12_VektoroveProstory2

1) determinant je roven –1 (nebo 1, podle pořadí vektorů), což je nenulové číslo, proto jsou vektory LN a tvoří bázi
2) M=Span$\langle -1+x, -1+x^2\rangle$
3) Ukáže se linearita, injektivita a surjektivita
4) a) Ne
   b) Ne
   c) Ano
   d) Ano
5) $f(o_U) = f(u-u) = f(u) - f(u) = o_V$
6) a) $\mathrm{Ker} f = \mathrm{Span}\langle 3-3x+2x^2\rangle, \mathrm{Im} f = \mathrm{Span}\langle 1, 1+2i\rangle = \mathbf{C}$
   b) $\mathrm{Ker} f = \mathrm{Span}\langle \emptyset\rangle = \left\{\begin{matrix}0\\0\end{matrix}\right\}, \mathrm{Im} f = \mathrm{Span}\left\langle \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}\right\rangle = \mathbf{R}^2$
   c) $\mathrm{Ker} f = \mathrm{Span}\left\langle \begin{pmatrix}0\\1\end{pmatrix}\right\rangle, \mathrm{Im} f = \mathrm{Span}\left\langle \begin{pmatrix}1\\0\end{pmatrix}\right\rangle$
   d) $\mathrm{Ker} f = \mathrm{Span}\left\langle \begin{pmatrix}-1 & 0\\2 & 0\end{pmatrix}, \begin{pmatrix}-3 & 2\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\right\rangle, \mathrm{Im} f = \mathrm{Span}\langle 2\rangle$
   e) $\mathrm{Ker} f = \mathrm{Span}\left\langle \begin{pmatrix}-2 & 2\\1 & 0\end{pmatrix}\right\rangle, \mathrm{Im} f = \mathrm{Span}\left\langle \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\rangle = \mathbf{R}^3$
   f) (paralela e)) $\mathrm{Ker} f = \mathrm{Span}\left\langle \begin{pmatrix}-2 & 2\\1 & 0\end{pmatrix}\right\rangle, \mathrm{Im} f = \mathrm{Span}\langle 1+x, 1, x^3\rangle = \mathrm{P}_2(x)$
7) 1) $f\left(\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\right) = \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad f\left(\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\right) = \begin{pmatrix}1\\-2\\1\end{pmatrix}, \quad f\left(\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\right) = \begin{pmatrix}0\\3\\0\end{pmatrix}, \quad f\left(\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right) = \begin{pmatrix}2\\3\\2\end{pmatrix}$

   2) $(f)_{\varepsilon_3\varepsilon_4} = \begin{pmatrix}0 & 1 & 0 & 2\\1 & -2 & 3 & 3\\0 & 1 & 0 & 2\end{pmatrix}$

   3) $f\left(\begin{pmatrix}a\\b\\c\\d\end{pmatrix}\right) = \begin{pmatrix}b+2d\\a-2b+3c+3d\\b+2d\end{pmatrix}$

   d) $(f)_{\alpha\varepsilon_4} = \begin{pmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{pmatrix}^{-1} \cdot \begin{pmatrix}0 & 1 & 0 & 2\\1 & -2 & 3 & 3\\0 & 1 & 0 & 2\end{pmatrix}$

   e) $\mathrm{Ker}\ f = \mathrm{Span}\left\langle \begin{pmatrix}-7\\-2\\0\\1\end{pmatrix}, \begin{pmatrix}-3\\0\\1\\0\end{pmatrix}\right\rangle, \quad \mathrm{Im}\ f = \mathrm{Span}\left\langle \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\-2\\1\end{pmatrix}\right\rangle$

8) a) $(f)_{\gamma\gamma} = \begin{pmatrix}6 & 10\\3 & 2\end{pmatrix}^{-1} \cdot \begin{pmatrix}1 & 1\\0 & 1\end{pmatrix} \cdot \begin{pmatrix}6 & 10\\3 & 2\end{pmatrix}$

$$(\mathrm{id})_{\varepsilon\alpha} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (\mathrm{id})_{\alpha\varepsilon} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{-1},$$

$$(\mathrm{id})_{\beta\alpha} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

b) $$(\mathrm{id})_{\alpha\beta} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

c) $(f)_{\beta\alpha} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ (v zadání chybí index na druhou: $3bx^2$)

9) $(\mathrm{id})_{\varepsilon\alpha} = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & -1 \\ 1 & 1 & 3 \end{pmatrix}^{-1}$, $\quad [2+3x+6x^2]_\alpha = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & -1 \\ 1 & 1 & 3 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$