Name: **Room:** **Coordinates:**

0007 ***sheet*** 1 ***učo*** ***score***

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0 1 2 3 4 5 6 7 8 9

Question 1
10 points

fixed point combinators

In the lecture, you have seen a proof that $\boldsymbol{Y} := \lambda f.(\lambda x.f(x\ x))\ (\lambda x.f(x\ x))$ is a fixed point combinator. Prove that

$$\boldsymbol{D} := \lambda f.(\lambda xy.f\ (x\ x\ y))\ (\lambda xy.f\ (x\ x\ y))\ (d\ r\ a\ g\ o\ n)$$

(where $\boldsymbol{D}$ stands for „Dragon“) is also a fixed point combinator.

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Church numerals

Question 2
10 points

In the lecture you have seen the following terms for multiplication and addition (for Church numerals):

$$\texttt{plus} := \lambda m.\lambda n.\lambda f.\lambda x.m\ f\ (n\ f\ x)$$

$$\texttt{times} := \lambda m.\lambda n.\lambda f.m\ (n\ f)$$

a) Show that `times` $\underline{3}$ $\underline{4}$ evaluates to $\underline{12}$.

b) Give an alternative definition of the term `times` using `plus`.

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Question 3
12 points

simply typed lambda calculus $\lambda^{\rightarrow}$

Lists can be easily added to the simply typed lambda calculus $\lambda^{\rightarrow}$. Here is the additional syntax (note that these lists are parameterized by the type of elements T):

t ::=	...	*terms*
	`nil`[T]	empty list
|	`cons`[T] t t	list constructor
|	`isnil`[T] t	test for emptiness
|	`hd`[T] t	head of a list
|	`tl`[T] t	tail of a list
v ::=	...	*values*
	`nil`[T]	empty list
|	`cons`[T] v v	list constructor
T ::=	...	*types*
|	List T	type of lists

a) You task is to write down the typing rules for lists (one has been provided for your convenience): *(5 rules, including the one provided)*

$$\frac{\Gamma \vdash t_1 : T \qquad \Gamma \vdash t_2 : \text{List } T}{\Gamma \vdash \texttt{cons}[T]\ t_1\ t_2 : \text{List } T}\ (\text{T-Cons})$$

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(continuation of the previous sheet)

b) Also write down the evaluation rules for lists, using the *call-by-value* semantics (again, the first one has been provided for you convenience):

(9 rules, including the one provided)

$$\frac{t_1 \to t_1'}{\texttt{cons[T]}\ t_1\ t_2 \to \texttt{cons[T]}\ t_1'\ t_2}\ (\text{E-Cons1})$$

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System HM

Question 4
10 points

In the System HM, prove that the term

$$\texttt{let}\ i\ =\ \lambda x.x\ \texttt{in}\ i\ i$$

is well typed (and give its type).