Příklad 41. *Pomocí vhodné transformace souřadnic vypočtěte integrál $\iint_A \sqrt{xy}\,dx\,dy$, kde množina A je ohraničena křivkami*

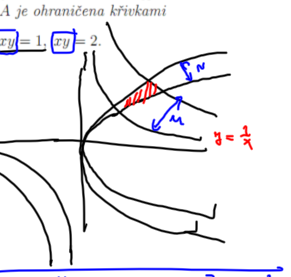
$$y^2 = 2x,\ y^2 = x,\ \boxed{xy} = 1,\ \boxed{xy} = 2.$$

$\frac{y^2}{x} = 2 \quad \frac{y^2}{x} = 1$

F^{-1}: $u = xy$, $v = \frac{y^2}{x}$

$1 \le u \le 2$

$1 \le v \le 2$



$$\iint_A \sqrt{u}\,\frac{1}{3v}\,du\,dv =$$

$$= \int_1^2\int_1^2 \frac{\sqrt{u}}{3v}\,du\,dv =$$

$$= \int_1^2 \frac{1}{3v}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^2 dv =$$

$$DG = \begin{vmatrix} y & x \\ -\frac{y^2}{x^2} & \frac{2y}{x} \end{vmatrix} = \frac{2y^2}{x} + \frac{y^2}{x} = 3\frac{y^2}{x}$$

$$DF = (DG)^{-1} = \frac{x}{3y^2} = \frac{1}{3v}$$

$$= \int_1^2 \frac{1}{3v}\cdot\frac{2}{3}(2\sqrt{2}-1)\,dv =$$

$$= \frac{2}{3}(2\sqrt{2}-1)\cdot\frac{1}{3}\left[\ln|v|\right]_1^2 = \frac{2}{9}(2\sqrt{2}-1)\ln 2.$$

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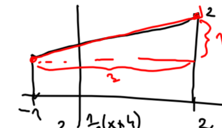
Příklad 42. *Vypočtěte*

1. *těžiště,*
2. *momenty setrvačnosti vzhledem k souřadným osám*

tenké homogenní rovinné lichoběžníkové desky s vrcholy v bodech $[-1, 0]$, $[2, 0]$, $[2, 2]$ *a* $[-1, 1]$.

Řešení. *Hustota* $\rho(x, y) = 1$ *pro libovolná* x, y. *Integrujeme přes množinu* $A: -1 \le x \le 2, 0 \le y \le \frac{1}{3}(x + 4)$.

Dostaneme $M = \iint_A dA = \frac{9}{2}$, $T_x = \frac{1}{M}\iint_A x\cdot 1\,dA = \frac{2}{3}$, $T_y = \frac{1}{M}\iint_A y\cdot 1\,dA = \frac{7}{9}$, $J_x(A) = \iint_A y^2\,dA = \frac{15}{4}$, $J_y(A) = \iint_A x^2\,dA = \frac{21}{4}$.



$y = \frac{1}{3}\cdot x + \frac{4}{3}$

$[T_x, T_y]$

$T_x = \frac{1}{M}\cdot\iint_A x\cdot\rho(x,y)\,dA$

$T_y = \frac{1}{M}\iint_A y\cdot\rho(x,y)\,dA$

$$M = \int_{-1}^{2}\int_0^{\frac{1}{3}(x+4)} 1\,dy\,dx = \int_{-1}^{2}\frac{1}{3}(x+4)\,dx =$$

$$= \frac{1}{3}\left[\frac{x^2}{2} + 4x\right]_{-1}^{2} = \frac{1}{3}\left(10 - \left(\frac{1}{2} - 4\right)\right) = \frac{9}{2}$$

$T_x = \ldots = \frac{6}{9} = \frac{2}{3}$

$T_y = \frac{7}{9}$

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$$J_x = \iint_A y^2\,dx\,dy = \int_{-1}^{2}\int_0^{\frac{1}{3}(x+4)} y^2\,dy\,dx =$$

$$= \frac{1}{3}\int_{-1}^{2}\left[y^3\right]_0^{\frac{1}{3}(x+4)}dx = \frac{1}{3}\cdot\int_{-1}^{2}\frac{1}{27}(x+4)^3\,dx =$$

$$= \frac{1}{81}\left[\frac{(x+4)^4}{4}\right]_{-1}^{2} = \frac{1}{81}\left(\frac{6^4}{4} - \frac{3^4}{4}\right) =$$

$$= \frac{1}{\cancel{81}}\left(4\cdot\cancel{3^4} - \frac{\cancel{3^4}}{4}\right) = \frac{15}{4}$$

$$J_y = \iint_A x^2\,dx\,dy = \ldots = \frac{21}{4}.$$

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Příklad 43. *Hodnotu integrálu*

$$I = \int_0^1 \frac{1}{1+x^2}\,dx = \left[\operatorname{arctg} x\right]_0^1 = \frac{\pi}{4} - 0 \doteq 0{,}785398$$

odhadněte pomocí

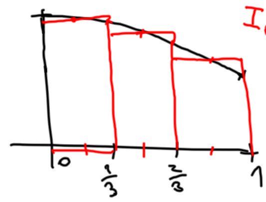
(a) *obdélníkového pravidla,*

(b) *lichoběžníkového pravidla*

(c) *Simpsonova pravidla,*

přičemž zadaný interval rozdělte na 3 intervaly téže délky. Dosažené výsledky porovnejte s přesnou hodnotou a pomocí obrázku zdůvodněte míru nepřesnosti jednotlivých pravidel.

(a)



$$I_o = f\left(\tfrac{1}{6}\right)\cdot\tfrac{1}{3} + f\left(\tfrac{1}{2}\right)\cdot\tfrac{1}{3} + f\left(\tfrac{5}{6}\right)\cdot\tfrac{1}{3} =$$

$$= \frac{1}{3}\left(f\left(\tfrac{1}{6}\right) + f\left(\tfrac{1}{2}\right) + f\left(\tfrac{5}{6}\right)\right)$$

$$= \frac{1}{3}\left(\frac{36}{37} + \frac{4}{5} + \frac{36}{61}\right) \doteq$$

$$\doteq 0{,}787712$$

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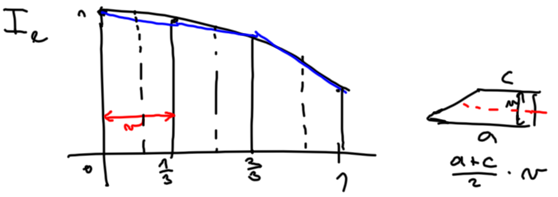
I_l

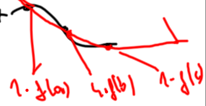


$$I_l = \frac{1}{6}\left(f(0) + 2f\left(\tfrac{1}{3}\right) + 2f\left(\tfrac{2}{3}\right) + f(1)\right) =$$

$$= \frac{1}{6}\left(1 + 2\cdot\frac{9}{10} + 2\cdot\frac{9}{13} + \frac{1}{2}\right) \doteq 0{,}780769$$

$$c)\ I_s = \frac{1}{3}\cdot\frac{1}{6}\left(f(0) + 4\cdot f\left(\tfrac{1}{6}\right) + 2\cdot f\left(\tfrac{1}{3}\right) + 4f\left(\tfrac{1}{2}\right) + 2f\left(\tfrac{2}{3}\right) + 4\cdot f\left(\tfrac{5}{6}\right) + f(1)\right) =$$

$$= \frac{1}{18}\left(1 + 4\cdot\frac{36}{37} + 2\cdot\frac{9}{10} + 4\cdot\frac{4}{5} + 2\cdot\frac{9}{13} + 4\cdot\frac{36}{61} + \frac{1}{2}\right) \doteq 0{,}7853979$$



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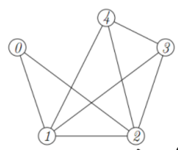
Příklad 44. *Pomocí matice sousednosti určete počet sledů délky 4 z vrcholu 0 do vrcholu 1 v následujícím grafu:*

G:



$$A_G\cdot A_G = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 & 1 & 1 & 2 & 2 \\ 1 & 4 & 3 & 2 & 2 \\ 1 & 3 & 4 & 2 & 2 \\ 2 & 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 2 & 3 \end{pmatrix}$$

$$(A^4)_{0,1} = 2\cdot 1 + 1\cdot 4 + 1\cdot 3 + 2\cdot 2 + 2\cdot 2 = 17$$

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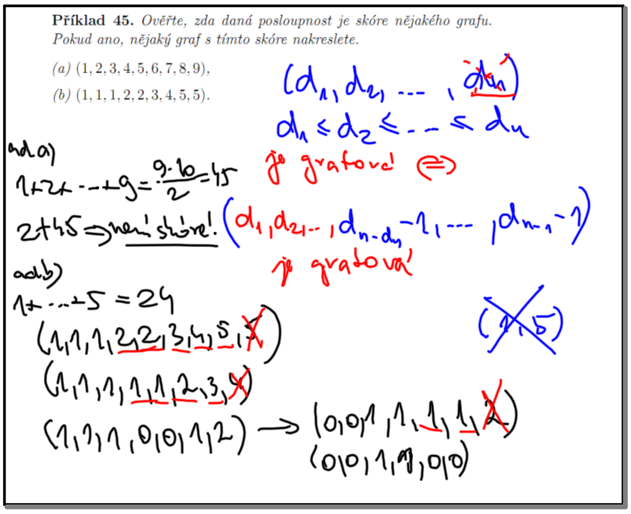

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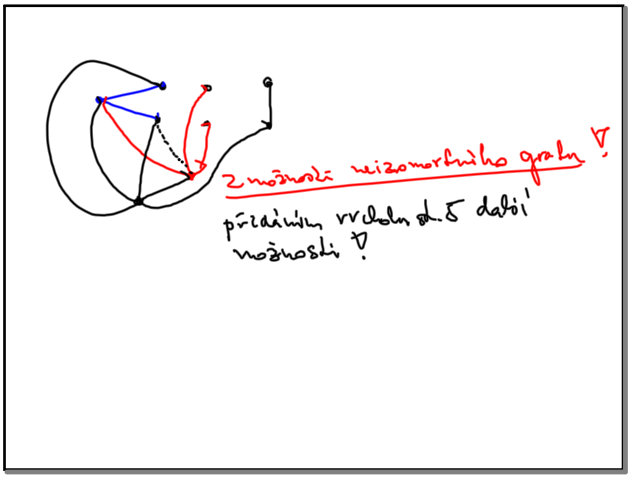

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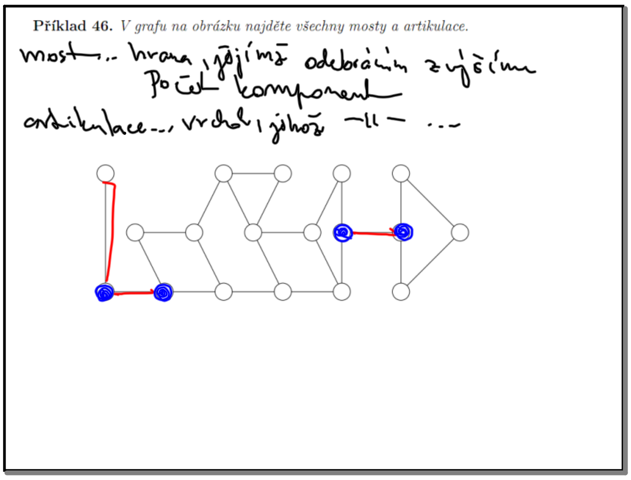

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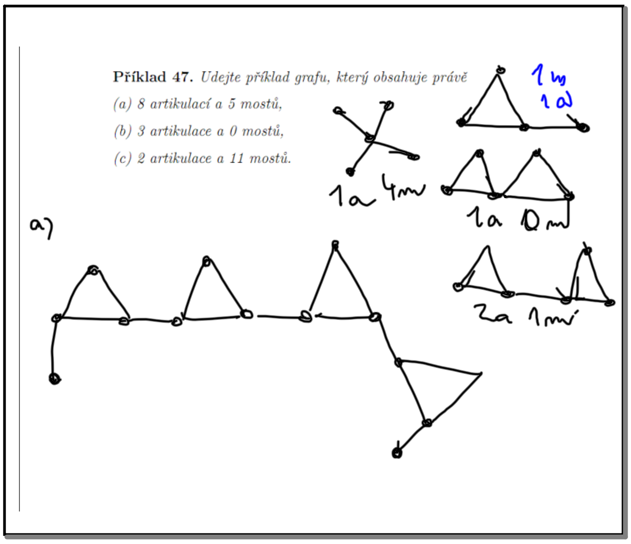

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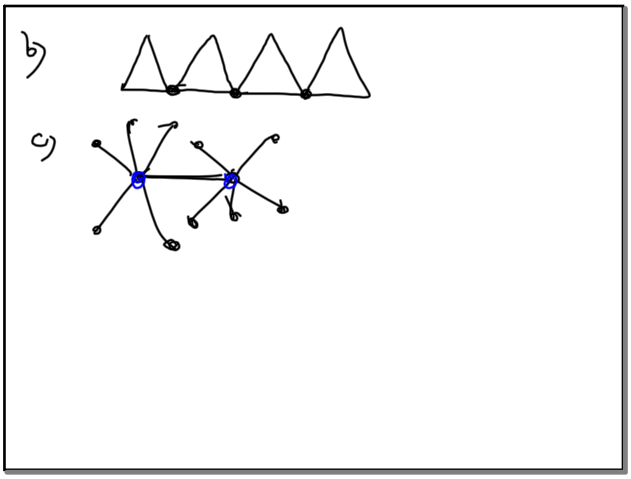

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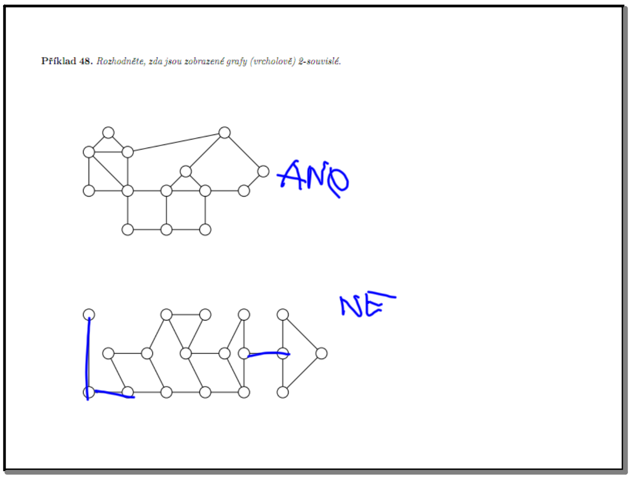

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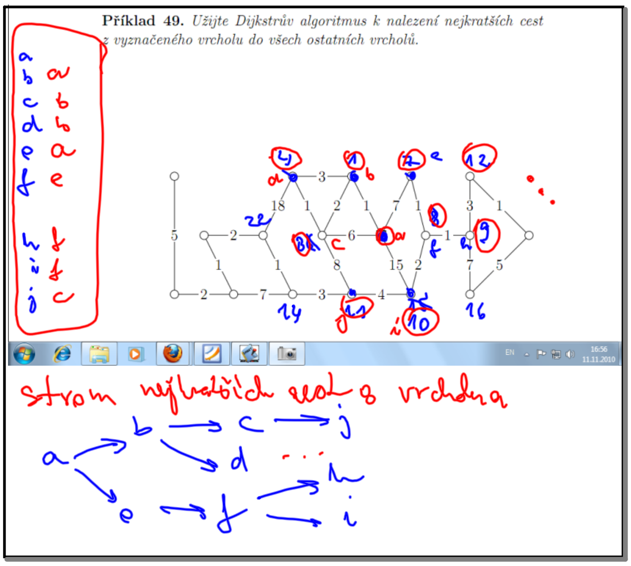

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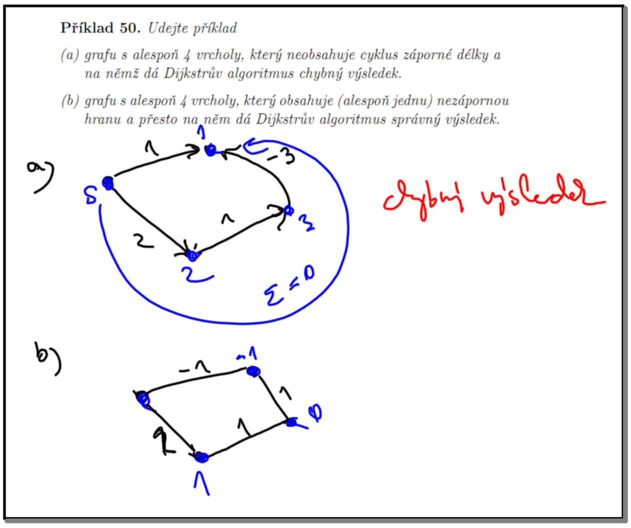

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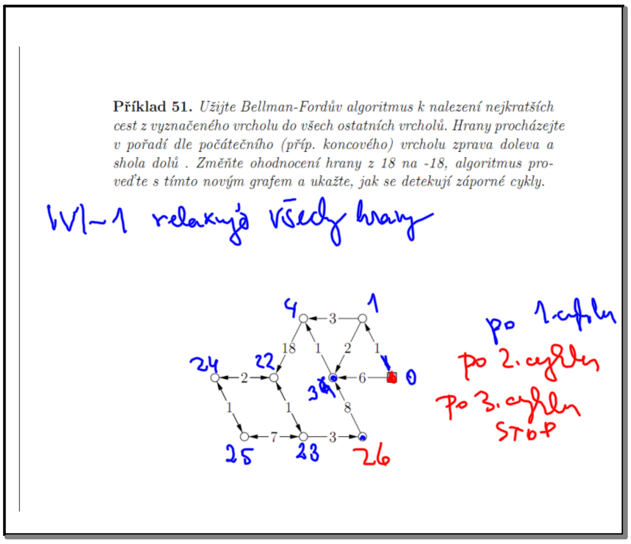

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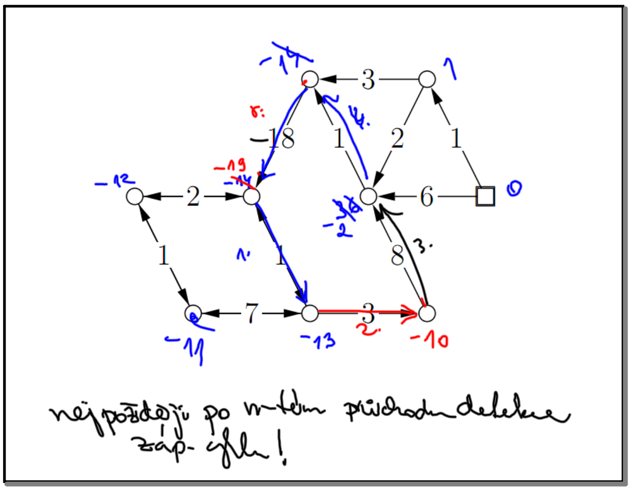

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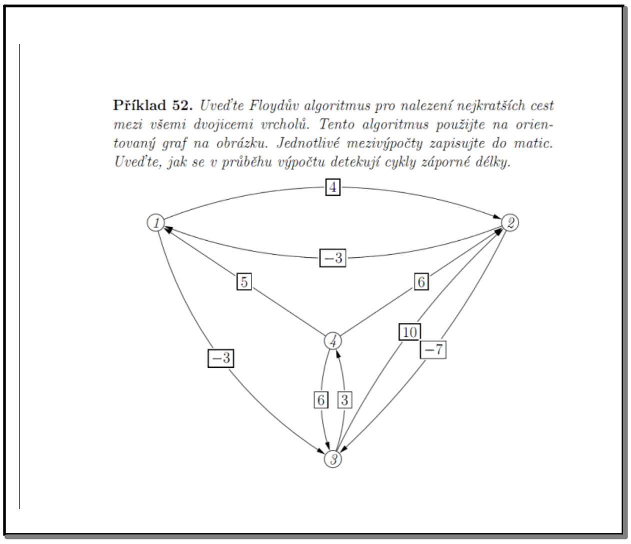

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