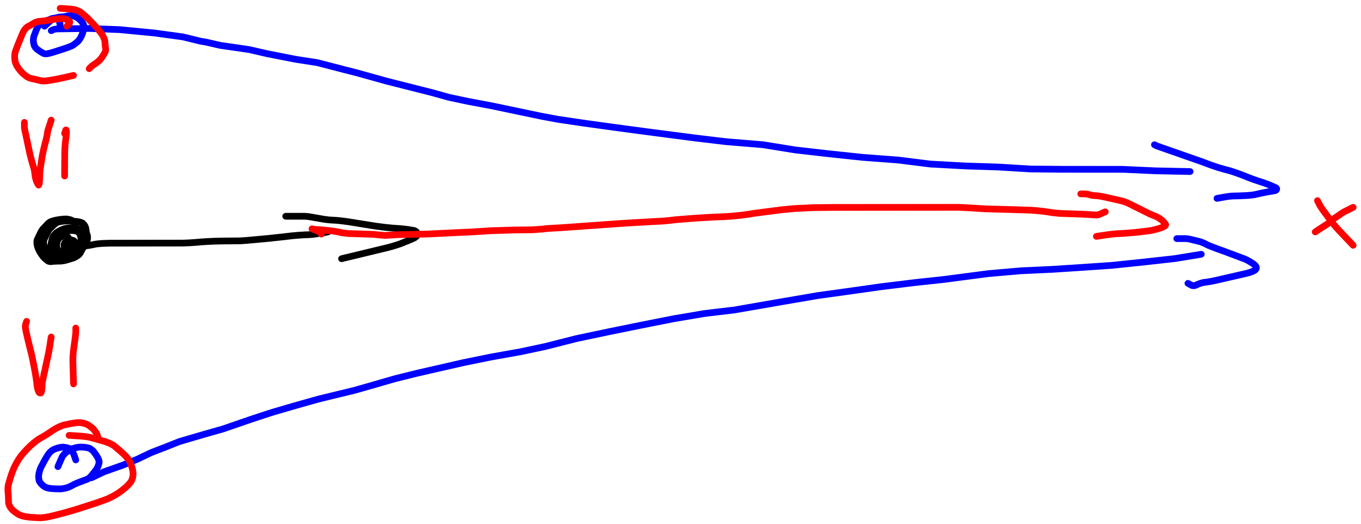
V1
V1
X

$$\lim_{n\to\infty} \sqrt[n]{2} = ?$$

$$\sqrt[n]{2} = 1 + a_n \quad \Big/^{\,n}, \quad a_n \geq 0$$

$$\underbrace{2 = (1+a_n)^n = \binom{n}{0}\cdot 1^n + \binom{n}{1}\cdot 1^{n-1}\cdot a_n +}$$

$$\underbrace{+ \binom{n}{2}\cdot 1^{n-2}\cdot a_n^2 + \ldots}_{\text{KLADNÉ}}$$

$$2 \geq \binom{n}{0}\cdot 1^n + \binom{n}{1}\cdot a_n$$

$$2 \geq 1 + n\cdot a_n$$

$a_n \leq \dfrac{1}{n}$ & víme $a_n \geq 0$

$$0 \leq a_n \leq \frac{1}{n}$$

$$0$$

$\Rightarrow a_n \to 0$

$$\lim_{n\to\infty} \sqrt[n]{2} = \lim_{n\to\infty} (1 + a_n) =$$

$$= \lim_{n\to\infty} 1 + \lim_{n\to\infty} a_n = 1 + 0 = \underline{\underline{1}}$$

Př. 2.2/ $\lim\limits_{n \to \infty} \sqrt[n]{n} = ?$

$$\sqrt[n]{n} = 1 + a_n \quad /^{n} \qquad , \quad a_n \geq 0$$

$$n = (1 + a_n)^n =$$

$$= \binom{n}{0} \cdot 1^n \cdot a_n^0 + \binom{n}{1} \cdot 1^{n-1} \cdot a_n^1 + \underbrace{\binom{n}{2} \cdot 1^{n-2} \cdot a_n^2} + \ldots$$

$$n \geq \binom{n}{2} \cdot 1^{n-2} \cdot a_n^2$$

$$n \geq \frac{n \cdot (n-1)}{2 \cdot 1} \cdot a_n^2$$

$$1 \geq \frac{n-1}{2} \cdot a_n^2$$

$$0 \leq a_n^2 \leq \frac{2}{n-1}$$

$$0 \quad \Rightarrow \quad a_n \to 0$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1 + 0 = \underline{\underline{1}}.$$

2.3 (i)

$$\lim_{n\to\infty} \frac{2n^3+3n^2+2}{4n^3-n} = \left[\frac{\infty}{\infty}\right] \Big/ \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} =$$

$$= \lim_{n\to\infty} \frac{\underset{\to 2}{2} + \underset{\to 0}{\frac{3}{n}} + \underset{\to 0}{\frac{2}{n^3}}}{\underset{\to 4}{4} - \underset{\to 0}{\frac{1}{n^2}}} = \frac{2}{4} = \frac{1}{2}$$

(ii) $\lim\limits_{n \to \infty} \left(\sqrt{n^2+n} - n \right) = [\infty - \infty] =$

$/ \cdot \dfrac{\sqrt{n^2+n} + n}{\sqrt{n^2+n} + n} = \lim\limits_{n \to \infty} \dfrac{\cancel{n^2} + n - \cancel{n^2}}{\sqrt{n^2+n} + n} =$

$= \lim\limits_{n \to \infty} \dfrac{n}{\sqrt{n^2+n} + n} = \left[\dfrac{\infty}{\infty} \right] / \cdot \dfrac{\frac{1}{n}}{\frac{1}{n}} =$

$= \lim\limits_{n \to \infty} \dfrac{1}{\sqrt{1 + \frac{1}{n}} + 1} = \left(\dfrac{1}{\sqrt{1+0} + 1} \right) = \underline{\underline{\dfrac{1}{2}}}$

(iii)

$$\lim_{n\to\infty} \frac{3^n + n^5 - 4n}{2^n + 3^n + n^2} = \left[\frac{\infty}{\infty}\right] \Big/ \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} =$$

$$= \lim_{n\to\infty} \frac{\overset{\to 1}{1} + \frac{n^5}{3^n}\to 0 - \frac{4n}{3^n}\to 0}{\underset{\to 0}{\left(\frac{2}{3}\right)^n} + \underset{\to 1}{1} + \underset{\to 0}{\frac{n^2}{3^n}}} = \underline{\underline{1}}$$

2. 5(i)

$$\lim_{x\to 0} \frac{\sin 5x}{\sin 7x} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0} \frac{\sin 5x}{\sin 7x}\cdot\frac{5x}{5x}\cdot\frac{7x}{7x} =$$

$$= \lim_{x\to 0}\left(\frac{\sin 5x}{5x}\cdot\frac{7x}{\sin 7x}\cdot\frac{5x}{7x}\right) =$$

$$= \frac{5}{7}\cdot\lim_{x\to 0}\left(\frac{\sin 5x}{5x}\cdot\frac{1}{\frac{\sin 7x}{7x}}\right) =$$

$$= \frac{5}{7}\cdot 1\cdot 1 = \underline{\underline{\frac{5}{7}}}$$

(ii) $$\lim_{x\to\infty} \frac{\pi x + \sin x}{2x + \cos x} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to\infty} \frac{\pi + \frac{\sin x}{x}}{2 + \frac{\cos x}{x}} = \frac{\pi}{2}$$

$$\frac{\text{OHR.}}{\infty} \to 0$$

(iii) $\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin 2x} =$

$$= \lim_{x \to 0} \left(\frac{e^x - e^{-x}}{x} \cdot \frac{x}{\sin 2x} \right) = ⊛ = 2 \cdot \frac{1}{2} = \underline{\underline{1}}$$

⊛ $\lim_{x \to 0} \frac{x}{\sin 2x} = \lim_{x \to 0} \frac{1}{\frac{2 \sin 2x}{2x}} =$

$$= \lim_{x \to 0} \left(\frac{1}{\frac{\sin 2x}{2x}} \cdot \frac{1}{2} \right) = \underline{\underline{\frac{1}{2}}}$$

(*)

$$\lim_{x\to 0}\frac{e^x-e^{-x}}{x}=\lim_{x\to 0}\frac{e^x-1-e^{-x}+1}{x}=$$

$$=\lim_{x\to 0}\left(\frac{e^x-1}{x}+\frac{e^{-x}-1}{-x}\right)=1+1=\underline{\underline{2}}$$

(iv)
$$\lim_{x \to 2} \frac{x^2+x-6}{x^2-3x+2} = \left[\frac{4+2-6}{4-6+2} = \frac{0}{0} \right] =$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)} \cdot (x+3)}{\cancel{(x-2)} \cdot (x-1)} = \left[\frac{2+3}{2-1} = \frac{5}{1} \right] = \underline{\underline{5}}$$

(w) $\lim\limits_{x \to 2} \dfrac{x^2}{x^2 - 3x + 2} = \left[\dfrac{4}{0} \right] =$

$= \lim\limits_{x \to 2} \dfrac{x^2}{(x-2)\cdot(x-1)} = ?$

$\lim\limits_{x \to 2^+} \dfrac{x^2}{\underline{\underline{(x-2)}}\cdot(x-1)} = \left[\dfrac{4}{0^+ \cdot 1} \right] = \underline{\underline{+\infty}}$

$\neq$

$\lim\limits_{x \to 2^-} \dfrac{x^2}{\underline{\underline{(x-2)}}\cdot(x-1)} = \left[\dfrac{4}{0^- \cdot 1} \right] = \underline{\underline{-\infty}}$

LIM. NEEX.

(vi) $$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = \left[\frac{0}{0}\right] =$$

$$/ \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$$

$$= \lim_{x \to 0} \frac{1+x-(1-x)}{x \cdot \left(\sqrt{1+x} + \sqrt{1-x}\right)} =$$

$$= \lim_{x \to 0} \frac{2\cancel{x}}{\cancel{x} \cdot (\sim)} = \lim_{x \to 0} \frac{2}{\sqrt{1+x} + \sqrt{1+x}} = \left[\frac{2}{1+1}\right]$$

$$= \underline{\underline{1}}$$

2.6/ $f(x) = \dfrac{\ln(2x^3+4x^2-x)}{1+x}$, $[1, f(1)]$

$$f(1) = \frac{\ln(2+4-1)}{2} = \frac{\ln 5}{2}$$

$$\Rightarrow \left[1, \frac{\ln 5}{2}\right]$$

$t: y - y_0 = f'(x_0)\cdot(x - x_0)$ $[x_0, y_0]$

$n: y - y_0 = -\dfrac{1}{f'(x_0)}\cdot(x - x_0)$ $y_0 = f(x_0)$

$$f'_{(x)} = \frac{\frac{1}{2x^3+4x^2-x}\cdot(6x^2+8x-1)\cdot(1+x) - \ln(2x^3+4x^2-x)\cdot 1}{(1+x)^2} =$$

$$= \frac{6x^2+8x-1}{(2x^3+4x^2-x)\cdot(1+x)} - \frac{\ln(2x^3+4x^2-x)}{(1+x)^2}$$

$$f'_{(1)} = \frac{13}{10} - \frac{\ln 5}{4}$$

$\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2}$	$(f\cdot g)' = f'\cdot g + f g'$	$(\ln x)' = \frac{1}{x}$
$(x^n)' = n\cdot x^{n-1}$	$f'(g_{(x)}) = f'_{(g_{(x)})}\cdot g'_{(x)}$	$(k\cdot f)' = k\cdot f'$

$$t: \quad y - \frac{\ln 5}{2} = \left(\frac{13}{10} - \frac{\ln 5}{4}\right) \cdot (x-1)$$

$$n: \quad y - \frac{\ln 5}{2} = -\frac{1}{\frac{13}{10} - \frac{\ln 5}{4}} \cdot (x-1)$$

$$y - \frac{\ln 5}{2} = \frac{5\ln 5 - 26}{20} \cdot (x-1)$$