

$\mathbb{Z}_2 = \{0, 1\}$

| |
|-----------|
| $0+1 = 1$ |
| $1+1 = 0$ |
| $0+0 = 0$ |

$\mathbb{Z}_3 = \{0, 1, 2\}$

| | | | |
|---|---|---|---|
| - | 0 | 1 | 2 |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 |
| 2 | 0 | 2 | 1 |

| | | | |
|---|---|---|---|
| + | 0 | 1 | 2 |
| 0 | 0 | 1 | 2 |
| 1 | 1 | 2 | 0 |
| 2 | 2 | 0 | 1 |

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| | | | | |
|---|---|---|---|---|
| . | 0 | 1 | 2 | 3 |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 |
| 2 | 0 | 2 | 0 | 2 |
| 3 | 0 | 3 | 2 | 1 |

$a \cdot b = 0 \quad a \neq 0, b \neq 0$

$\underbrace{a^{-1}}_1 \cdot a \cdot b = b \Rightarrow b = 0$

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funktion:

$f(x) = x!$

| | | | | | | |
|--------|---|---|---|---|----|-----|
| $f(x)$ | 0 | 1 | 2 | 3 | 4 | 5 |
| | 1 | 1 | 2 | 6 | 24 | 120 |

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$n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1 = n!$

$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

$(a+b)^n = \underbrace{(a+b) \cdot (a+b) \cdot \dots \cdot (a+b)}_{n \text{ mal}}$

$= \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$

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$\mathbb{Q}(\mathbb{Z})$: rekursiv:

$\binom{n}{k} + \binom{n}{k+1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!}$

$= \frac{(k+1)n! + (n-k)n!}{(k+1)!(n-k)!} = \frac{(n+1)!}{(k+1)!(n-k)!} = \binom{n+1}{k+1}$

$\binom{0}{0} = \frac{0!}{0! \cdot 0!} = 1$

$\binom{n}{k} := 0 \quad \begin{matrix} k < 0 \\ k > n \end{matrix}$

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$\sum_{k=0}^n \binom{n}{k} = 2^n \quad \checkmark$

$\sum_{k=0}^{n+1} \binom{n+1}{k} = \sum_{k=0}^n \left(\binom{n}{k-1} + \binom{n}{k} \right)$

$= \sum_{k=1}^n \binom{n}{k-1} + \sum_{k=0}^n \binom{n}{k} = 2^n + 2^n = 2^{n+1} \quad \checkmark$

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$$F(n, f(n)) = (n+1) \cdot f(n)$$

$$\text{recl } \leftarrow f(n) = n!$$

$$f(n+1) = \left(\frac{a}{12}\right) \cdot f(n) + b$$

↑
multiplier
const

↑
offset

$$f(n) = a \cdot f(n-1) + b$$

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