Př. 4

$$\begin{vmatrix} 2 & -2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ -1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1.(-1)^{4+4} \begin{vmatrix} 2 & -2 & 0 \\ 0 & 3 & 0 \\ -1 & -2 & 3 \end{vmatrix} = 2 \cdot 3 \cdot 3 = \underline{\underline{18}}$$

$|A| \neq 0 \Rightarrow A$ <u>je regulární</u>, $|A^{-1}| = \frac{1}{|A|} = \underline{\underline{\frac{1}{18}}}$

Př. 5

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ -2 & 1 & 0 & 3 \\ 1 & 1 & 3 & 3 \\ 0 & 2 & 4 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{array}{l} x_4 = t \\ x_3 = s \\ x_2 = -3t - 2s \\ x_1 = -s \end{array}$$

$$\Rightarrow \operatorname{Ker} A = \underline{\underline{\left((-1,-2,1,0)^T, (0,-3,0,1)^T\right)}} \to \dim(\operatorname{Ker} A) = \underline{\underline{2}}$$

$$B^T \underbrace{Bx}_{v} = B^T w, \quad B = \begin{pmatrix} -1 & 0 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad w = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 4 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} -1 & -2 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\begin{pmatrix} 6 & 6 \\ 6 & 10 \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & -2 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 0 \\ 4 \end{pmatrix}}_{\begin{pmatrix} -6 \\ -2 \end{pmatrix}}$$

$$\to \left(\begin{array}{cc|c} 6 & 6 & -6 \\ 6 & 10 & -2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 3 & 5 & -1 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & 2 & 2 \end{array}\right) \to \begin{array}{l} x_2 = 1 \\ x_1 = -2 \end{array}$$

$$\Rightarrow v = Bx = \begin{pmatrix} -1 & 0 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 \\ 1 \\ -2 \\ 1 \end{pmatrix}}} \to \|v - w\| = \|(0,-1,-2,-3)^T\| = \sqrt{0+1+4+9} =$$

$$= \underline{\underline{\sqrt{14}}}$$

Př. 1

② ② ②

a) rozlišitelné koule i přihrádky $\rightarrow V(\overset{\text{přihrádky}}{n}, \overset{\text{předměty}}{k}) = V(2,3) = \underline{\underline{2^3}}$ ← pro každou kouli máme 2 různé možnosti

b) nerozlišitelné koule, rozliš. přihrádky $\rightarrow K(n,k) = K\binom{n+k-1}{k} = \binom{5-1}{3} = \binom{4}{1} = \underline{\underline{4}}$

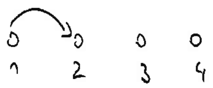
c) rozliš. koule, nerozliš. přihr.

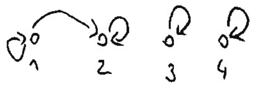
$$\left.\begin{matrix} \boxed{abc} & \boxed{} & \\ \boxed{ab} & \boxed{c} & \cdot 3 \end{matrix}\right\} \underline{\underline{4}}$$

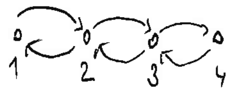
d) nerozliš. koule i přihrádky

$$\left.\begin{matrix} \boxed{oo} & \boxed{o} \\ \boxed{ooo} & \boxed{} \end{matrix}\right\} \underline{\underline{2}}$$

Př. 2 $\{1,2,3,4\}$

a) není reflexivní 

b) je reflexivní ∧ není symetrická 

c) je symetrická ∧ není tranzitivní 

např.

Př. 3

$$\left(\begin{array}{cccc|c} -4 & -3 & -2 & -1 & a \\ 0 & 1 & 2 & 3 & 2 \\ 4 & 5 & 6 & 7 & 2 \\ 8 & 9 & 10 & 11 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 4 & 5 & 6 & 7 & 2 \\ 8 & 9 & 10 & 11 & 2 \\ 0 & 1 & 2 & 3 & 2 \\ -4 & -3 & -2 & -1 & a \end{array}\right) \sim \left(\begin{array}{cccc|c} 4 & 5 & 6 & 7 & 2 \\ 0 & -1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 2 & 4 & 6 & 2+a \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 4 & 5 & 6 & 7 & 2 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & 0 & a-2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

→ řeš. neex. pro $a-2 \neq 0$ $\boxed{a \neq 2}$

→ ∞ mnoho řeš. pro $a-2=0 \rightarrow \boxed{a=2}$

→ jediné řeš. neex.

$\rightarrow a=2$:

$$\left(\begin{array}{cccc|c} 4 & 5 & 6 & 7 & 2 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{array}{l} x_4 = t \\ x_3 = s \\ x_2 = 2-3t-2s \end{array}$$

$4x_1 = 2-7t-6s-5(2-3t-2s) = -8+8t+4s \Rightarrow x_1 = -2+2t+s$

$$\Rightarrow \underline{\underline{\{(-2+2t+s,\ 2-3t-2s,\ s,\ t),\ t,s \in \mathbb{R}\}}}$$

Př. 6

$$\left.\begin{aligned} V_{t+1} &= 0{,}6V_t + 0{,}2Z_t \\ Z_{t+1} &= -0{,}4V_t + 1{,}2Z_t \end{aligned}\right\} \begin{pmatrix} V_t \\ Z_t \end{pmatrix} = \begin{pmatrix} 0{,}6 & 0{,}2 \\ -0{,}4 & 1{,}2 \end{pmatrix}^t \begin{pmatrix} V_0 \\ Z_0 \end{pmatrix}, \quad \begin{aligned} V_0 &= 50 \\ Z_0 &= 210 \end{aligned}$$

$$A = \begin{pmatrix} 0{,}6 & 0{,}2 \\ -0{,}4 & 1{,}2 \end{pmatrix} = P \cdot D \cdot P^{-1}$$

$$\begin{vmatrix} 0{,}6-\lambda & 0{,}2 \\ -0{,}4 & 1{,}2-\lambda \end{vmatrix} = (0{,}6-\lambda)(1{,}2-\lambda) + 0{,}2\cdot 0{,}4 = 0{,}72 - 0{,}6\lambda - 1{,}2\lambda + \lambda^2 + 0{,}08 =$$

$$= \lambda^2 - 1{,}8\lambda + 0{,}8 = 0$$

$$\rightarrow \lambda_{1,2} = \frac{1{,}8 \pm \sqrt{1{,}8^2 - 4\cdot 0{,}8}}{2} = \frac{1{,}8 \pm \sqrt{0{,}04}}{2} = \frac{1{,}8 \pm 0{,}2}{2} = \begin{cases} \underline{\underline{1}} \\ \underline{\underline{0{,}8}} \end{cases}$$

$\lambda_1 = 1$:

$$\begin{pmatrix} -0{,}4 & 0{,}2 \\ -0{,}4 & 0{,}2 \end{pmatrix} \sim \begin{pmatrix} -0{,}4 & 0{,}2 \\ 0 & 0 \end{pmatrix} \rightarrow \left.\begin{aligned} x_2 &= t \\ x_1 &= \frac{0{,}2}{0{,}4}t = \frac{1}{2}t \end{aligned}\right\} u_1 = (1,2)^T$$

$\lambda_2 = 0{,}8$:

$$\begin{pmatrix} -0{,}2 & 0{,}2 \\ -0{,}4 & 0{,}4 \end{pmatrix} \rightarrow \left.\begin{aligned} x_2 &= t \\ x_1 &= t \end{aligned}\right\} u_2 = (1,1)^T$$

$$\lim_{t\to\infty} \begin{pmatrix} V_t \\ Z_t \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \lim_{t\to\infty} \begin{pmatrix} 1^t & 0 \\ 0 & 0{,}8^t \end{pmatrix} \cdot \frac{1}{-1} \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} V_0 \\ Z_0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 50 \\ 210 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 160 \\ -110 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 160 \\ 320 \end{pmatrix}}}$$

Populace vlků a zajíců se z dlouhodobého hlediska ustálí na 160 vlcích a 320 zajících.