IA168 Algorithmic Game Theory

Survey

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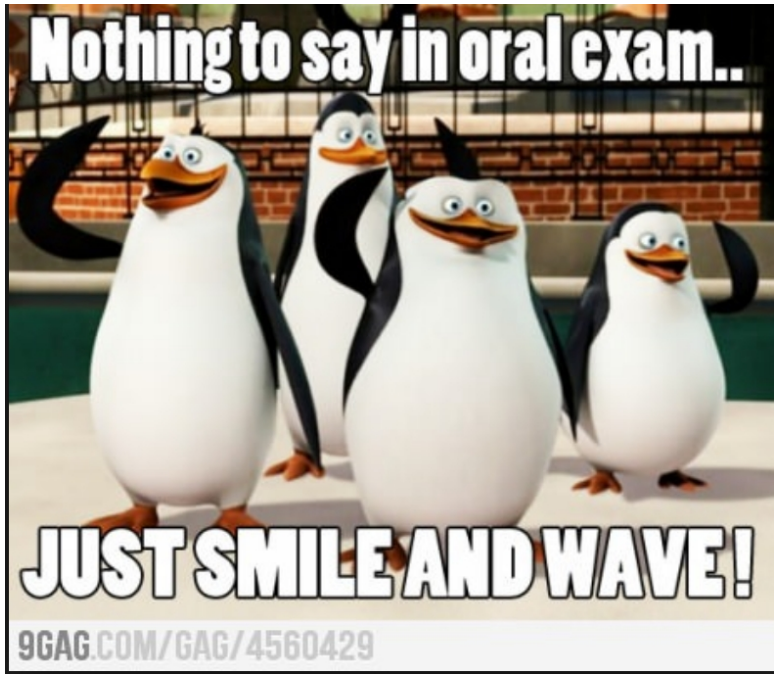
Evaluation

- **Oral exam**
- Homework (occasionally)



Strictly dominated strategies for the exam:

- No preparation (skim-through)
- Learn only a strict subset

THE strictly dominant strategy:

Learn all definitions, algorithms, theorems and proofs.

What we did ...

Types of games:

- strategic-form games
- extensive-form games
- (strict) incomplete information games & Bayesian games

Types of strategies:

- pure
- mixed

What we did ... strategic-form games

Solution concepts:

- strictly dominant strategy equilibrium
- iterated elimination of strictly dominated strategies
- rationalizability
- Nash equilibria

We studied all these concepts in both pure and mixed strategies.

We studied computational complexity of solving strategic-form games w.r.t. all above concepts.

In particular, we considered classical algorithms for computing mixed Nash equilibria for *two-player games*:

- support enumeration
- Lemke-Howson

For zero-sum two-player games a polynomial time algorithm based on von Neumann's theorem was presented.

What we did ... extensive-form games

We considered three levels of expressiveness:

- perfect-information extensive-form games
- imperfect-information extensive-form games
- perfect and imperfect-information extensive-form games with chance nodes

In all cases we considered the following types of strategies:

- pure
- mixed
- behavioral

Solution concepts:

- Nash equilibria
- subgame perfect equilibria (SPE)

What we did ... extensive-form games ... results

For finite perfect-information extensive-form games:

- there always exists a pure strategy SPE (in pure as well as behavioral strategies)
- backward induction for computing SPE (can be used also for perfect-information games with chance nodes)
- equivalence of mixed and behavioral strategies

For finite imperfect-information extensive-form games:

- there always exists a behavioral strategy Nash equilibrium
- backward induction on "perfect information" nodes
- mixed and behavioral strategies are not equivalent in general, they are equivalent for games with perfect recall

What we did ... repeated games

Strategic-form games played repeatedly for either finitely many, or infinitely many rounds.

Behavior of players may depend (arbitrarily) on the history of the play.

They are a special case of imperfect-information extensive-form games.

Solution concepts:

- For finitely repeated: average payoff (sum of payoffs)
- For infinitely repeated:
  - discounted payoff
  - long-run average payoff

We have considered only pure strategies.

What we did ... repeated games ... results

For finitely repeated:

- There is a unique SPE if the strategic-form game has a unique pure str. NE
- SPE obtained by iterating a NE from the strategic-form game
- other SPE (punishing equilibria)

For infinitely repeated:

- discounted payoff:
  - one-shot deviation property iff SPE
    (for bounded payoff functions)
  - grim trigger strategy profiles & simple Folk theorem for SPE
    (for bounded payoff functions)
  - an approximate version of general Folk theorem for SPE
    (repeated finite strategic-form games only)
    (feasible payoffs)
- long-run average payoff:
  - (almost) general Folk theorems for SPE and NE
    (repeated finite strategic-form games only)
    (feasible and individually rational payoffs)

What we did ... incomplete information games

- strict incomplete information games
  - solution concepts: weak dominance, ex-post-Nash equilibrium
- Bayesian games
  - solution concepts: weak dominance, Bayesian Nash equilibrium

Only pure strategies.

Auctions:

- Second-price auction:
  - truth telling strategies are weakly dominant in both strict imperfect information as well as Bayesian model
- First-price auction:
  - Bayesian games needed to obtain a solution, solved for uniform common prior

Revenue equivalence.