Exercise 1

Let A be a propositional variable. Decide (and prove) whether there is a formula φ such that

1. $\varphi \to (\varphi \to A) \approx A$
2. $\varphi \to (\varphi \to A) \approx \neg A$
3. $\varphi \to (A \to \varphi) \approx A$
4. $\varphi \to (A \to \varphi) \approx \neg A$

Exercise 2

Given a formula $\psi \approx A \to \Big(B \to \big(C \to (D \to E)\big)\Big)$, find an equivalent formula φ of the system $\mathcal{L}(|)$. (we define $|$ by $\xi_1 \mid \xi_2 \approx \neg(\xi_1 \wedge \xi_2)$ for all ξ_1, ξ_2)

Exercise 3

Prove that the system $\mathcal{L}(\neg)$ is not functionally complete (i.e. "plnohodnotný").

Exercise 4

Decide and prove whether the system $\mathcal{L}(\to)$ is functionally complete.

Exercise 5

Decide and prove whether the system $\mathcal{L}(\leftrightarrow, \neg)$ is functionally complete.

Exercise 6

Let A, B be countable family and let $R \subseteq A \times B$ be a relation such that for all $a \in A$ the set $B_a = \{b \mid (a, b) \in R\}$ is a finite nonempty family. Find a family of propositional formulae T such that T is satisfiable if and only if there is an injective function $f : A \to B$ such that $f \subseteq R$.

Exercise 7

Give a polynomial time algorithm that to any finite directed graph $G = (V, E)$ (i.e. $E \subseteq V \times V$, see `http://en.wikipedia.org/wiki/Graph_(mathematics)`) for detailed information) returns a formula φ_G such that φ_G is satisfiable if and only if G is strongly connected (see `http://en.wikipedia.org/wiki/Strongly_connected_component`). It suffices to describe the formula φ_G and argue why it can be constructed in time polynomial in the size of G.

(Note: The algorithm itself *must not* compute whether the graph is strongly connected. In particular, the solution "decide whether G is strongly connected and return A if yes and $A \wedge \neg A$ otherwise" does not count)

Exercise 8

Give a polynomial time algorithm that to any finite directed graph $G = (V, E)$ returns a formula φ_G such that φ_G is satisfiable if and only if G contains a Hamiltonian cycle (`http://en.wikipedia.org/wiki/Hamiltonian_path`).

It suffices to describe the formula φ_G and argue why it can be constructed in time polynomial in the size of G.