to implement this, we need to devise some form of variable length encoding.
- Variable length encoding uses few bits for small gaps and many bits for large gaps.

Variable byte (VB) code

- Used by many commercial/research systems
- Good low-tech blend of variable-length coding and sensitivity to alignment matches (bit-level codes, see later).
- Dedicate 1 bit (high bit) to be a continuation bit c.
- If the gap G fits within 7 bits, binary-encode it in the 7 available bits and set $c = 1$.
- Else: encode lower-order 7 bits and then use one or more additional bytes to encode the higher order bits using the same algorithm.
- At the end set the continuation bit of the last byte to 1 ($c = 1$) and of the other bytes to 0 ($c = 0$).

VB code examples

docIDs	824	829	215406
gaps		5	214577
VB code	00000110 10111000	10000101	00001101 00001100 10110001

VB code encoding algorithm

```
VBEncodeNumber(n)
bytes ← ⟨⟩
while true
do Prepend(bytes, n mod 128)
    if n < 128
      then Break
    n ← n div 128
bytes[Length(bytes)] += 128
return bytes
```

```
VBEncode(numbers)
bytestream ← ⟨⟩
for each n ∈ numbers
do bytes ← VBEncodeNumber(n)
    bytestream ← Extend(bytestream, bytes)
return bytestream
```

VB code decoding algorithm

```
VBDecode(bytestream)
numbers ← ⟨⟩
n ← 0
for i ← 1 to Length(bytestream)
do if bytestream[i] < 128
      then n ← 128 × n + bytestream[i]
      else n ← 128 × n + (bytestream[i] − 128)
           Append(numbers, n)
           n ← 0
return numbers
```

Other variable codes

- Instead of bytes, we can also use a different “unit of alignment”: 32 bits (words), 16 bits, 4 bits (nibbles), etc.
- Variable byte alignment wastes space if you have many small gaps – nibbles do better on those.
- There is work on word-aligned codes that efficiently “pack” a variable number of gaps into one word – see resources at the end

Codes for gap encoding

- You can get even more compression with another type of variable length encoding: bitlevel code.
- Gamma code is the best known of these.
- First, we need unary code to be able to introduce gamma code.
- Unary code
  - Represent *n* as *n* 1s with a final 0.
  - Unary code for 3 is 1110
  - Unary code for 1 is 10, for 0 is 0, for 30 is 1111111111111111111111111111110

Gamma code

- Represent a gap G as a pair of length and offset.
- Offset is the gap in binary, with the leading bit chopped off.
- For example $13 \rightarrow 1101 \rightarrow 101 =$ offset
- Length is the length of offset.
- For 13 (offset 101), this is 3.
- Encode length in unary code: 1110.
- Gamma code of 13 is the concatenation of length and offset: 1110101.

Another Gamma code (γ) examples

number	unary code	length	offset	γ code
0	0			
1	10	0		0
2	110	10	0	10,0
3	1110	10	1	10,1
4	11110	110	00	110,00
9	1111111110	1110	001	1110,001
13		1110	101	1110,101
24		11110	1000	11110,1000
511		111111110	11111111	111111110,11111111
1025		11111111110	0000000001	11111111110,0000000001

The universal coding of the integers: Elias codes

- unary code $\alpha(N) = \underbrace{11\ldots1}_{N}0$. $\alpha(4) = 11110$
- binary code $\beta(1) = 1, \beta(2N + j) = \beta(N)j$, $j = 0, 1$. $\beta(4) = 100$
- β is not uniquely decodable (it is not a prefix code).
- ternary $\tau(N) = \beta(N)\#$. $\tau(4) = 100\#$
- $\beta'(1) = \epsilon$, $\beta'(2N) = \beta'(N)0$, $\beta'(2N + 1) = \beta'(N)1$, $\tau'(N) = \beta'(N)\#$. $\beta'(4) = 00$.
- $\gamma(N) = \alpha|\beta'(N)|\beta'(N)$. $\gamma(4) = 11000$
- alternatively, γ': every bit $\beta'(N)$ is inserted between a pair from $\alpha(|\beta'(N)|)$. the same length as γ (bit permutation $\gamma(N)$), but less readable
- example: $\gamma'(4) = 1\bar{0}1\bar{0}0$
- $C_\gamma = \{\gamma(N) : N > 0\} = (1\{0, 1\})^*0$ is regular and therefore it is decodable by finite automaton.

Elias codes: gamma, delta, omega: formal definitions II

- $\delta(N) = \gamma(|\beta(N)|)\beta'(N)$
- example: $\delta(4) = \gamma(3)00 = 01100$
- decoder δ: $\delta(?) = 1001?$
- ω:

  $K := 0;$

  while $\lfloor \log_2(N) \rfloor > 0$ do

  begin $K := \beta(N)K;$

  $N := \lfloor \log_2(N) \rfloor$

  end.

Exercise

- Compute the variable byte code of 130
- Compute the gamma code of 130
- Compute $\delta(42)$

Length of gamma code

- The length of *offset* is $\lfloor \log_2 G \rfloor$ bits.
- The length of *length* is $\lfloor \log_2 G \rfloor + 1$ bits,
- So the length of the entire code is $2 \times \lfloor \log_2 G \rfloor + 1$ bits.
- γ codes are always of odd length.
- Gamma codes are within a factor of 2 of the optimal encoding length $\log_2 G$.
    - (assuming the frequency of a gap G is proportional to $\log_2 G$ – only approximately true)

Gamma code: Properties

- Gamma code is prefix-free: a valid code word is not a prefix of any other valid code.
- Encoding is optimal within a factor of 3 (and within a factor of 2 making additional assumptions).
- This result is independent of the distribution of gaps!
- We can use gamma codes for any distribution. Gamma code is universal.
- Gamma code is parameter-free.

Gamma codes: Alignment

- Machines have word boundaries – 8, 16, 32 bits
- Compressing and manipulating at granularity of bits can be slow.
- Variable byte encoding is aligned and thus potentially more efficient.
- Another word aligned scheme: Anh and Moffat 2005
- Regardless of efficiency, variable byte is conceptually simpler at little additional space cost.

Compression of Reuters

data structure	size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
$\sim$, with blocking, $k = 4$	7.1
$\sim$, with blocking & front coding	5.9
collection (text, xml markup, etc.)	3600.0
collection (text)	960.0
T/D incidence matrix	40,000.0
postings, uncompressed (32-bit words)	400.0
postings, uncompressed (20 bits)	250.0
postings, variable byte encoded	116.0
postings, γ encoded	101.0

Term-document incidence matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
ANTHONY	1	1	0	0	0	1	
BRUTUS	1	1	0	1	0	0	
CAESAR	1	1	0	1	1	1	
CALPURNIA	0	1	0	0	0	0	
CLEOPATRA	1	0	0	0	0	0	
MERCY	1	0	1	1	1	1	
WORSER	1	0	1	1	1	0	

...

Entry is 1 if term occurs.

Example: CALPURNIA occurs in *Julius Caesar*.

Entry is 0 if term does not occur.

Example: CALPURNIA doesn't occur in *The tempest*.

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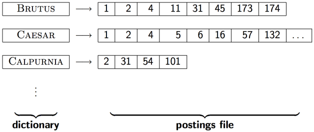
Summary

- We can now create an index for highly efficient Boolean retrieval that is very space efficient.
- Only 4% of the total size of the collection.
- Only 10–15% of the total size of the text in the collection.
- However, we've ignored positional and frequency information.
- For this reason, space savings are less in reality.

Take-away today

For each term t, we store a list of all documents that contain t.



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- How can we compress the dictionary component of the inverted index?
- How can we compress the postings component of the inverted index?
- Term statistics: how are terms distributed in document collections?

Resources

`http://ske.fi.muni.cz`

- Chapter 5 of IIR
- Resources at `https://www.fi.muni.cz/~sojka/PV211/` and `http://cislmu.org`, materials in MU IS and FI MU library
  - Original publication on word-aligned binary codes by Anh and Moffat (2005); also: Anh and Moffat (2006a).
  - Original publication on variable byte codes by Scholer, Williams, Yiannis and Zobel (2002).
  - More details on compression (including compression of positions and frequencies) in Zobel and Moffat (2006).