1.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}' = \begin{pmatrix} 16 & 14 & 38 \\ -9 & -7 & -18 \\ -4 & -4 & -11 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$p(\lambda) = -(\lambda+1)(\lambda-2)(\lambda+3) = 0$$

$$\underline{\lambda_1 = -1, \quad \lambda_2 = 2, \quad \lambda_3 = -3}$$

METÓDA VLASTNÝCH VEKTOROV (NEDEFEK. VLAS. ČÍSLA)

$\lambda_1 = -1$

$$(A + I) = \begin{pmatrix} 17 & 14 & 38 \\ -9 & -6 & -18 \\ -4 & -4 & -10 \end{pmatrix} \approx \begin{pmatrix} 0 & 0 & 0 \\ -9 & -6 & -18 \\ -4 & -4 & -10 \end{pmatrix}$$

$v = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ VLAST. VEKTOR :

$$\begin{pmatrix} 17 & 14 & 38 \\ -9 & -6 & -18 \\ -4 & -4 & -10 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \approx \begin{pmatrix} 0 & 0 & 0 \\ -9 & -6 & -18 \\ -4 & -4 & -10 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

$$\ldots \quad 9a + 6b + 18c = 0 \Rightarrow 3a + 2b + 6c = 0$$

$$+4a + 4b + 10c = 0 \Rightarrow \underline{2a + 2b + 5c = 0}$$

$$a + c = 0 \ldots \boxed{a = -c}$$

$$\boxed{b} = -\frac{1}{2}(2a + 5c) = -\frac{1}{2} \cdot 3c = \boxed{-\frac{3}{2} c} \quad \ldots \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -c \\ -3/2 \cdot c \\ c \end{pmatrix}$$

$$c = 2 \ldots \quad v = (-2, -3, 2)^T \quad \ldots \quad e^{-t} \cdot v = \boxed{(-2e^{-t}, -3e^{-t}, 2e^{-t})^T}$$

$\lambda_2 = 2$

$$(A - 2I) = \begin{pmatrix} 14 & 14 & 38 \\ -9 & -9 & -18 \\ -4 & -4 & -13 \end{pmatrix} \approx \begin{pmatrix} 0 & 0 & 0 \\ -1 & -1 & -2 \\ -4 & -4 & -13 \end{pmatrix}$$

$$v = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \rightsquigarrow \begin{pmatrix} 0 & 0 & 0 \\ -1 & -1 & -2 \\ -4 & -4 & -13 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \, . \quad \begin{matrix} a + b + 2c = 0 \\ 4a + 4b + 13c = 0 \end{matrix} \rightsquigarrow \begin{matrix} \underline{\underline{c = 0}} \\ \underline{\underline{a = -b}} \end{matrix}$$

$v = (-b, b, 0)^T$ ... pre $b = 1$ ... $v = (-1, 1, 0)^T$

$e^{2t} \cdot v = \boxed{(-e^{2t}, e^{2t}, 0)^T}$

$\lambda_3 = -3$ $\qquad (A + 3 \cdot I) = \begin{pmatrix} 19 & 14 & 38 \\ -9 & -4 & -18 \\ -4 & -4 & -8 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ -9 & -4 & -18 \\ +1 & +1 & +2 \end{pmatrix}$

$v = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ ... $\begin{pmatrix} 0 & 0 & 0 \\ -9 & -4 & -18 \\ 1 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$ ...

$$9a + 4b + 18c = 0$$
$$a + b + 2c = 0$$
$$5a + 10c = 0$$

$\Rightarrow \underline{\underline{a = -2c}}$

$\underline{\underline{b}} = -2c - a = \underline{\underline{0}}$

$v = (-2c, 0, c)^T$ ... $c = 1$ ... $v = (-2, 0, 1)^T$

$e^{-3t} \cdot v = \boxed{(-2e^{-3t}, 0, e^{-3t})^T}$

FUNDAMENTÁLNA MATICA: $Y(t) = \begin{pmatrix} -2e^{-t} & -e^{2t} & -2e^{-3t} \\ -3e^{-t} & e^{2t} & 0 \\ 2e^{-t} & 0 & e^{-3t} \end{pmatrix}$

2.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}' = \begin{pmatrix} 0 & 1 & 0 \\ 4 & 3 & -4 \\ 1 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad p(\lambda) = -\lambda \cdot (\lambda - 1)^2 = 0$$

$$\underline{\lambda_1 = 0, \quad \lambda_{2,3} = 1}$$

METÓDA NEURČITÝCH KOEF.

$$\lambda_1 = 0 \quad \dots \quad \begin{pmatrix} a \cdot e^{0 \cdot t} \\ b \cdot e^{0 \cdot t} \\ c \cdot e^{0 \cdot t} \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$x' = y' = z' = 0 \quad ; \quad 0 = b \quad \dots \quad \underline{\underline{b = 0}}$$

$$0 = 4a + 3b - 4c$$

$$0 = a + 3b - c \Rightarrow \underline{\underline{a = c}}$$

$$\dots \quad a = c = 1, \quad b = 0 \quad \dots \quad \boxed{(1,0,1)^T} \quad \dots \text{ 1 LNZ RIEŠENIE}$$

$$\lambda_2 = 1 \quad \dots \quad \begin{pmatrix} (at+b)e^t \\ (ct+d)e^t \\ (ft+g)e^t \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad ; \quad \begin{aligned} x' &= ae^t + (at+b)e^t \\ y' &= ce^t + (ct+d)e^t \\ z' &= fe^t + (ft+g)e^t \end{aligned}$$

DOSADÍME A VYKRÁTIME e^t:

$$a + at + b = ct + d$$

$$c + ct + d = 4(at+b) + 3 \cdot (ct+d) - 4(ft+g)$$

$$f + ft + g = at + b + 2(ct+d) - (ft+g)$$

$$t^1: \quad \begin{aligned} a &= c \\ c &= 4a + 3c - 4f \\ f &= a + 2c - f \end{aligned} \Rightarrow \boxed{a = c \; ; \; f = \frac{3}{2}c}$$

t^0: $a + b = d$

$c + d = 4b + 3d - 4g \rightsquigarrow c = 4b + 2d - 4g$

$f + g = b + 2d - g \rightsquigarrow f = b + 2d - 2g$

DOSADÍME $a = c$, $f = \frac{3}{2} c$

$c + b = d$; $\underbrace{c = 4b + 2d - 4g; \frac{3}{2} c = b + 2d - 2g}$

$\frac{1}{2} c = -3b + 2g$ ... $g = \frac{3}{2} b + \frac{1}{4} c$

2 VOĽNÉ PREMENNÉ: b, c:

$a = c$, $d = b + c$, $f = \frac{3}{2} c$, $g = \frac{3}{2} b + \frac{1}{4} c$

2 LNZ RIEŠENIA:

$b = 2$, $c = 0$ ... $a = 0$, $b = 2$, $c = 0$, $d = 2$, $f = 0$, $g = 3$

$$\boxed{(2e^t, 2e^t, 3e^t)^T}$$

$b = 0$, $c = 4$ ... $a = 4$, $b = 0$, $c = 4$, $d = 4$, $f = 6$,
$g = 1$

$$\boxed{(4te^t, (4t+4)e^t, (6t+1)e^t)^T}$$

FUND. MATICA: $Y(t) = \begin{pmatrix} 1 & 2e^t & 4te^t \\ 0 & 2e^t & (4t+4)e^t \\ 1 & 3e^t & (6t+1)e^t \end{pmatrix}$

3. $$\begin{pmatrix} x \\ y \\ z \end{pmatrix}' = \begin{pmatrix} -2 & 1 & -2 \\ 1 & -2 & 2 \\ 1 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad p(\lambda) = -(\lambda+1)^3$$

$$\underline{\lambda_{1,2,3} = -1}$$

METÓDA ZOVŠEOBEC. VLAST. VEKTOROV (WEYR)

$\underline{\lambda = -1}$ $\quad n = 3 \, , \quad m(\lambda) = 3 \leadsto 3 - 3 = \underline{\underline{0}}$

$(A+I)^0 = I \quad \dots \quad \underline{h_0 = 3}$

$$(A+I)^1 = \begin{pmatrix} -1 & 1 & -2 \\ 1 & -1 & 2 \\ 1 & -1 & +2 \end{pmatrix} \dots \underline{h_1 = 1}$$

$$(A+I)^2 = \begin{pmatrix} -1 & 1 & -2 \\ 1 & -1 & 2 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} -1 & 1 & -2 \\ 1 & -1 & 2 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \dots \underline{\underline{h_2 = 0}}$$

$\downarrow$

$\underline{\underline{k = 2}}$

nulity : $\nu_0 = 0 < \nu_1 = 2 < \nu_2 = 3$

Weyrove charakteristiky : $\sigma_1 = \nu_1 - \nu_0 = 2$

$\sigma_2 = \nu_2 - \nu_1 = 1$

Weyrova tabuľka :

$$\begin{array}{c} k \\ \| \\ 2 \end{array} \left\{ \begin{array}{|c|c|} \hline v_{11} & v_{12} \\ \hline v_{21} & \\ \cline{1-1} \end{array} \right. \begin{array}{l} \dots \sigma_1 = 2 \\ \dots \sigma_2 = 1 \end{array}$$

$\dots\quad v_{21}:\quad (A+I)^2\cdot v_{21}=0\quad ;\quad (A+I)\,v_{21}\neq 0$

$v_{11}:\quad v_{11}=(A+I)\cdot v_{21}$

$v_{12}:\quad (A+I)\,v_{12}=0\quad ;\quad v_{12}\neq 0$

$v_{21}=\begin{pmatrix}a\\b\\c\end{pmatrix}\ \dots\ (A+I)^2\cdot v_{21}=0$

$0\cdot\begin{pmatrix}a\\b\\c\end{pmatrix}=0\ \dots$ platí pre $\forall\begin{pmatrix}a\\b\\c\end{pmatrix}$

$$(A+I)\cdot v_{21}=\begin{pmatrix}-1&1&-2\\1&-1&2\\1&-1&2\end{pmatrix}\cdot\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}-a+b-2c\\a-b+2c\\a-b+2c\end{pmatrix}\neq 0$$

$\dots\ \underline{-a+b-2c\neq 0}\ \dots$ napr. $a=1,\ b=c=0$

$\boxed{v_{21}=(1,0,0)^T}$

$v_{11}=(A+I)\cdot v_{21}=\boxed{(-1,1,1)^T}$

$v_{12}=\begin{pmatrix}u\\v\\w\end{pmatrix}\dots\ (A+I)\cdot v_{12}=0\ \dots\ \begin{pmatrix}-1&1&-2\\1&-1&2\\1&-1&2\end{pmatrix}\cdot\begin{pmatrix}u\\v\\w\end{pmatrix}=0$

$\Longleftrightarrow\ u-v+2w=0$

napr. $w=0,\ u=+1,\ v=1$

$\dots\boxed{v_{12}=(1,1,0)^T}$

(v_{12}, v_{11} musia byť LNZ !!!)

v) LNZ riešenia:

$$e^{-t} \cdot v_{11} = \begin{pmatrix} -e^{-t} \\ e^{-t} \\ e^{-t} \end{pmatrix} \; ;$$

$$e^{-t} \cdot \left[v_{21} + t \cdot v_{11} \right] = \begin{pmatrix} (1-t)e^{-t} \\ t e^{-t} \\ t e^{-t} \end{pmatrix}$$

$$e^{-t} \cdot v_{12} = \begin{pmatrix} e^{-t} \\ e^{-t} \\ 0 \end{pmatrix}$$

FUND. MATICA : $Y(t) = \begin{pmatrix} -e^{-t} & (1-t)e^{-t} & e^{-t} \\ e^{-t} & t e^{-t} & e^{-t} \\ e^{-t} & t e^{-t} & 0 \end{pmatrix}$