06.11.2014	IA014 Advan	nced Functional Prog	ramming	Duration: 60 minute
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fixed point combinators

Question 1

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In the lecture, you have seen a proof that $\mathbf{Y} := \lambda f.(\lambda x.f(x x)) (\lambda x.f(x x))$ is a **10 points** fixed point combinator. Prove that

 $\boldsymbol{D} := \lambda f.(\lambda xy.f (x \ x \ y)) \ (\lambda xy.f (x \ x \ y)) \ (d \ r \ a \ g \ o \ n)$

(where \boldsymbol{D} stands for "Dragon") is also a fixed point combinator.

Church numerals

Question 2

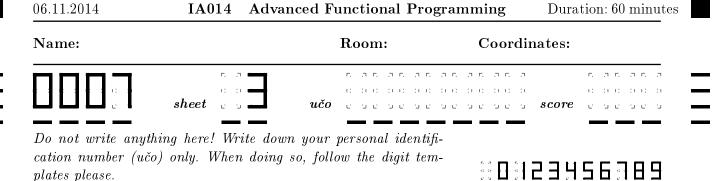
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In the lecture you have seen the following terms for multiplication and addition **10 points** (for Church numerals):

$$\begin{split} \texttt{plus} &:= \lambda m.\lambda n.\lambda f.\lambda x.m \ f \ (n \ f \ x) \\ \texttt{times} &:= \lambda m.\lambda n.\lambda f.m \ (n \ f) \end{split}$$

a) Show that times $\underline{3} \underline{4}$ evaluates to $\underline{12}$.

b) Give an alternative definition of the term times using plus.



simply typed lambda calculus λ^{\rightarrow}

Question 3 12 points

Lists can be easily added to the simply typed lambda calculus λ^{\rightarrow} . Here is the additional syntax (note that these lists are parameterized by the type of elements T):

t ::=		terms
	$\mathtt{nil}[T]$	empty list
	cons[T] t t	list constructor
	isnil[T] t	test for emptiness
	hd[T] t	head of a list
	tl[T] t	tail of a list
v ::=		values
	$\mathtt{nil}[T]$	empty list
	cons[T] v v	list constructor
T ::=		types
	List T	type of lists

a) You task is to write down the typing rules for lists (one has been provided for your convenience): (5 rules, including the one provided)

$$\frac{\Gamma \vdash t_1 : T \qquad \Gamma \vdash t_2 : \text{List } T}{\Gamma \vdash \text{cons}[T] \ t_1 \ t_2 : \text{List } T} (T\text{-}\text{Cons})$$

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(continuation of the previous sheet)

b) Also write down the evaluation rules for lists, using the *call-by-value* semantics (again, the first one has been provided for you convenience):

(9 rules, including the one provided)

 $\frac{t_1 \rightarrow t_1'}{\left[\text{cons}[T] \ t_1 \ t_2 \rightarrow \text{cons}[T] \ t_1' \ t_2 \right]} \ (\text{E-Cons1})$

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System HM

In the System HM, prove that the term

 $\texttt{let} \ i \ = \ \lambda x.x \texttt{in} \ i \ i$

is well typed (and give its type).

Question 4 10 points