Untyped lambda calculus

terms and values

$$M ::= x \mid M\ M' \mid \lambda x.M \qquad \text{terms}$$
$$V ::= \lambda x.M \qquad \text{value}$$

full β-reduction evaluation rules

$$\frac{}{(\lambda x.M)\ N \to_\beta M[x := N]} \qquad \frac{M_1 \to_\beta M_2}{\lambda x.M_1 \to_\beta \lambda x.M_2}$$
$$\frac{M_1 \to_\beta M_2}{M_1\ N \to_\beta M_2\ N} \qquad \frac{N_1 \to_\beta N_2}{M\ N_1 \to_\beta M\ N_2}$$

call-by-value evaluation rules

$$\frac{}{(\lambda x.M)\ V \to_{cbv} M[x := V]} \qquad \frac{M_1 \to_{cbv} M_2}{M_1\ N \to_{cbv} M_2\ N} \qquad \frac{N_1 \to_{cbv} N_2}{V\ N_1 \to_{cbv} V\ N_2}$$

Church numerals and booleans

$\underline{0} := \lambda f.\lambda x.x$

$\underline{1} := \lambda f.\lambda x.f\ x$

$\underline{n} := \lambda f.\lambda x.\underbrace{f(f \ldots (f}_{\text{n-times}}(x)\ldots) = \lambda f.\lambda x.f^n(x)$

$\mathsf{true} := \lambda x.\lambda y.x$

$\mathsf{false} := \lambda x.\lambda y.y$

Simply-typed lambda calculus $\lambda^{\to}$

with Booleans

terms and values

$$M ::= x \mid M\ M' \mid \lambda x:\sigma.M \mid \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}\ M\ \mathsf{then}\ M'\ \mathsf{else}\ M'' \qquad \text{terms}$$
$$V ::= \lambda x:\sigma.M \mid \mathsf{true} \mid \mathsf{false} \qquad \text{values}$$

types

$$\sigma ::= \sigma \to \sigma \mid \text{Bool}$$

typing rules

$$\Gamma \vdash \mathsf{true} : \text{Bool}\ (\text{T-TRUE})$$
$$\Gamma \vdash \mathsf{false} : \text{Bool}\ (\text{T-FALSE})$$
$$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma}\ (\text{T-VAR})$$
$$\frac{\Gamma, x : \sigma_1 \vdash M : \sigma_2}{\Gamma \vdash \lambda x : \sigma_1.M : \sigma_1 \to \sigma_2}\ (\text{T-ABS})$$
$$\frac{\Gamma \vdash M_1 : \sigma_1 \to \sigma_2 \qquad \Gamma \vdash M_2 : \sigma_1}{\Gamma \vdash M_1\ M_2 : \sigma_2}\ (\text{T-APP})$$
$$\frac{\Gamma \vdash M_1 : \text{Bool} \qquad \Gamma \vdash M_2 : \sigma \qquad \Gamma \vdash M_3 : \sigma}{\Gamma \vdash \mathsf{if}\ M_1\ \mathsf{then}\ M_2\ \mathsf{else}\ M_3 : \sigma}\ (\text{T-IF})$$

Hindley-Milner type system

terms and values

$$M ::= x \mid M_1\ M_2 \mid \lambda x.M \mid \textsf{let}\ x = M_1\ \textsf{in}\ M_2$$

types

$$\sigma ::= \alpha \mid \sigma \to \sigma \qquad \text{monotypes}$$

$$\tau ::= \sigma \mid \forall \vec{\alpha}.\tau \qquad \text{type schemes}$$

typing rules

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau}\ (\text{T-VAR})$$

$$\frac{\Gamma, x : \sigma_1 \vdash M : \sigma_2}{\Gamma \vdash \lambda x.M : \sigma_1 \to \sigma_2}\ (\text{T-ABS})$$

$$\frac{\Gamma \vdash M_1 : \sigma_1 \to \sigma_2 \qquad \Gamma \vdash M_2 : \sigma_1}{\Gamma \vdash M_1\ M_2 : \sigma_2}\ (\text{T-APP})$$

$$\frac{\Gamma \vdash M_1 : \tau \qquad \Gamma, x : \tau \vdash M_2 : \sigma}{\Gamma \vdash \textsf{let}\ x = M_1\ \textsf{in}\ M_2 : \sigma}\ (\text{T-LET})$$

$$\frac{\Gamma \vdash M : \tau' \qquad \tau' \sqsubseteq \tau}{\Gamma \vdash M : \tau}\ (\text{T-INST})$$

$$\frac{\Gamma \vdash M : \tau \qquad \alpha \notin FV(\Gamma)}{\Gamma \vdash M : \forall \alpha.\tau}\ (\text{T-GEN})$$

System F

terms and values

$$M ::= x \mid M\ M' \mid \lambda x : \sigma.M \mid \Lambda\alpha.M \mid M\ [\sigma] \qquad \text{terms}$$

$$V ::= \lambda x : \sigma.M \mid \Lambda\alpha.M \qquad \text{values}$$

types

$$\sigma ::= \alpha \mid \sigma \to \sigma \mid \forall \vec{\alpha}.\sigma$$

typing rules *(in addition to $\lambda^{\to}$)*

$$\frac{\Gamma \vdash M : \sigma \qquad \alpha \notin FV(\Gamma)}{\Gamma \vdash \Lambda\alpha.M : \forall\alpha.\sigma}\ (\text{T-TABS}) \qquad (\text{T-GEN})$$

$$\frac{\Gamma \vdash M : \forall\alpha.\sigma_1}{\Gamma \vdash M\ [\sigma_2] : \{\sigma_2/\alpha\}\sigma_1}\ (\text{T-TAPP}) \qquad (\text{T-INST})$$

evaluation rules *(in addition to $\lambda^{\to}$)*

$$\frac{M \to M'}{M\ [\sigma] \to M'\ [\sigma]}\ (\text{E-TAPP})$$

$$(\Lambda\alpha.M)\ [\sigma] \to \{\sigma/\alpha\}M\ (\text{E-TAPPTABS})$$