$a + aq + aq^2 + aq^3 + \dots$

geometrická řada

A.P. $\frac{x+y}{2}$

$\frac{x_1+\dots+x_n}{n}$

geometrický průměr

$\sqrt{x\cdot y}$, $\sqrt[n]{x_1 \cdots x_n}$

$x_i \in \mathbb{R}^+$

$a, a+d, a+2d$

$a\cdot q^{n-1}, a\cdot q^n, a\cdot q^{n+1}$

G.P. $a\cdot q^n = \sqrt{a\cdot q^{n-1}\cdot a\cdot q^{n+1}}$

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$\sqrt{2} = 1{,}41421\ldots$

$= 1 + \frac{4}{10} + \frac{1}{100} + \frac{4}{1000} + \frac{2}{10^4} + \dots$

$s = a + aq + aq^2 + \dots$

$|q| < 1 \Rightarrow s = \frac{a}{1-q}$

$q \geq 1 \Rightarrow s_n = a \cdot \underbrace{\frac{1-q^{n+1}}{1-q}}_{+\infty}$, $q^{n+1} \to +\infty$, < 0

$q \neq 1$

$a > 0$ $s = +\infty$

$a < 0$ $s = -\infty$

11 23-12:10

$A(x_1,\dots,x_n) \geq G(x_1,\dots,x_n) \geq H(x_1,\dots,x_n)$

$\frac{x_1+\dots+x_n}{n} \geq \sqrt[n]{x_1\cdots x_n} \geq \frac{n}{\frac{1}{x_1}+\dots+\frac{1}{x_n}}$

AG-nerovnost

$\sum_{n=1}^{\infty} \frac{1}{n}$ .. Harmonická řada

$\left(\frac{1}{n}\right)^{-1} = \frac{1}{2}\left(\left(\frac{1}{n-1}\right)^{-1} + \left(\frac{1}{n+1}\right)^{-1}\right) =$

$= \frac{1}{2}(n-1+n+1) = n$

11 23-12:20

$\underbrace{1}_{\geq \frac12} + \underbrace{\frac{1}{2}}_{\geq \frac12} + \underbrace{\frac{1}{3}+\frac{1}{4}}_{\geq \frac12} + \underbrace{\frac{1}{5}+\dots+\frac{1}{8}}_{\geq \frac48 = \frac12} + \underbrace{\frac{1}{9}+\dots+\frac{1}{16}}_{\geq \frac{8}{16} = \frac12} + \underbrace{\frac{1}{17}+\dots \frac{1}{32}}_{\geq \frac{16}{32} = \frac12} + \dots$

$= \infty \cdot \frac{1}{2} = \infty$

H. řada diverguje

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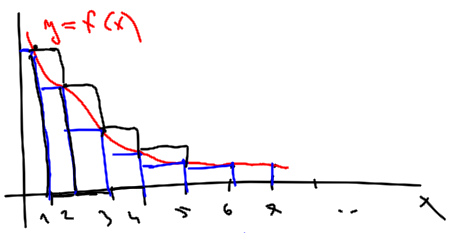

$a_1 \; a_2 \; a_3$

$f(1) = a_1$

$f(2) = a_2, \dots$

$\int_1^{\infty} f(x)\,dx \geq 1\cdot a_2 + 1\cdot a_3 + \dots + 1\cdot a_n + \dots$

konv. $\Rightarrow \sum a_n$ konv.

$\int_1^{\infty} f(x)\,dx \leq 1\cdot a_1 + 1\cdot a_2 + \dots$

div. $\Rightarrow$ div.

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$S = a_0 + a_1 + a_2 + a_3 + \dots$

$\frac{a_{n+1}}{a_n} \leq q < 1$

$a_0, a_1 \leq a_0\cdot q, a_2 \leq a_1 \cdot q \leq a_0 q^2, a_3 \leq a_2 \cdot q, \dots$

$S \leq a_0 + a_0 q + a_0 q^2 + a_0 q^3 + \dots$ konv. $\Leftrightarrow |q| < 1$

$0 < q < 1$

$\frac{a_{n+1}}{a_n} > 1$

$a_1 > a_0 \cdot q, a_2 > a_0 q^2$

11 23-12:52

$\{1+\frac{1}{n}\}^n \to e$

$\{1+\frac{1}{n+1}\}^n \to e$

odmocninové kritérium

$\lim_{n\to\infty} \sqrt[n]{a_n} = q$

$\forall n \ge n_0: \quad q-\varepsilon \le \sqrt[n]{a_n} \le q+\varepsilon$

$(q-\varepsilon)^n \le a_n \le (q+\varepsilon)^n$

$(q-\varepsilon)+(q-\varepsilon)^2+(q-\varepsilon)^3+\dots$ konv. $\Leftrightarrow q-\varepsilon<1$

$(q+\varepsilon)+\dots$ konv. $\Rightarrow q+\varepsilon<1$

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Základní pojmy | Nekonečné řady s nezápornými členy | Alternující řady | Řady absolutně a relativně konvergentní | Součin řad

Příklad

Pro nekonečnou řadu
$\sum_{n=0}^{\infty} a_n = \frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\frac{3}{2^3}+\frac{1}{2^4}+\frac{5}{2^5}+\frac{1}{2^6}+\frac{7}{2^7}+\dots$, tj.

$$a_n = \begin{cases} \frac{n}{2^n}, & \text{pro } n \text{ liché,} \\ \frac{1}{2^n}, & \text{pro } n \text{ sudé,} \end{cases}$$

s podílovým kritériem neuspějeme,

n sudé: $\frac{a_{n+1}}{a_n} = \frac{n+1}{2^{n+1}}\cdot 2^n = \frac{n+1}{2}$ b_n

n liché: $\frac{a_{n+1}}{a_n} = \frac{1}{2^{n+1}}\cdot\frac{2^n}{n} = \frac{1}{2n}$ b_n

$\lim b_n$ neexistuje

11 23-13:01

$(1-1)+(1-1)+(1-1)+\dots = 0$

$\sum_{n=0}^{\infty}(-1)^n$

$1\cancel{(-1}+1\cancel{)(-1}+1)(-1+1)-1$

$1+0+0+\dots = 1$

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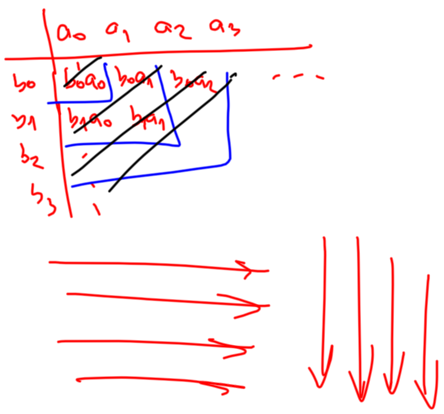

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