

Name:

Room:

Coordinates:



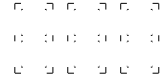
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**fixed point combinators****Question 1**

In the lecture, you have seen a proof that  $Y := \lambda f.(\lambda x.f(x x)) (\lambda x.f(x x))$  is a fixed point combinator. Prove that

**10 points**

$$D := \lambda f.(\lambda xy.f (x x y)) (\lambda xy.f (x x y)) \text{ (d r a g o n)}$$

(where  $D$  stands for „Dragon“) is also a fixed point combinator.



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### simply typed lambda calculus $\lambda^{\rightarrow}$

Lists can be easily added to the simply typed lambda calculus  $\lambda^{\rightarrow}$ . Here is the additional syntax (note that these lists are parameterized by the type of elements  $T$ ):

### Question 3 12 points

$t ::=$	...	<i>terms</i>
	$\text{nil}[T]$	empty list
	$\text{cons}[T] t t$	list constructor
	$\text{isnil}[T] t$	test for emptiness
	$\text{hd}[T] t$	head of a list
	$\text{tl}[T] t$	tail of a list
$v ::=$	...	<i>values</i>
	$\text{nil}[T]$	empty list
	$\text{cons}[T] v v$	list constructor
$T ::=$	...	<i>types</i>
	$\text{List } T$	type of lists

a) Your task is to write down the typing rules for lists (one has been provided for your convenience): (5 rules, including the one provided)

$$\frac{\Gamma \vdash t_1 : T \quad \Gamma \vdash t_2 : \text{List } T}{\Gamma \vdash \text{cons}[T] t_1 t_2 : \text{List } T} \text{ (T-CONS)}$$



