# PV227 GPU Rendering 

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## Points and Vectors

- points (before projection) are quadruples: $(x, y, z, 1.0)$,
- can be transformed with a $4 \times 4$ matrix,
- vectors are also quadruples: $(x, y, z, 0.0)$,
- can be transformed with a $4 \times 4$ or $3 \times 3$ matrix.


## Transformation

- points are transformed to eye space with modelview matrix,
- vectors constructed from points (e.g. $P_{2}-P_{1}$ ) are also transformed with this matrix,
- normals are not!


## Normal Transformation Error



Figure: Taken from lighthouse3d.com

## Normal Transformation Solution

- caused by non-uniform scale,
- $M$ is the modelview matrix, $\vec{t}$ is tangent vector $\left(P_{2}-P_{1}\right)$ and $I$ is identity,
- we need another matrix ( $N$ ) for transforming normal $\vec{n}$.

$$
\begin{aligned}
(M \times \vec{t}) \bullet(N \times \vec{n}) & =0 \\
(M \times \vec{t})^{T} \times(N \times \vec{n}) & =0 \\
\vec{t}^{T} \times M^{T} \times N \times \vec{n} & =0
\end{aligned}
$$

## Normal Transformation Solution (cont.)

$$
\begin{aligned}
\vec{t}^{T} \times M^{T} \times N \times \vec{n} & =0 \\
\vec{t} \bullet \vec{n}=0 \Rightarrow \vec{t}^{T} \times \vec{n}=0 \Rightarrow M^{T} \times N & =1 \\
M^{T} \times N & =1 \\
\left(M^{T}\right)^{-1} \times M^{T} \times N & =\left(M^{T}\right)^{-1} \\
N & =\left(M^{T}\right)^{-1}
\end{aligned}
$$

## Normal Transformation Result

- $N$ is inverse transpose of $\mathrm{M}(3 \times 3$ submatrix of $M)$,
- for orthogonal matrices: $A^{T}=A^{-1}$ (rotation is orthogonal),
- $M$ is orthogonal $M=\left(M^{T}\right)^{-1} \Rightarrow N=M$.


## Renormalization

- normals must be of unit length,
- can be destroyed by normal transformation $\rightarrow$ must be normalized in vertex shader,
- interpolation can also destroy vector length $\rightarrow$ must be normalized in fragment shader.


Figure: Taken from lighthouse3d.com

## Lighting

- computation of light's interaction with surfaces,
- huge cheat,
- ambient, diffuse and specular lighting,
- flat, gouraud and phong shading,
- directional, point and spot light,
- no shadow, no bouncing of light.


## Ambient Lighting

- approximates lighting after infinite number of bounces,
- homogeneous,
- prevents black areas that look unnatural,
- usually chosen as fraction of the diffuse (material) color,
- $I=K_{a}$.


Figure: Ambient spheres

## Directional Light



Figure: Taken from lighthouse3d.com

- far away light,
- defined by a direction vector (position is irelevant), - can represent e.g. the sun.


## Gouraud Shading

- per vertex shading,
- interpolation of vertex colors,
- unable to capture lighting details inside polygons.


## Diffuse Lighting

- simulate light's interaction with perfectly diffuse material,
- light angle dependent,
- significant color component,
- $I=\cos (\alpha) \cdot K_{d}$.


Figure: Diffuse spheres

## Diffuse Lighting



Figure: Taken from lighthouse3d.com

- amount of incoming light diminishes with increasing angle,
- $\cos (\alpha)=\frac{\vec{L} \bullet \vec{N}}{|\vec{L}| \cdot|\vec{N}|}$,
- normalized vectors: $I=(\vec{L} \bullet \vec{N}) \cdot K_{d}$,
- all vectors must be in same space (usually defined in world space, computation in camera space).


## Flat Shading

- per primitive shading,
- no interpolation,
- unable to capture smooth changes in light intensity.


Figure: Flat shading

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## Combined Lighting

- light from various sources can be combined (added),
- combination of ambient and diffuse prevents black areas,
- $I=K_{a}+\cos (\alpha) \cdot K_{d}$,
- value should not be outside the [0.0, 1.0] range.


Figure: Ambient + Diffuse spheres

## Specular Lighting

- simulate light's interaction with reflective material,
- view angle dependent,
- highlight of the light's color, not material color,
- $I=\cos (\beta)^{s} \cdot K_{s}, \boldsymbol{s}$ controls size of the highlight.


Figure: Specular spheres (Phong vs Blinn-Phong)

## Phong Lighting



Figure: Taken from lighthouse3d.com

- amount of reflected light diminishes with increasing angle,
- $\vec{R}=-\vec{L}+2 \cdot(\vec{N} \bullet \vec{L}) \cdot \vec{N}$,
- $\cos (\beta)=\vec{R} \bullet \overrightarrow{E y e}$,
- all vectors must be in same space, normalized.


## Blinn-Phong Lighting



Figure: Taken from lighthouse3d.com

- amount of reflected light diminishes with increasing angle,
- $\vec{H}=\vec{L}+\overrightarrow{E y e}$,
- $\cos (\beta)=\vec{H} \bullet \vec{N}$,
- all vectors must be in same space, normalized.


## Basic Lighting

- ambient, diffuse and specular form the baseline lighting,
- $I=K_{a}+\cos (\alpha) \cdot K_{d}+\cos (\beta)^{s} \cdot K_{s}$,
- light from various sources can be combined (added),
- value should not be outside the $[0.0,1.0]$ range.


Figure: Ambient + Diffuse + Specular pawns

## Phong Shading

- per pixel shading,
- smooth lighting including details,
- interpolation of vertex attributes (normal, eye, light).


Figure: Per pixel lighting pawns

## Point Light

- light source inside the scene,
- defined by a position vector (all directions),
- can represent e.g. a lightbulb.


Figure: Taken from lighthouse3d.com Point light pawns.

## Spot Light

- light source inside the scene,
- only a directed cone is illuminated,
- defined by a position vector, direction vector and angle,
- can represent e.g. a flashlight.


Figure: Spot light pawns

