Př. 1 (3b)

báži tvoří LNZ vektory ~> přepíšeme si matice $U_1, \dots U_5$ jako vektory a dáme je do sloupců do matice

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 2 \\ 2 & -1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 2 \\ 0 & -1 & -1 & -2 & 2 \\ 0 & 0 & -1 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 2 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \alpha = \{U_1, U_2, U_3\} \Rightarrow \dim = 3$$

Př. 2 (4b)

máme $A = (f)_{\varepsilon_2 \varepsilon_3}$

chceme $B = (f)_{\delta \varphi} = (id)_{\delta \varepsilon_2} \cdot (f)_{\varepsilon_2 \varepsilon_3} \cdot (id)_{\varepsilon_3 \varphi}$

$$(id)_{\varepsilon_3 \varphi} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (id)_{\delta \varepsilon_2} = (id)^{-1}_{\varepsilon_2 \delta} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

$$\Rightarrow B = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 6 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

Př. 3 (4b)

Ker f ... řešíme $f(x) = (0,0,0,0)$; Im f ... LNZ obrazy (= sloupce)

$$\Rightarrow \left.\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 0 \\ 4x_1 + 3x_2 + 2x_3 + x_4 &= 0 \\ x_1 - 2x_2 + 3x_3 - x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 0 \end{aligned}\right\} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ 1 & -2 & 3 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -5 & -10 & -15 \\ 0 & -4 & 0 & -5 \\ 0 & -1 & -2 & -3 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & +8 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow x_4 = t$$

$x_3 = -7/8\, t$

$x_2 = -3t + 7/4\, t = -5/4\, t$

$x_1 = -4t + 21/8\, t + 5/2\, t = 9/8\, t$

$\Rightarrow \operatorname{Ker} f = \{t(9/8, -5/4, -7/8, 1), t \in \mathbb{R}\}$

$= \operatorname{span}\langle (9/8, -5/4, -7/8, 1) \rangle$

$\Rightarrow \operatorname{Im} f = \operatorname{span}\langle (1,4,1,1); (2,3,-2,1); (3,2,3,1) \rangle$

Př. 4 (4b)

řešíme soustavu $A^T A \hat{x} = A^T b$

$$\rightarrow \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -2 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}}_{\begin{pmatrix} 3 & 2 & -2 \\ 2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix}}\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -2 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}_{\begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}}$$

$$\rightarrow \left(\begin{array}{ccc|c} 3 & 2 & -2 & 0 \\ 2 & 2 & 0 & -1 \\ -2 & 0 & 4 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 2 & 4 & -3 \\ 0 & 2 & 4 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 2 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left.\begin{array}{l} \Rightarrow \hat{z} = t \\ \hat{y} = \dfrac{-3-4t}{2} = -3/2 - 2t \\ \hat{x} = 1 + 2t \end{array}\right\} \Rightarrow \hat{x} = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} 1 \\ -3/2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

$$\|A\hat{x} - b\| \ldots t = 0 \Rightarrow \hat{x} = \begin{pmatrix} 1 \\ -3/2 \\ 0 \end{pmatrix} \rightarrow A\hat{x} = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -3/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix}$$

$$\Rightarrow \left\| \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 0 \\ +3/2 \\ -3/2 \end{pmatrix} \right\| = \sqrt{\frac{9}{4} + \frac{9}{4}} = \frac{\sqrt{18}}{2} = \frac{3\sqrt{2}}{2}$$