$\mathbb{R}^n$ $x=(x_1,\dots,x_n)^T$ $y=(y_1,\dots,y_n)^T$

$x\cdot y = \langle x,y\rangle = x_1y_1+\dots+x_ny_n = x^T\cdot y$

$\mathbb{C}^n$ $x\cdot y = x_1\bar y_1+\dots+x_n\bar y_n = \bar y^T\cdot x$

$\boxed{f: V\to V}$ $\boxed{\bar y^T A x} = (\bar A\cdot y)^T\cdot x = \boxed{\bar y^T\cdot \bar A^T\cdot x}$

$\langle f(u), v\rangle = \langle u, f(v)\rangle$

$A=\bar A^T =: A^*$

11 21-10:04

$\varphi: V\to V$ $\varphi(u)=\lambda\cdot u$ realno število

ortog. φ: $\langle\varphi(u),\varphi(v)\rangle = \langle u,v\rangle$

$\langle\varphi(u), w\rangle = \langle u,\varphi^{-1}(w)\rangle$

A $\bar A^T$

$\boxed{A^{-1}=A^*}$ $\varphi: U\to U\subset V$, $u\in U$, $v\in U^\perp$

$0=\langle\varphi(u),v\rangle = \langle u,\varphi^{-1}(v)\rangle = 0$

$\Rightarrow \varphi': U^\perp\to U^\perp$

U inv. $\Rightarrow$ $U^\perp$ inv.

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$A=A^*$ "simetrični" $\varphi: V\to V$

$\varphi: U\to U\subset V$ U invarianten, $u\in U$, $v\in U^\perp$

$0=\langle\varphi(u),v\rangle = \langle u,\varphi(v)\rangle \Rightarrow \varphi(v)\in U^\perp$

$A\cdot x = \lambda\cdot x$ $\lambda\in\mathbb{C}$ $x\neq 0$

$\langle A\cdot x, x\rangle = \lambda\cdot\cancel{\langle x,x\rangle}$

$\langle x, Ax\rangle = \langle x,\lambda x\rangle = \bar\lambda\cancel{\langle x,x\rangle}$

$\Rightarrow \lambda=\bar\lambda\in\mathbb{R}$.

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$c_0z^n+c_1z^{n-1}+\dots+c_n = P(z)$

$c_0A^n+c_1A^{n-1}+\dots+c_nE = P(A)$

$\boxed{A=T^{-1}\cdot D\cdot T}$ $D=\begin{pmatrix}\lambda_1&&\\&\ddots&\\&&\lambda_n\end{pmatrix}$ $T^*=T^{-1}$ $\lambda_i\in\mathbb{R}$

$\sqrt{x}=f(x)$ $x\geq 0$.

$B=A^*\cdot A$ velja:

$\boxed{\langle B\cdot x, x\rangle} = \langle A^*\cdot A\cdot x, x\rangle = \langle A\cdot x, A\cdot x\rangle \geq 0$

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$B=T^*\cdot D\cdot T$ $B\geq 0$ (≥)

$D=\begin{pmatrix}\lambda_1&0\\0&\lambda_n\end{pmatrix}$

$\boxed{\lambda_i\geq 0}$

$\sqrt{B}:=T^*\sqrt{D}\cdot T$

$(\sqrt{B})^2 = T^*\sqrt{D}\cdot\underbrace{T\cdot T^*}_{E}\sqrt{D}\,T = T^*\begin{pmatrix}\lambda_1&0\\0&\lambda_n\end{pmatrix}T = B$

$\sqrt{D}=\begin{pmatrix}\sqrt{\lambda_1}&0\\0&\sqrt{\lambda_n}\end{pmatrix}$

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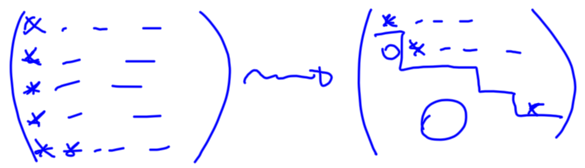


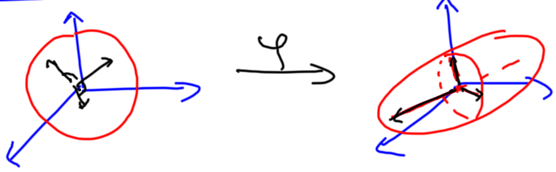
$A\mapsto P_1\cdot A\mapsto\dots\mapsto \underbrace{P_k\cdots P_1\cdot A}_{U}$

$A=P_1^{-1}\cdot P_2^{-1}\cdots P_k^{-1}\cdot U = P\cdot U$

[illegible] trikotna

11 21-10:58

$\boxed{\varphi: \mathbb{R}^n \to \mathbb{R}^m}$ A typu m/n



11 21-11:09

Důkaz: (Sing. rozklad) $\varphi: \mathbb{K}^n \to \mathbb{K}^m$

A^*A pozitivně-semidef (nezáporné)

$\langle A^*A x, x\rangle \geq 0$ $\quad B = A^*A$ samoadj.

$\langle Ax, Ax\rangle$

$\Rightarrow$ ortonorm. báze v_i na $\mathbb{K}^n$ ve složení vl. $\varphi^*\circ\varphi$ diag. tvar

$\Rightarrow$ existuje V, $V^* = V^{-1}$, $A^*A = \underbrace{V\cdot\begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_n\end{pmatrix}V^*}_{C}$

$d_1,\dots,d_r \neq 0$, $d_{r+1}=\dots=d_n=0$

$\Rightarrow C = V^*A^*A\cdot V = (A\cdot V)^*\cdot\boxed{(A\cdot V)}$

sloupce $v_1,\dots,v_r$ ortog. sloupce v $A\cdot V$

$\langle v_i, v_i\rangle = d_i \Rightarrow u_i = \frac{1}{\sqrt{d_i}}v_i$

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$u_1,\dots,u_r \to u_1,\dots,u_r,\boxed{u_{r+1},\dots,u_m}$ ortonormální báze $\mathbb{K}^m$

doplněno libovolně

$\varphi: \mathbb{K}^n \to \mathbb{K}^m$ v bázi $w_1,\dots,w_n$ a v bázi $u_1,\dots,u_m$ má matici $\sqrt{C}$.

(předp. $m \leq n$) ☒

Věta

11 21-11:26

Věta (polární rozklad)

A typu n/n. Pak $A = P\cdot V$, kde $P = P^*$, $P \geq 0$, $V^* = V^{-1}$.

$A = U\cdot S\cdot W^*$, S diag (sing. rozklad)

$= \underbrace{\boxed{U\cdot S\cdot U^*}}_{P}\underbrace{\boxed{U\cdot W^*}}_{V}$ $\quad S = \begin{pmatrix}\sqrt{d_1} & & & \\ & \ddots & & \\ & & \sqrt{d_r} & \\ & & & 0\ \ddots\ 0\end{pmatrix}$

$P^* = U\cdot S^*\cdot U^* = U\cdot S\cdot U^*$

pro A^{-1} ex. $\Rightarrow$ $P = \sqrt{A\cdot A^*}$

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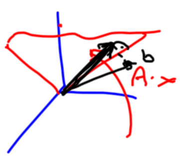
Kor. $A = USV^*$ $\quad S = \begin{pmatrix} d_1 & & 0\\ 0 & \ddots & d_n\end{pmatrix}$

$A^{-1} = V\cdot S^{-1}\cdot U^*$ $\quad S^{-1} = \begin{pmatrix} d_1^{-1} & & & \\ & \ddots & & \\ & & d_r^{-1} & \\ & & & 0\ \ddots\ 0\end{pmatrix}$

použití:

$A\cdot x = b$

$\boxed{x := A^{(-1)}\cdot b}$



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