Modelling of Living Systems: Medical Images and Physical Models

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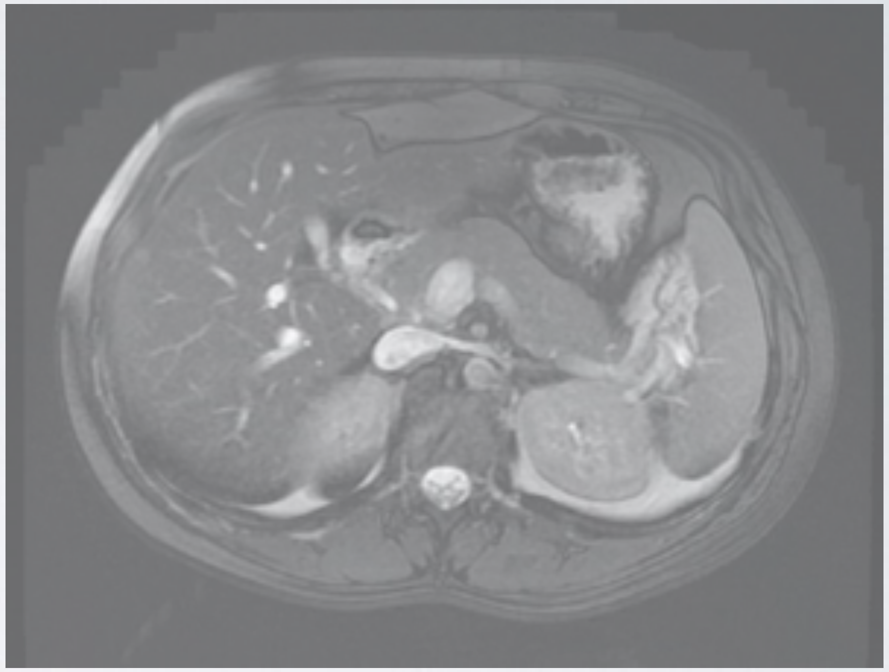
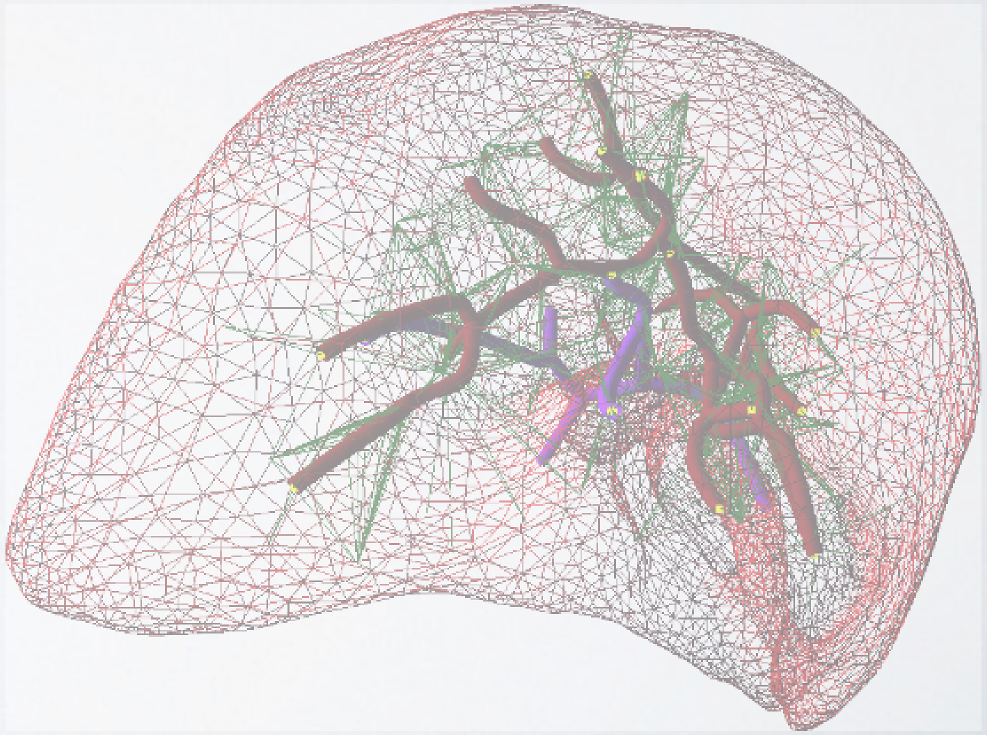
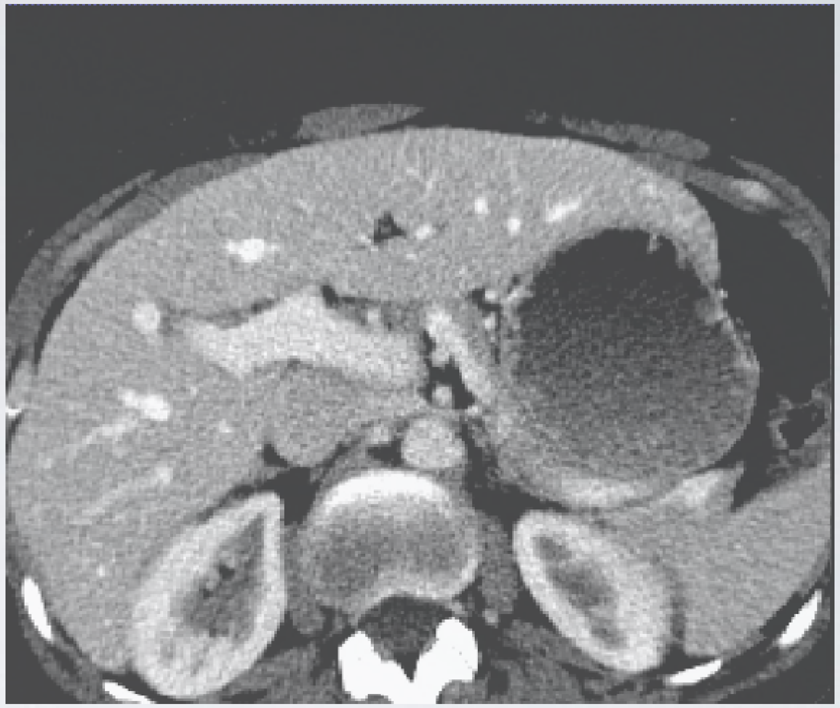
OUTLINE

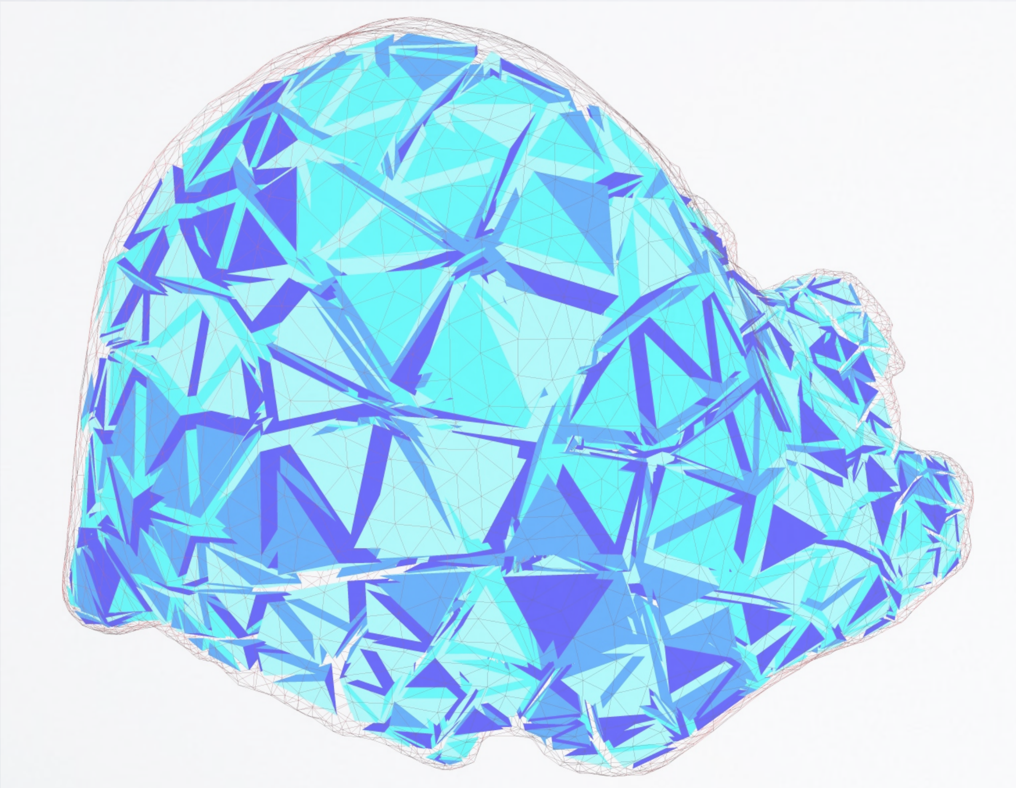
- **Motivation**
  - comparing images and models
- **Biomechanical modeling**
- **Building a Model: Geometry**
  - segmentation and meshing
- **Building a Model: Physics**
  - elastic formulation, parameters
  - discretization, finite elements
- **Example: Linear elasticity over P1 elements**
- **Numerical Solution**
  - direct and iterative solvers
  - towards dynamics







FROM **IMAGES** TO **MODELS** (AND BACK)



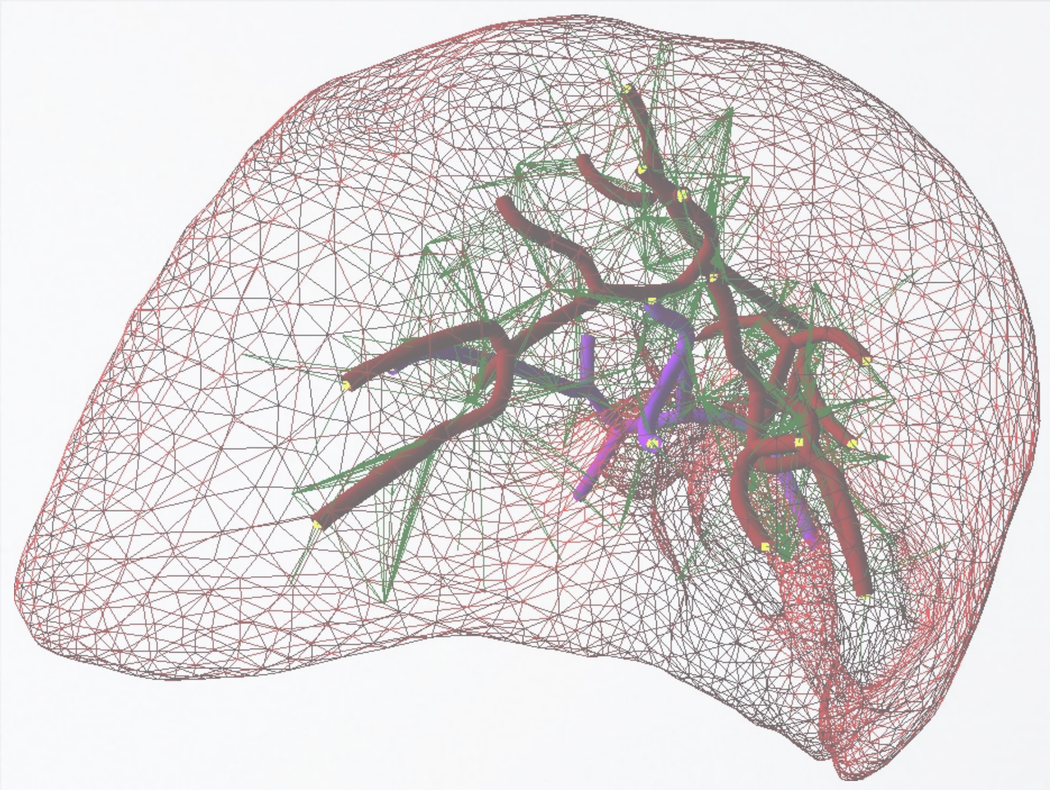
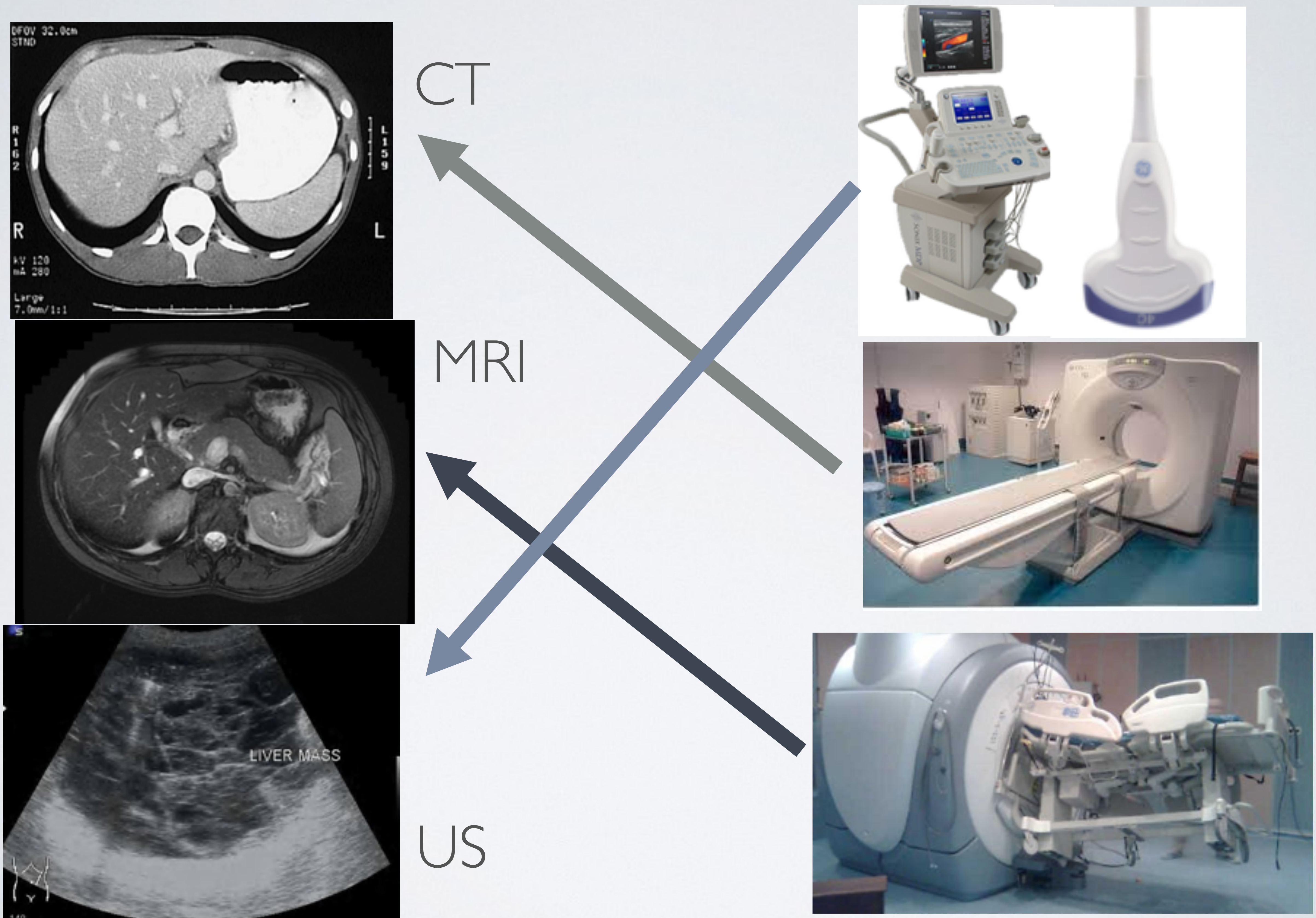
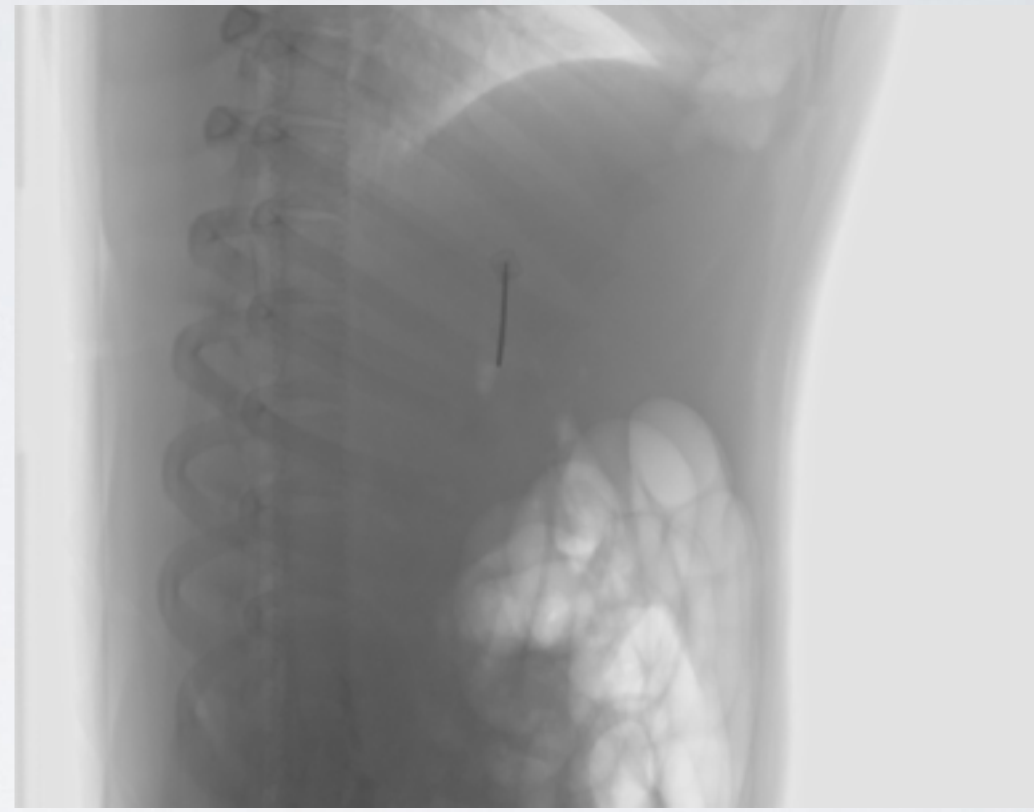
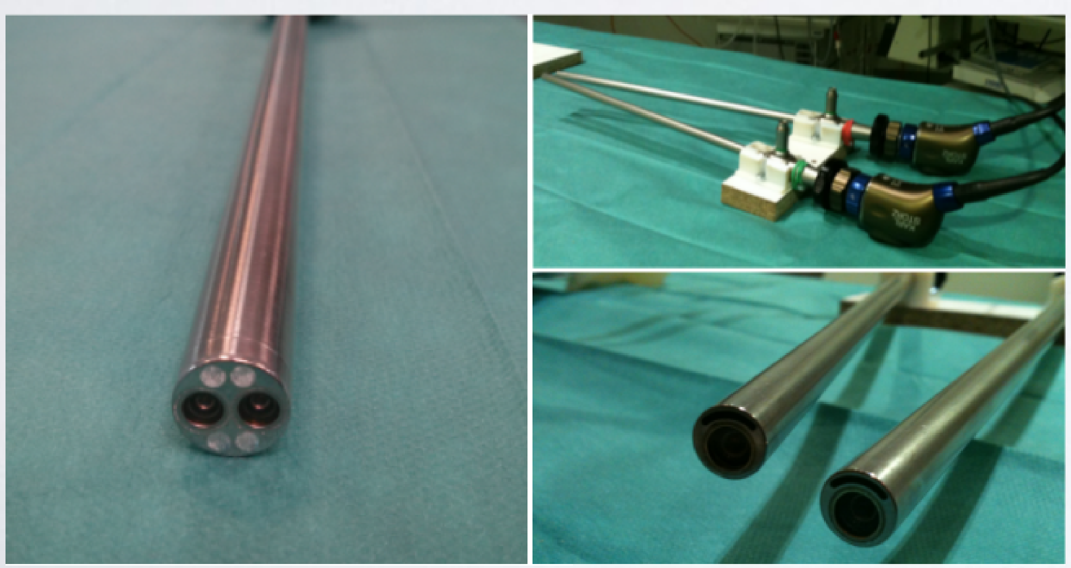


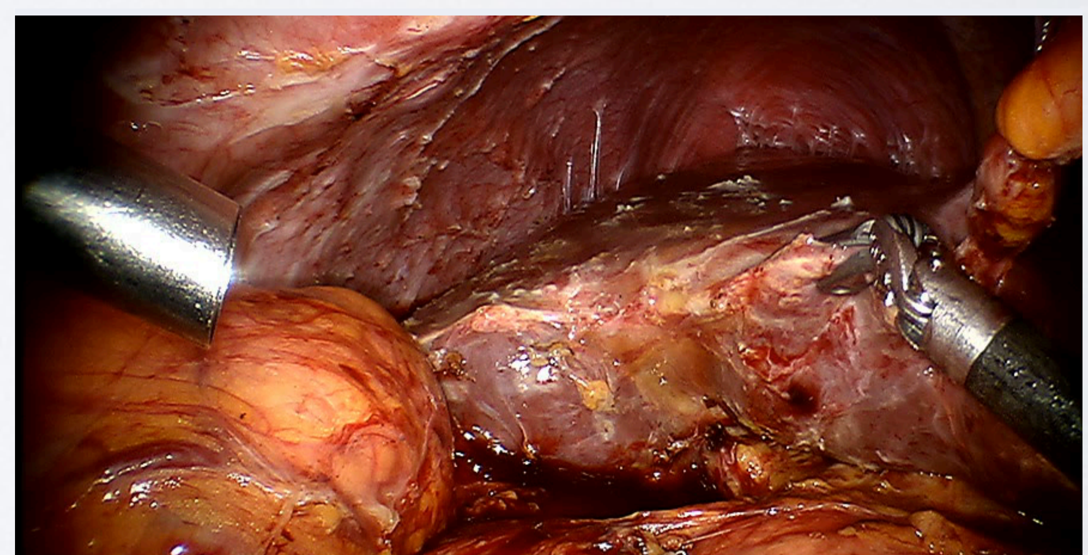
IMAGE ACQUISITION



DIFFERENT MODALITIES

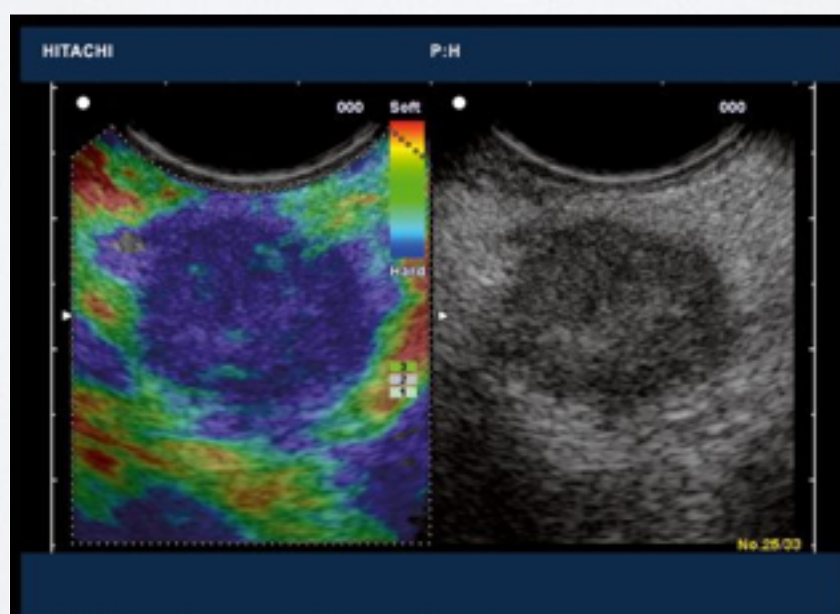


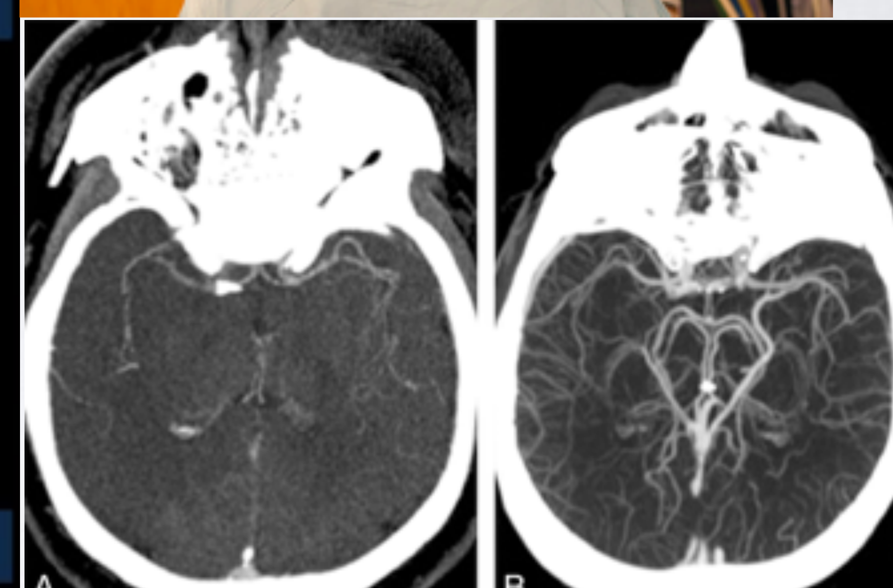




MEDICAL IMAGING: PROS

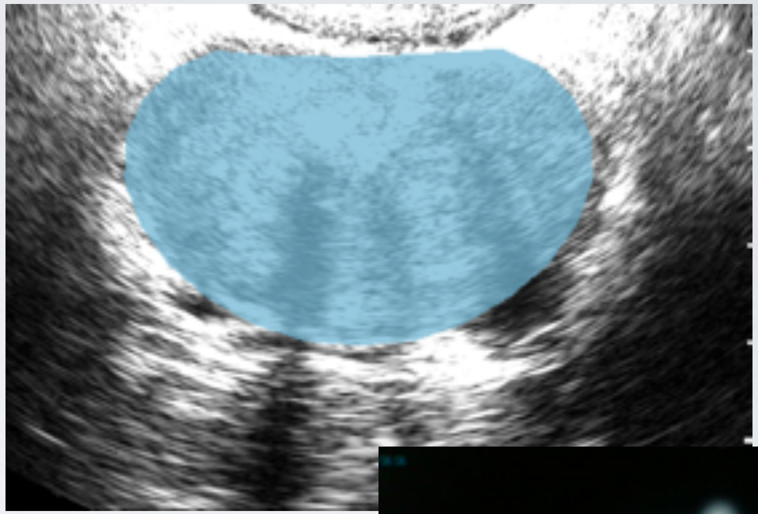
- direct output of a scanning machine
  - although already post-processed...
- doctors are used to look at images
  - familiar representation
- huge source of information
  - about geometry AND physics (elastography...)
- statistical evaluation
- (dis)-similarity
  - visually-based
  - stat/math-based

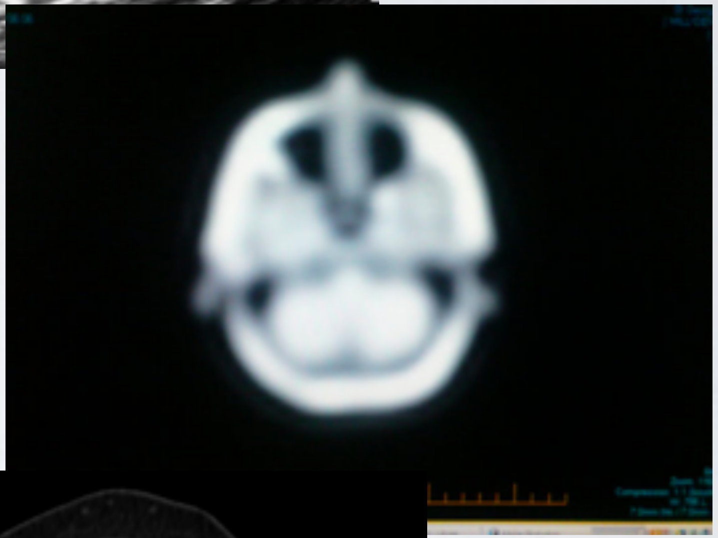




MEDICAL IMAGING: CONS

- noise
  - loosing information due to modality
- noise
  - loosing information due to motion
- noise
  - loosing information due to post-processing
- a set of pixels/voxels
  - no ***explicit*** physical meaning (although might carry enough information about physical parameters)





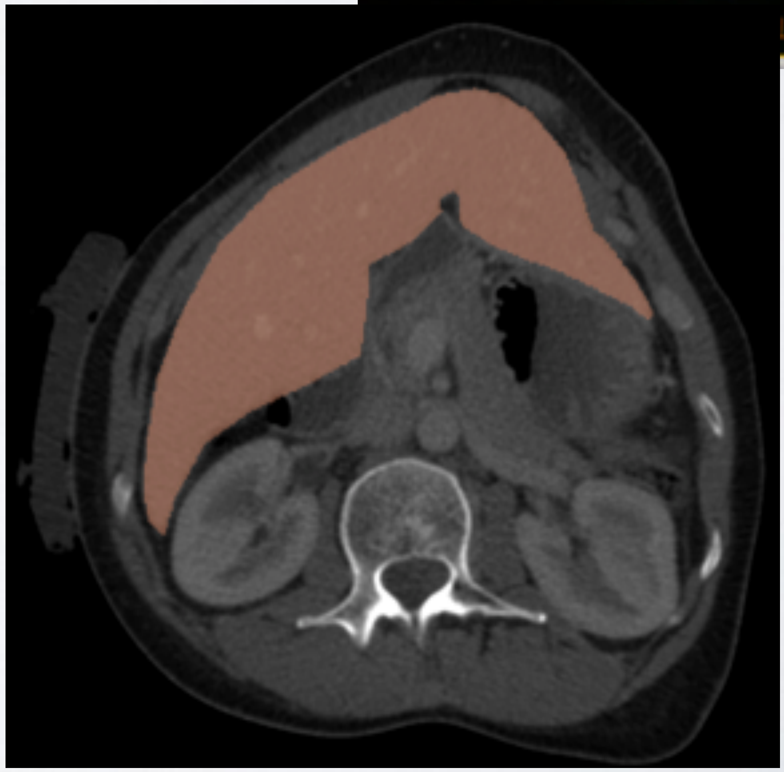
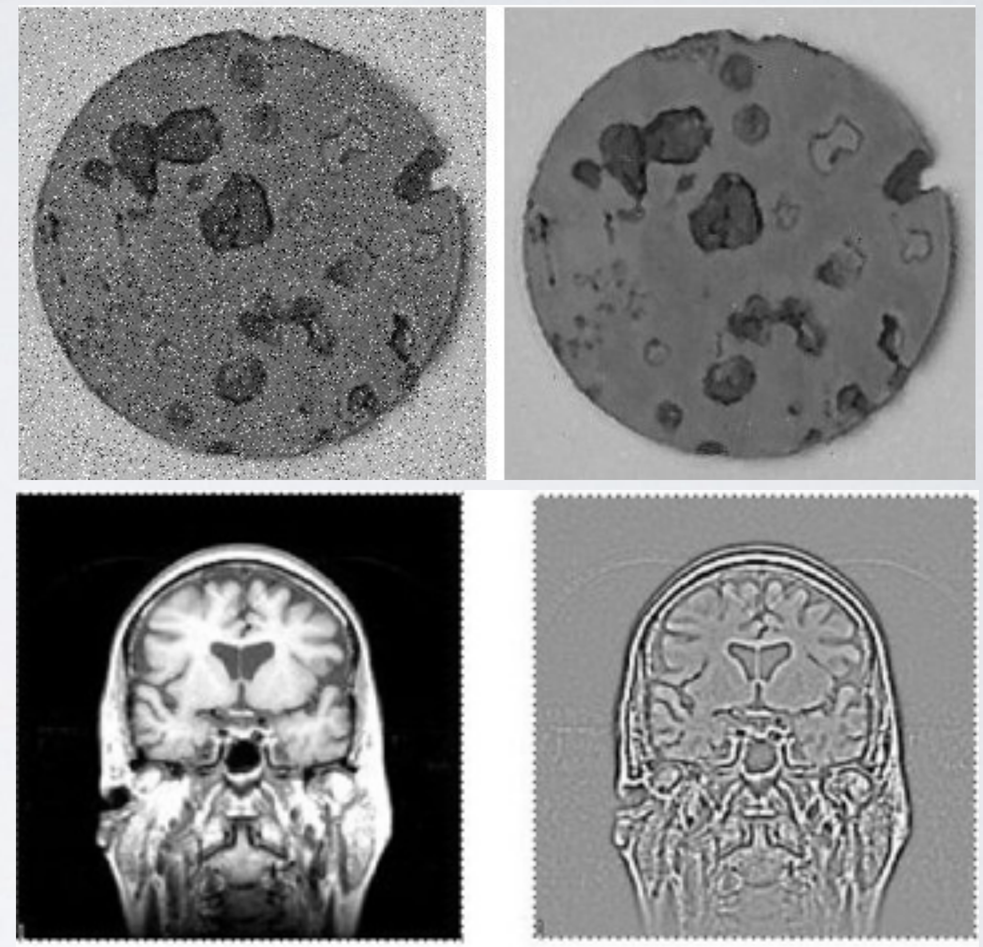


IMAGE PROCESSING

- filtering
  - smoothing, denoising, edge-detection
  - see **Slicer3D** (an open-source software)
- comparing: similarity metrics
  - mean absolute differences
  - summed squared differences
  - normalized cross-correlation
  - mutual information (different modalities)



$$MAD(I,J) = \frac{1}{N}\sum_{x,y}|I(x,y) - J(x,y)|$$

$$NCC(I,J) = \sum_{x,y}\frac{(I(x,y)-\bar{I})(J(x,y)-\bar{J})}{\sigma_I \sigma_J}$$

$$SSD(I,J) = \frac{1}{N}\sum_{x,y}(I(x,y) - J(x,y))^2$$

$$MI(I,J) = \sum_{x,y} p_{IJ}(x,y)\log\frac{p_{IJ}(x,y)}{p_I(x)p_J(y)}$$

MEDICAL IMAGE REGISTRATION

- minimizing dissimilarity
- many criteria
  - inter/intra-patient
  - single/multi-modal
  - slice-slice, volume-volume, slice-volume
  - rigid, affine, deformable
  - intensity/feature based
  - model-based
  - others...
- real-time in per-operational scenario

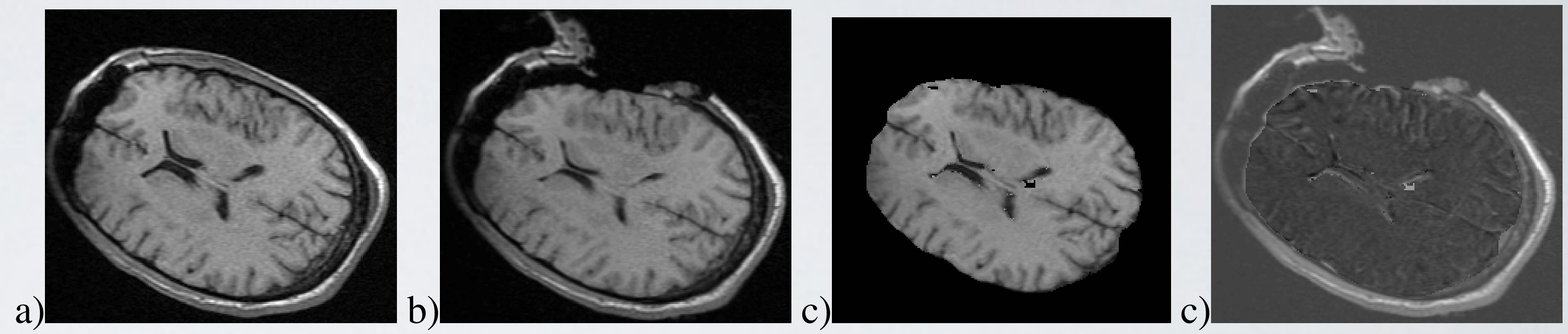
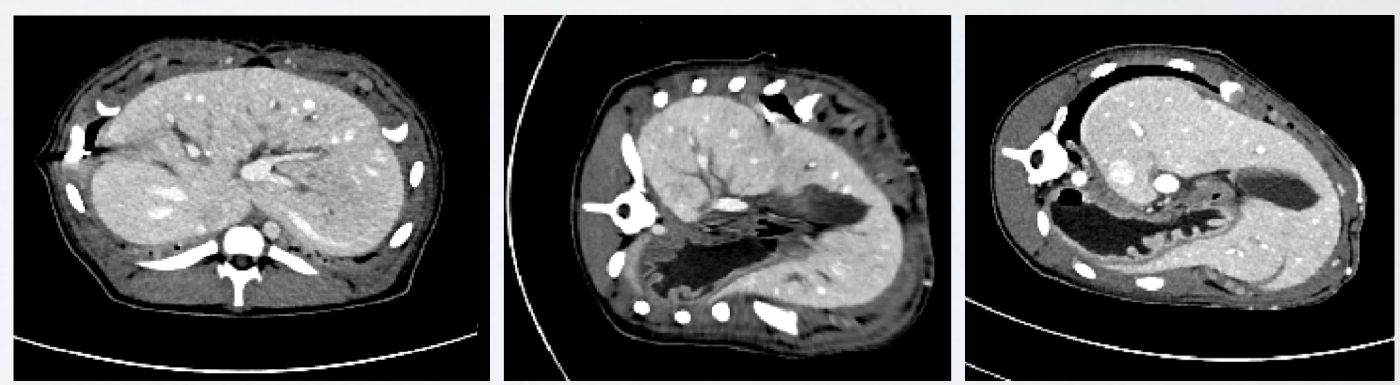


Fig. 3. Slice 29 of a) initial scan b) target scan c)initial scan deformed using our algorithm d)difference between target scan and deformed initial scan.

Ferrant et al (2001): Registration of 3-D intraoperative MR images of the brain using a finite-element biomechanical model.



(a) Source image (supine) (b) Warped image (c) Target image (flank)

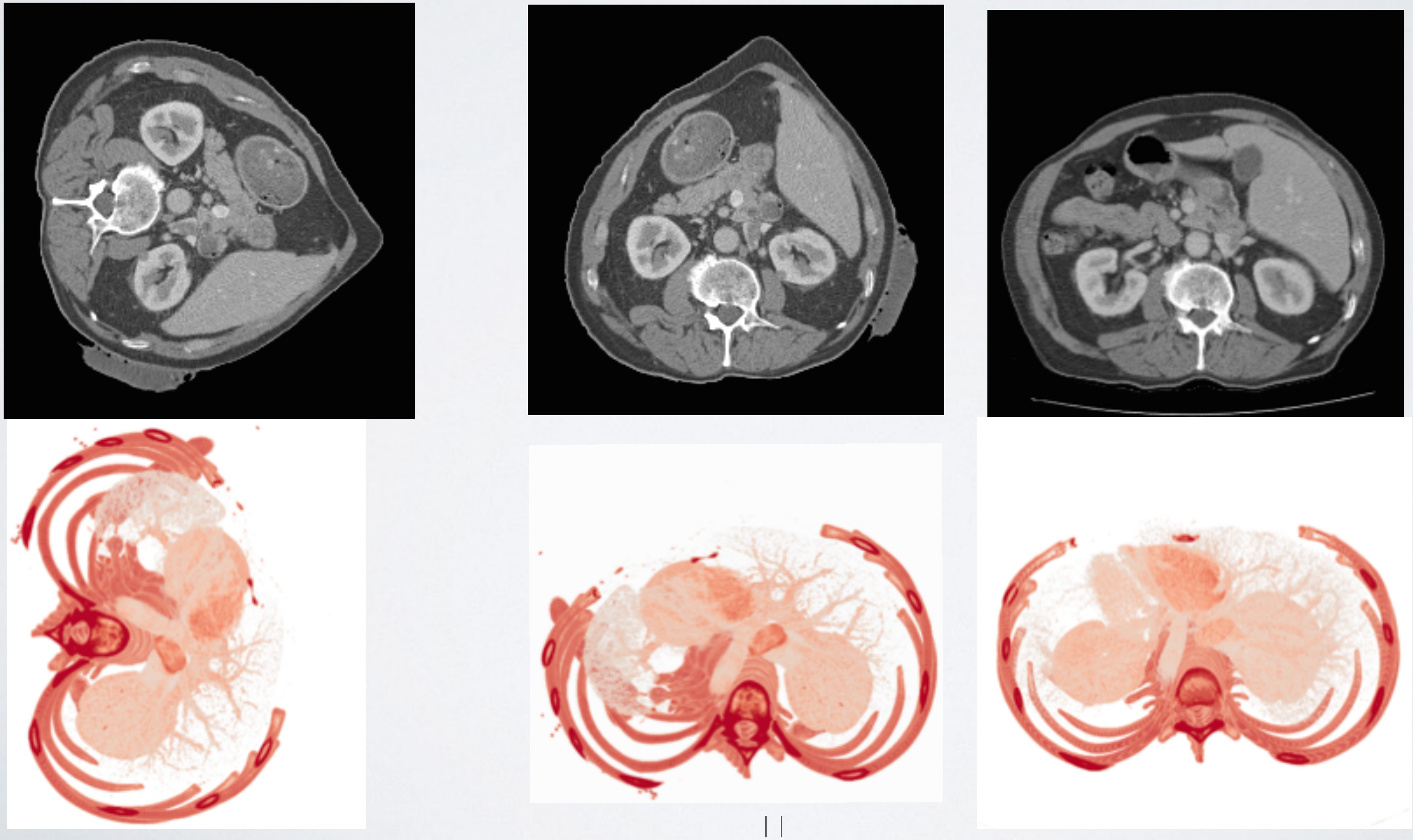
Fig. 5. Illustration of the accuracy of the registration for a cut in the source, warped and target volume.

MODELS

- **A model is an abstract structure that uses mathematical language to describe the behaviour of a system.**
- typical examples of *models of living systems*:
  - electrophysiological model: describes electrical properties of tissues
    - e.g. electrophysiological model of heart
  - model of fluid dynamics: describes behaviour of liquids
    - e.g. cardiovascular fluid mechanics (blood circulation)
  - **biomechanical model** of an organ: describes elastic (plastic) behaviour of tissues
    - e.g. hyperelastic model of liver
- the mathematical language is usually based on differential equation
  - since the behaviour usually means "a change of state"

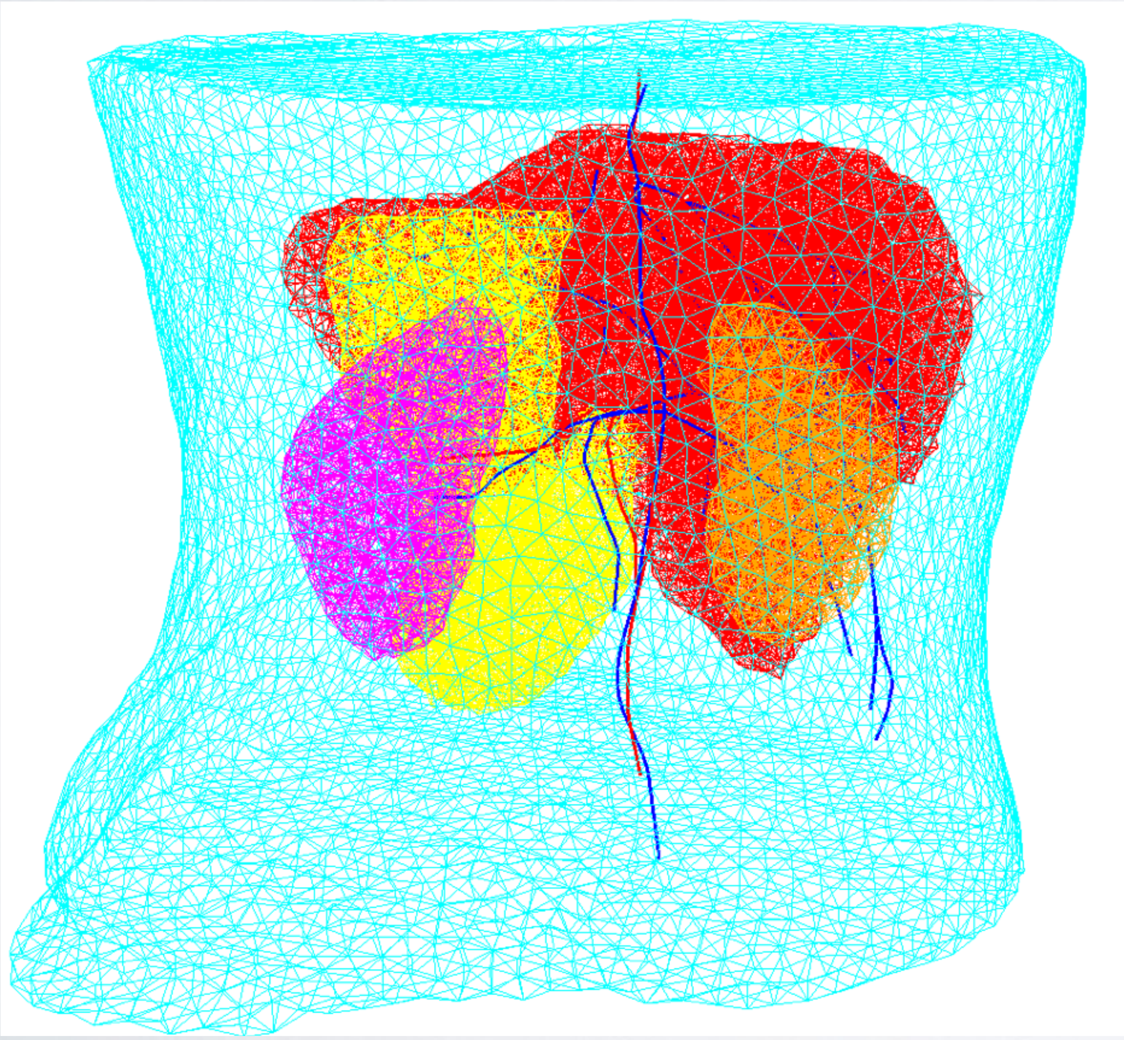
BIOMECHANICAL MODEL: EXAMPLE

- **medical image registration of volumes with important deformations**
  - two volumes taken at different configurations (pre-/intra- operational data)
  - the goal is to align (register) the two images, i.e. match the voxels



BIOMECHANICAL MODEL: EXAMPLE

- **model based solution: an "energy-minimization" problem:**
  - an "error energy" given by difference between the two data (similarity metric, difference in feature positions)
  - an "elastic energy" given by a *regularization term* provided by an elastic model



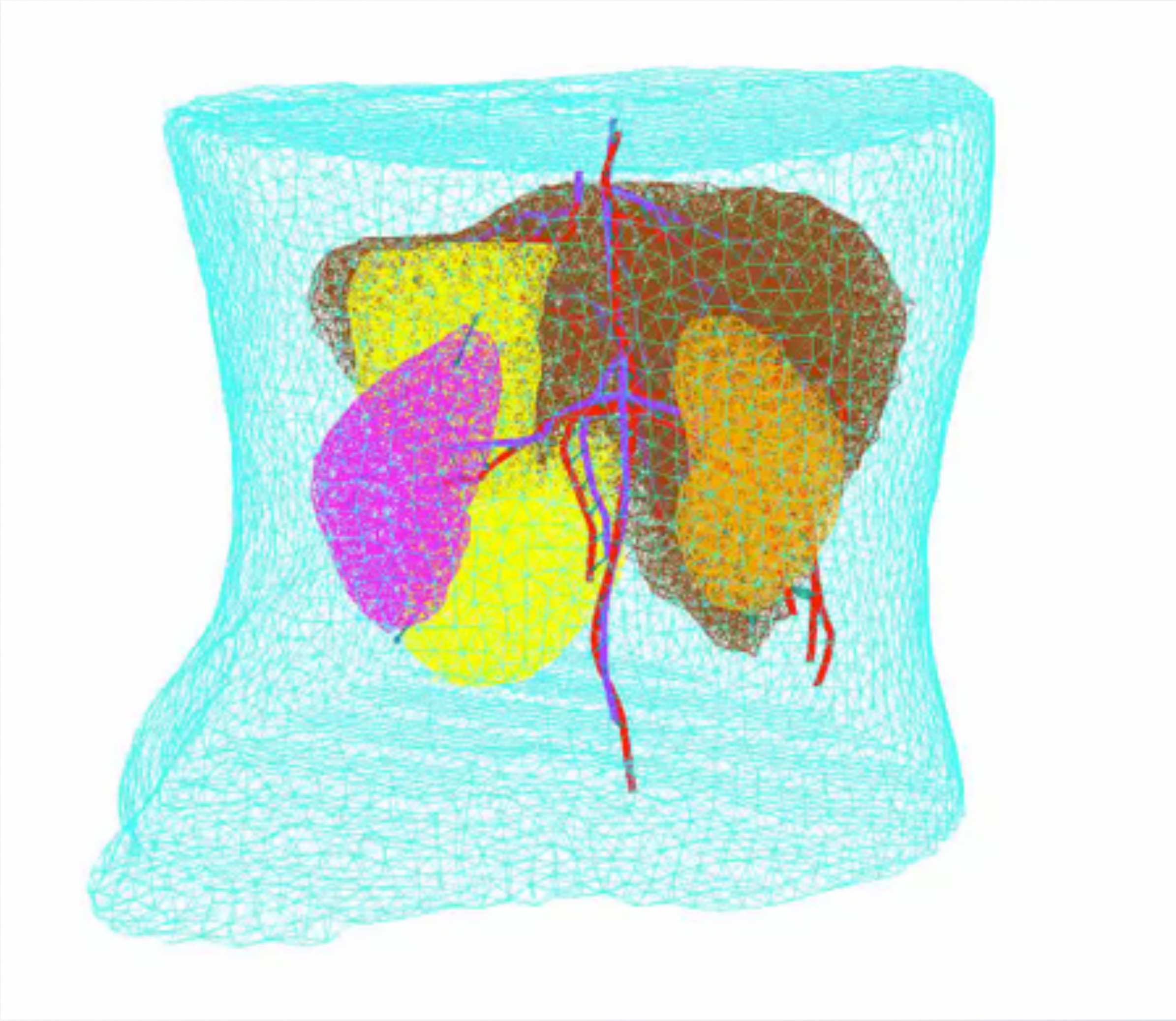
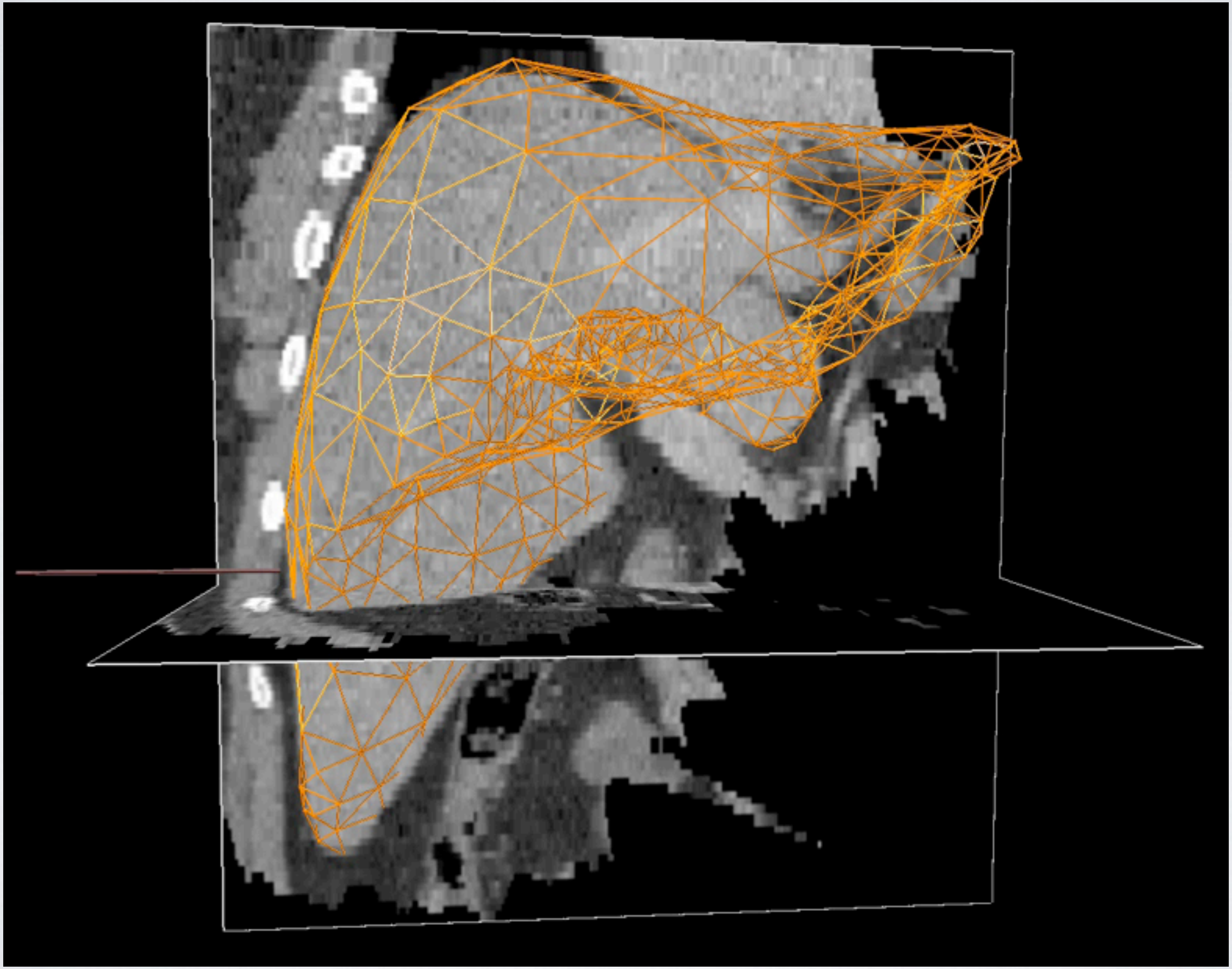
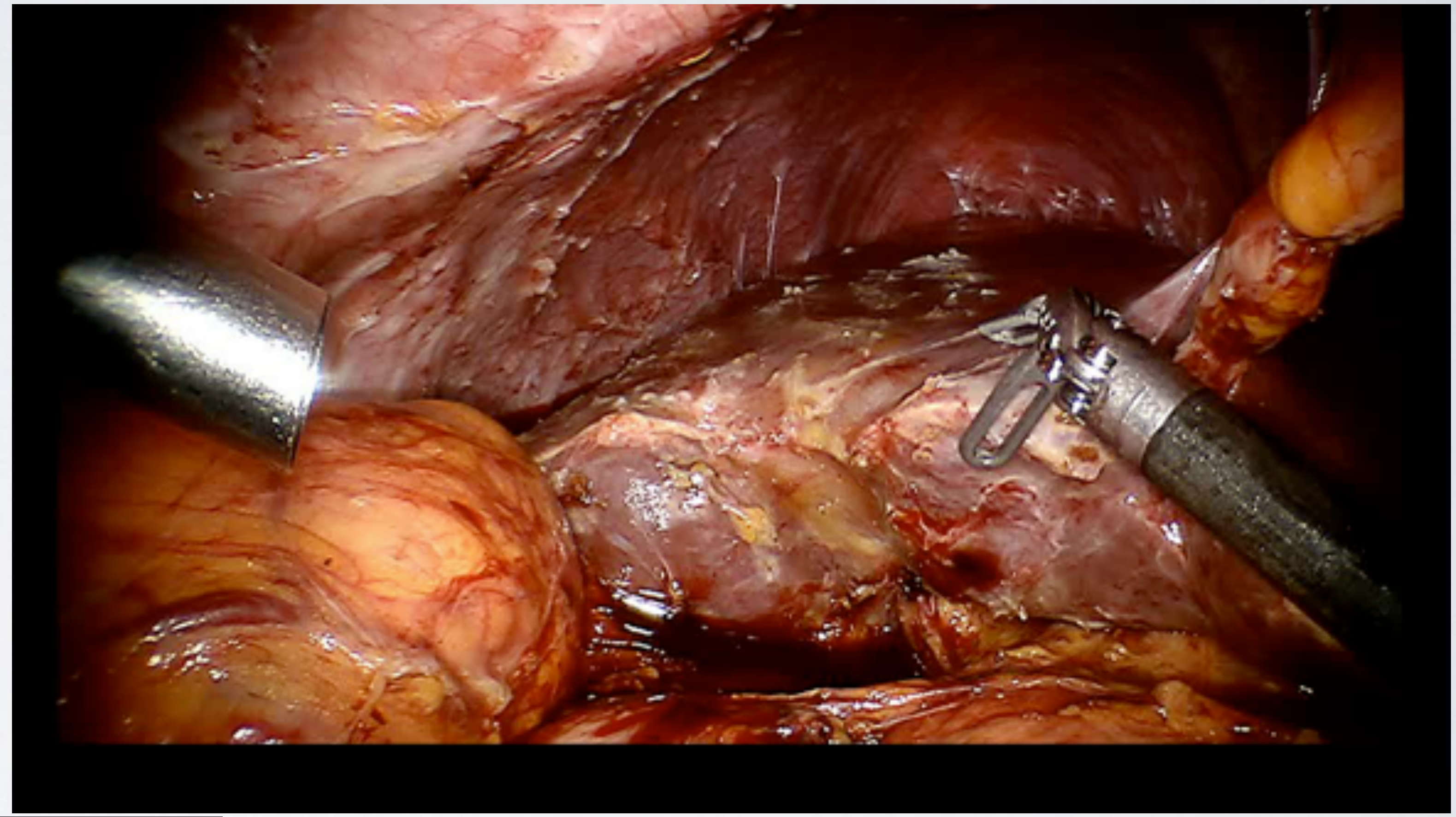
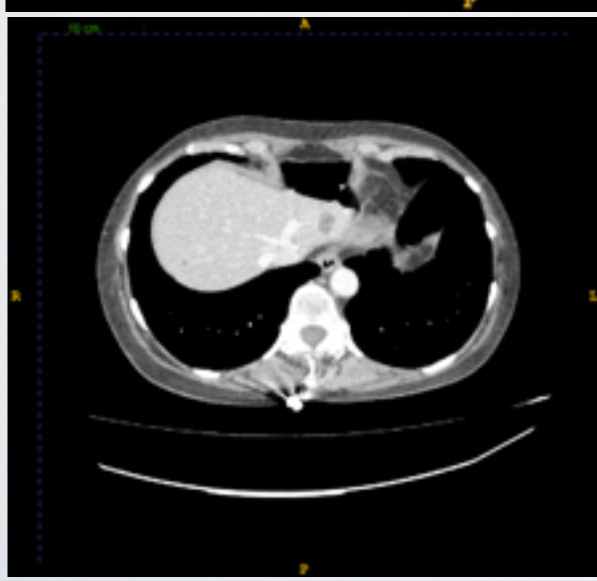


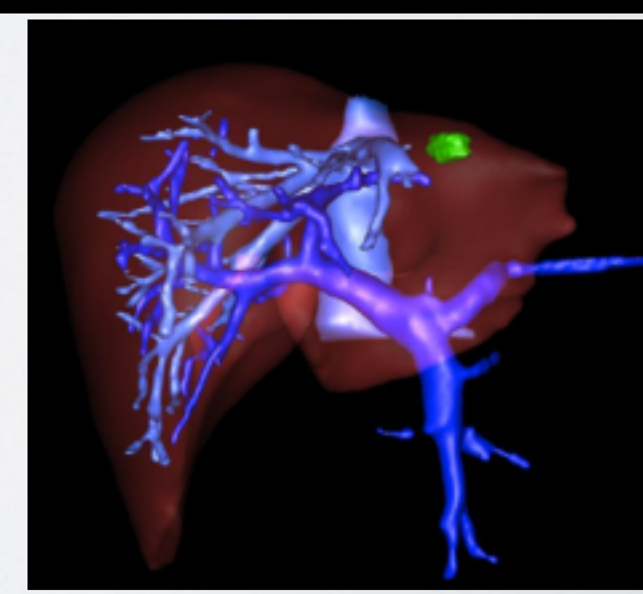
IMAGE-MODEL COUPLING



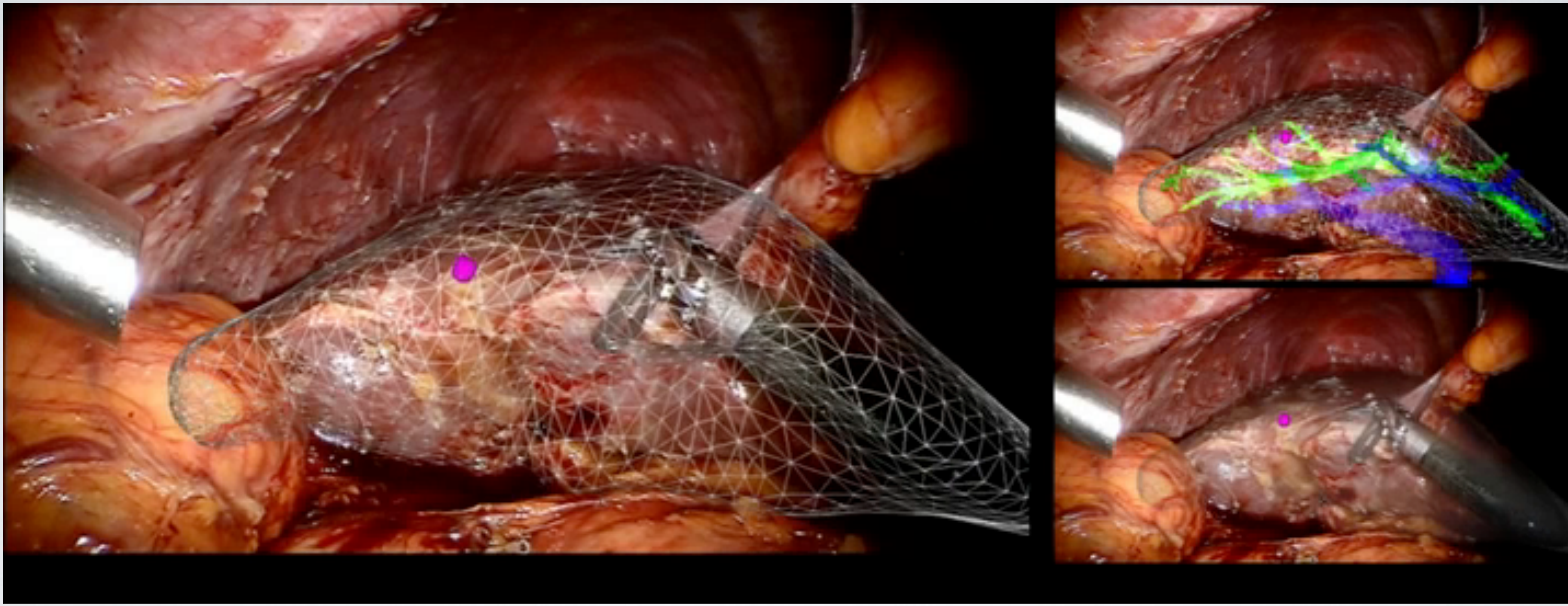
ADVANCED MODELLING







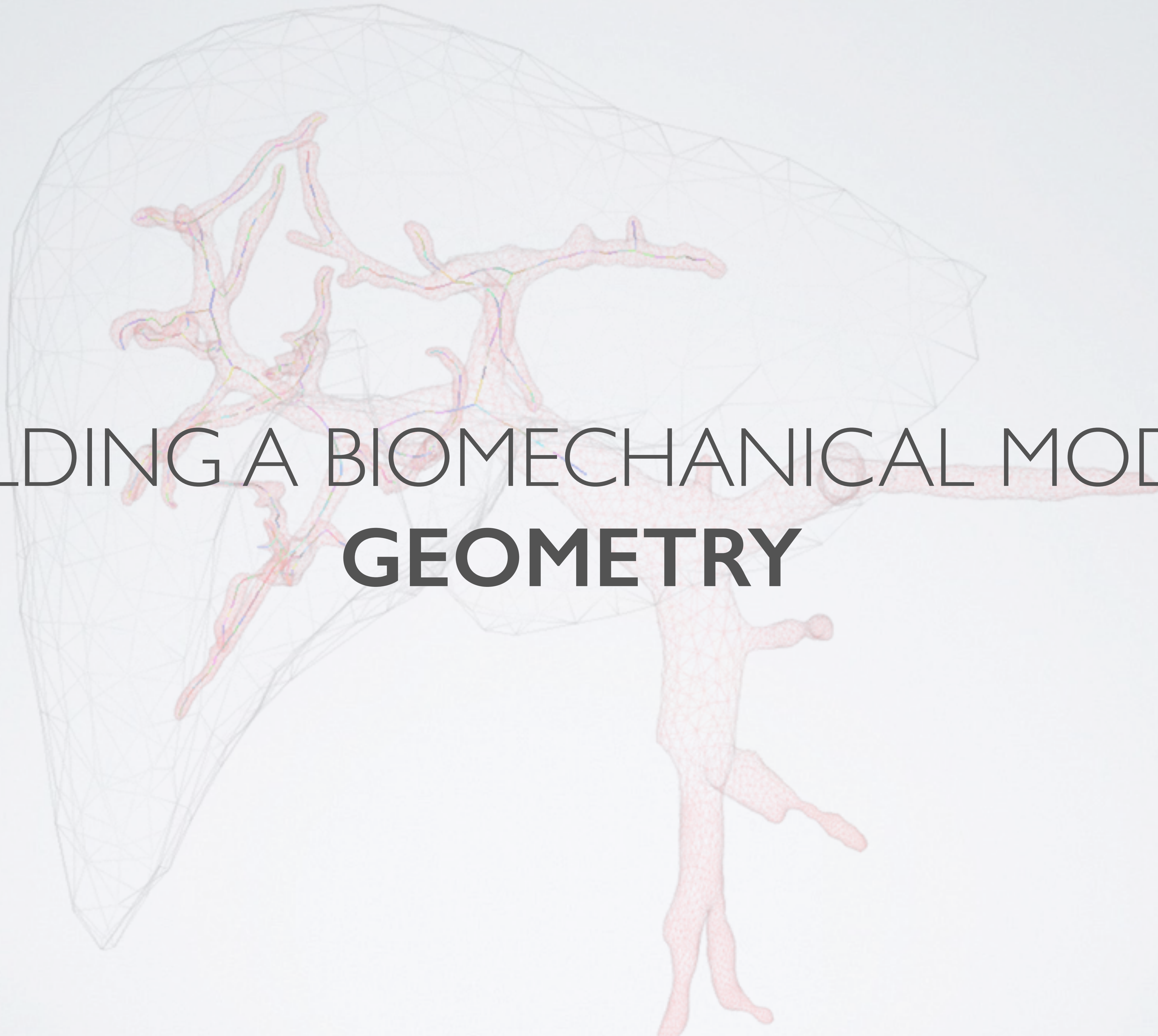
ADVANCED MODELLING



N. Haouchine, J. Dequidt, **I.P.**, E. Kerrien, M.-O. Berger, S. Cotin. Image-guided Simulation of Heterogeneous Tissue Deformation For Augmented Reality during Hepatic Surgery. In *ISMAR* proc. 2013

BUILDING A BIOMECHANICAL MODEL FROM THE IMAGE DATA

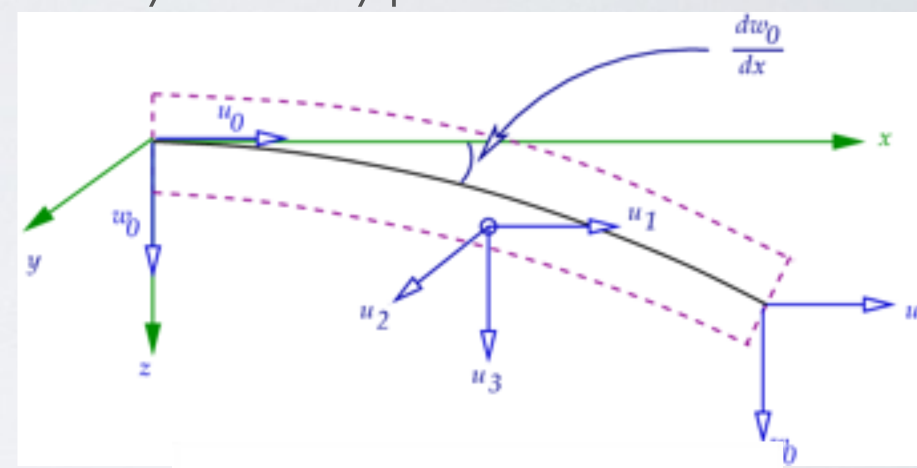
- two aspects: geometry (domain) and physics (formulation and parameters)
- the two aspects are closely interconnected
- **geometry**:
  - type of the geometry structure is given by the nature of the problem and the physical formulation (e.g. *the basic "unit" is a* ***tetrahedral element with 4 nodes***)
  - particular realisation is extracted from the image (e.g. *the domain covered by the elements is given by the* ***shape of the organ***)
- **physics:**
  - formulation is given by a set of differential equations solved over the geometric domain (e.g. ***finite element formulation of hyper-elasticity*** *over linear tetrahedra)*
  - particular behaviour is determined by the physical parameters, usually obtained by a measurement [invasive, non-invasive] (e.g. ***stiffness*** *of the liver parenchyma*)

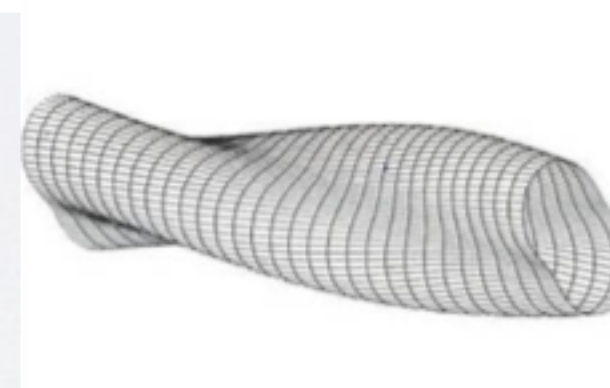


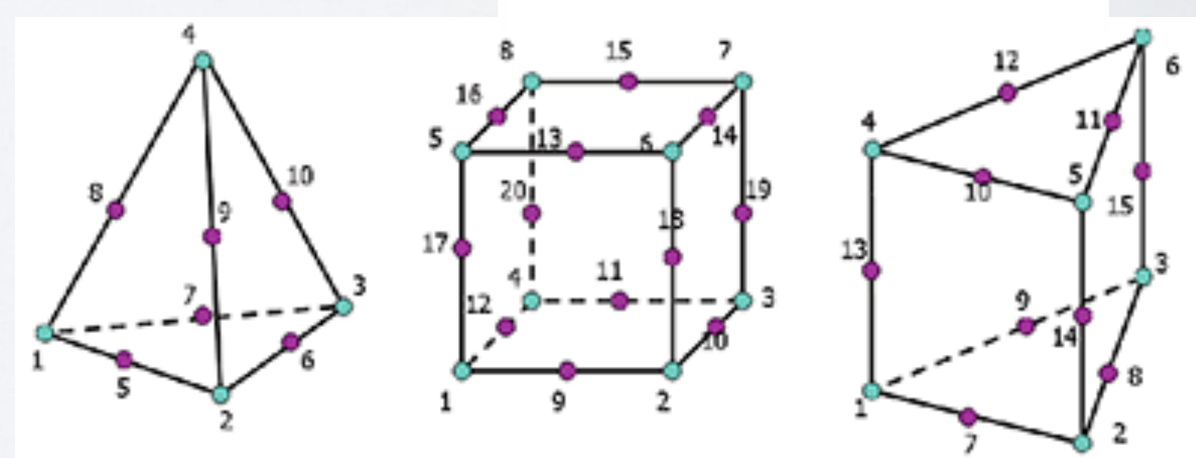
BUILDING A BIOMECHANICAL MODEL: **GEOMETRY**

GEOMETRY DISCRETIZATION: MESH

- usual geometric representation is given by a **mesh** (discretization of domain)
  - a set of (connected) elements of given dimensionality and type
  - 1D: line mesh, beam mesh, spline mesh
  - 2D: triangular- and quad-mesh, shell mesh
  - 3D: tetrahedral mesh, hexahedral mesh
  - mixed meshes
- used in computer-aided design (CAD) for decades
  - many mesh generators from CADs
  - commercial solutions (Ansys)
  - open-source: GMsh, TetGen





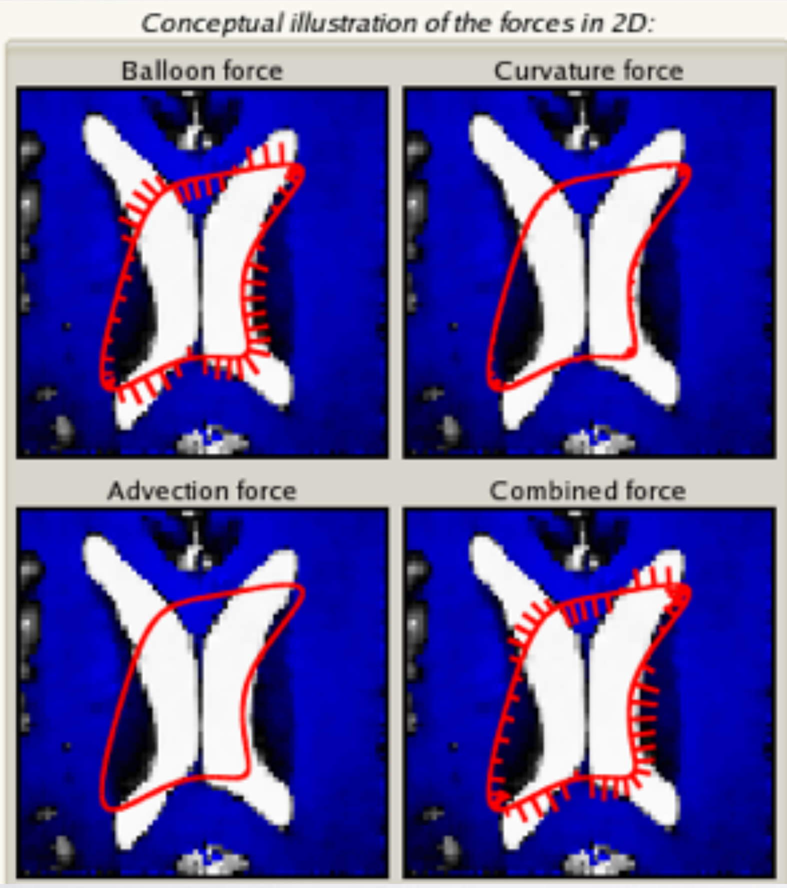


MESHING OF MEDICAL IMAGES

- classical mesh generation from images consists of two steps
- **segmentation**: delimitation of the *domain of interest* in the image
- **meshing**: discretization of the segmented domain

IMAGE SEGMENTATION I

- manual segmentation (time consuming in 3D)
- semi-automatic methods:
  - basic: histogram-based, edge detection, region-growing
  - PDE-based: active contours (snakes, subject of TP), level-sets methods
  - graph-based segmentation: using graphs flows and cuts



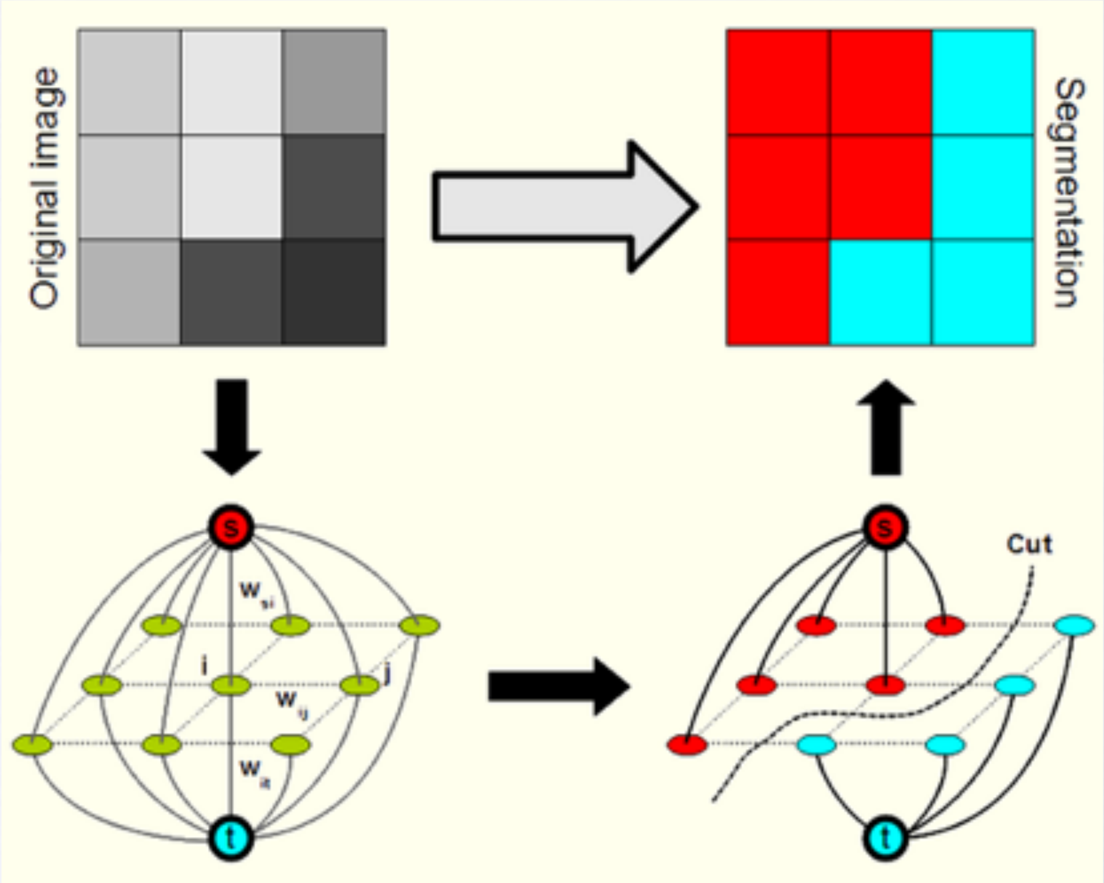
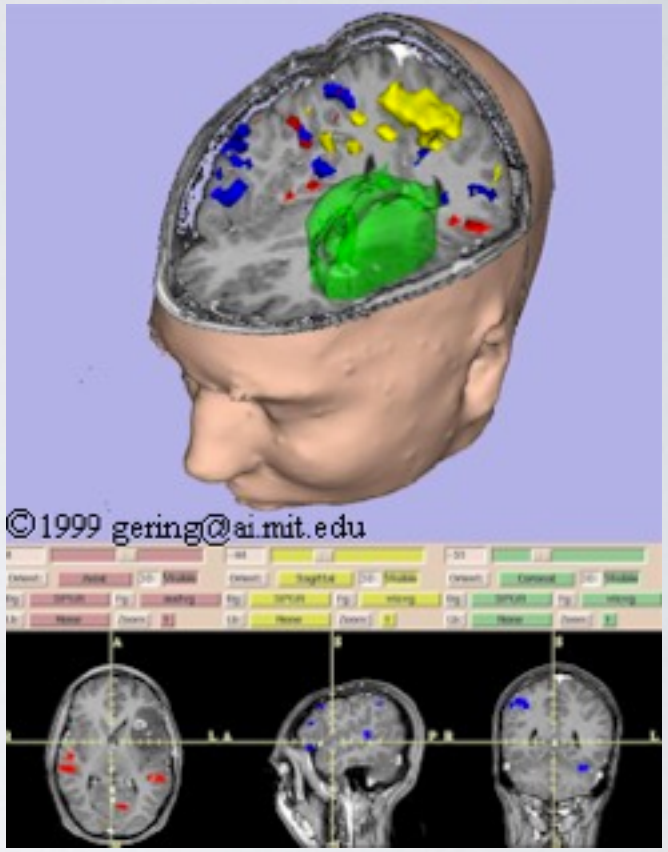
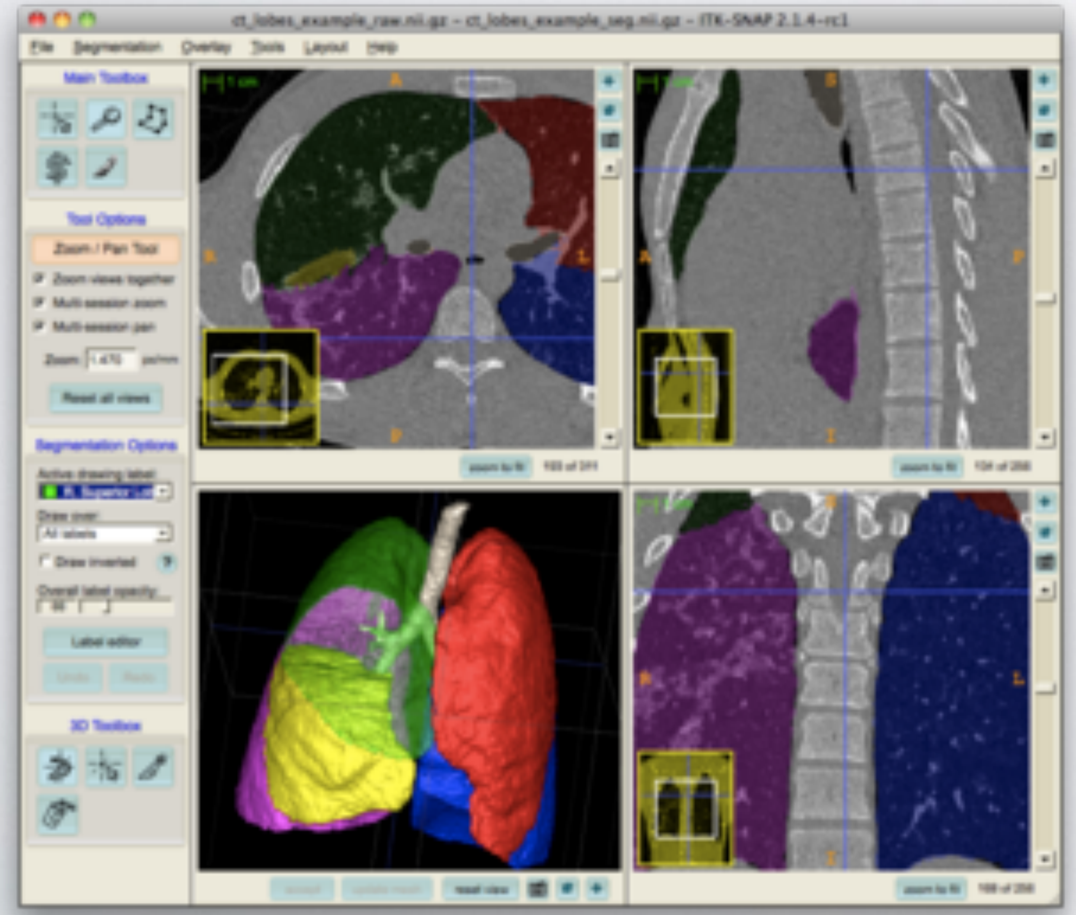
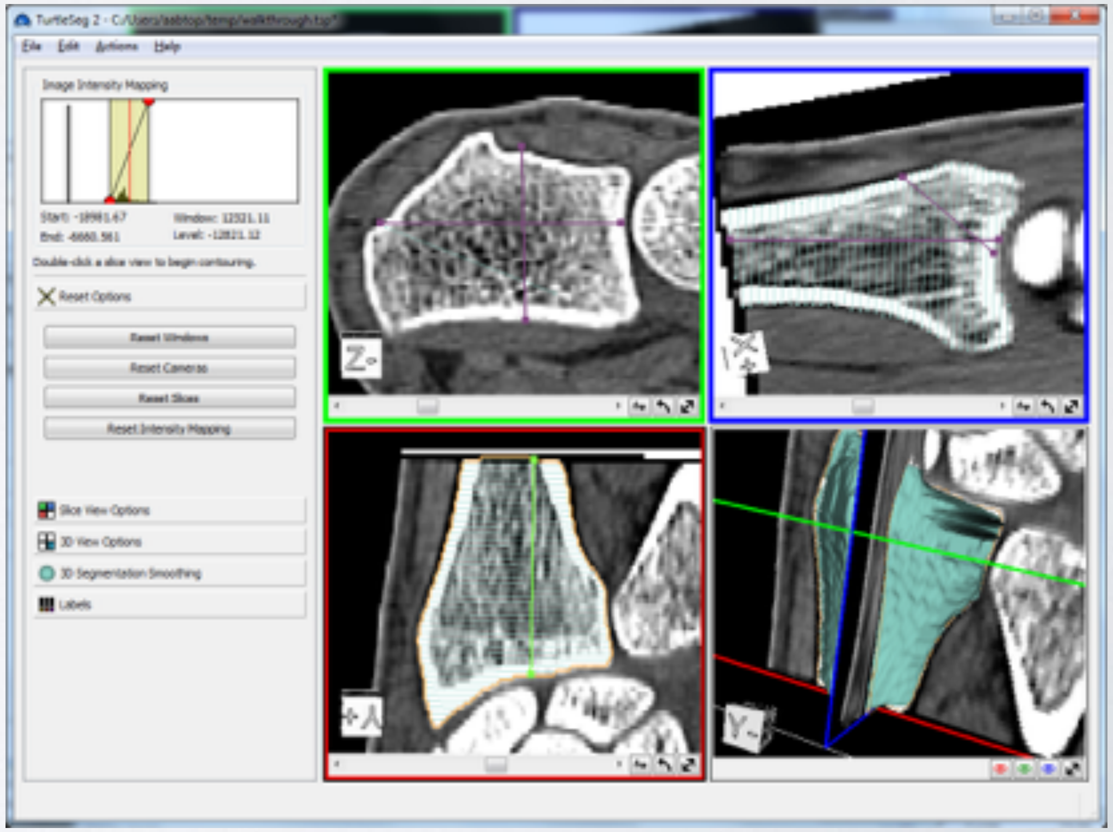


IMAGE SEGMENTATION II

- atlas-based methods
  - probabilistic methods (mean shape and possible variations)
- methods based on training
  - neural-networks
- many open-source programs: ITKSnap, 3DSlicer, TurtleSeg



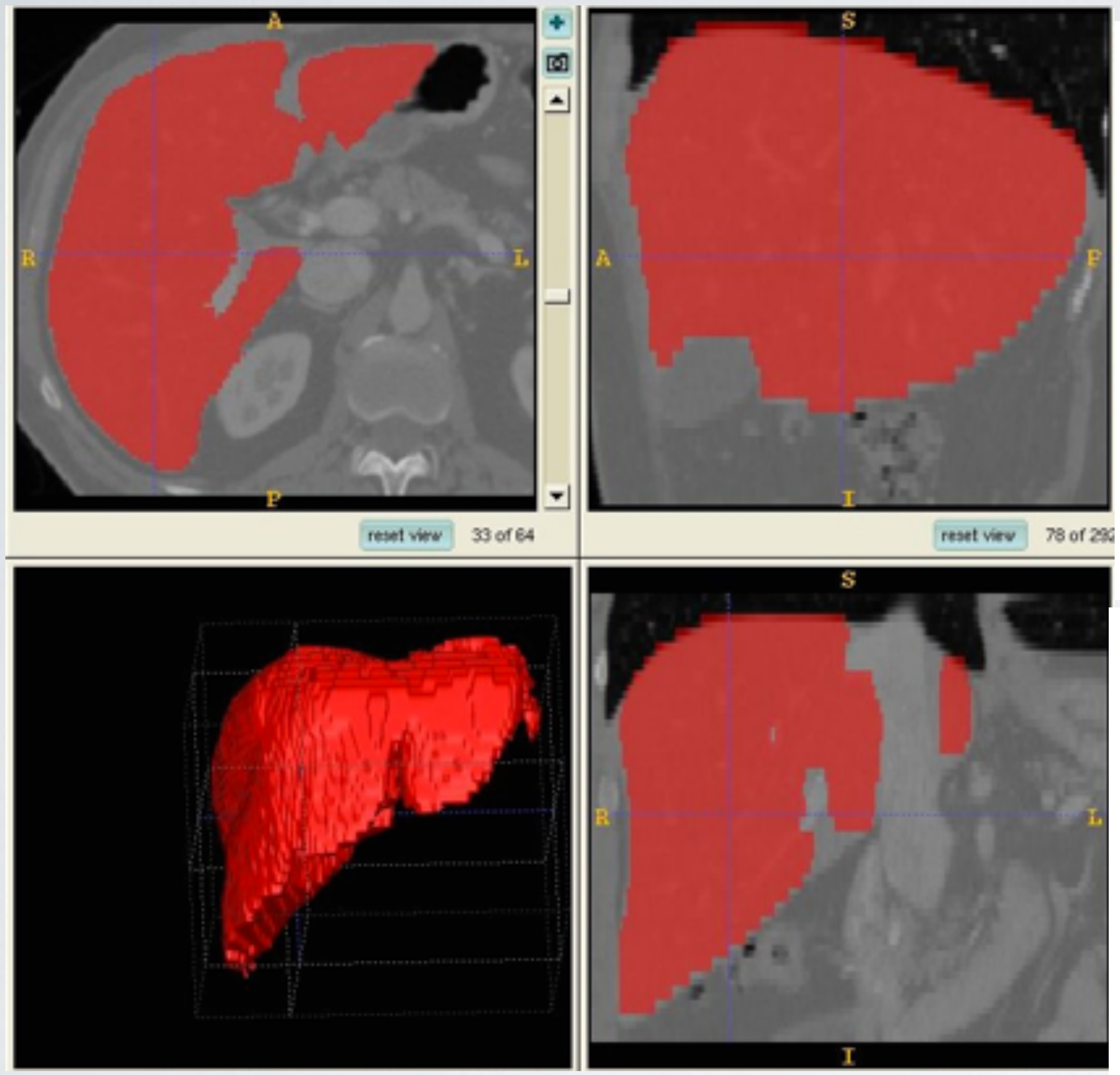


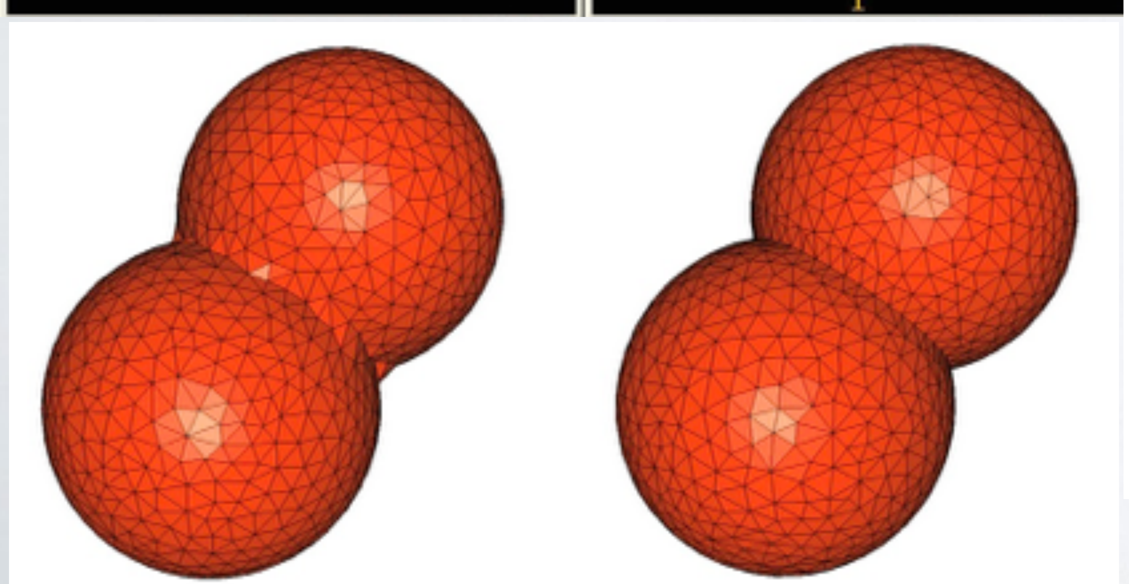


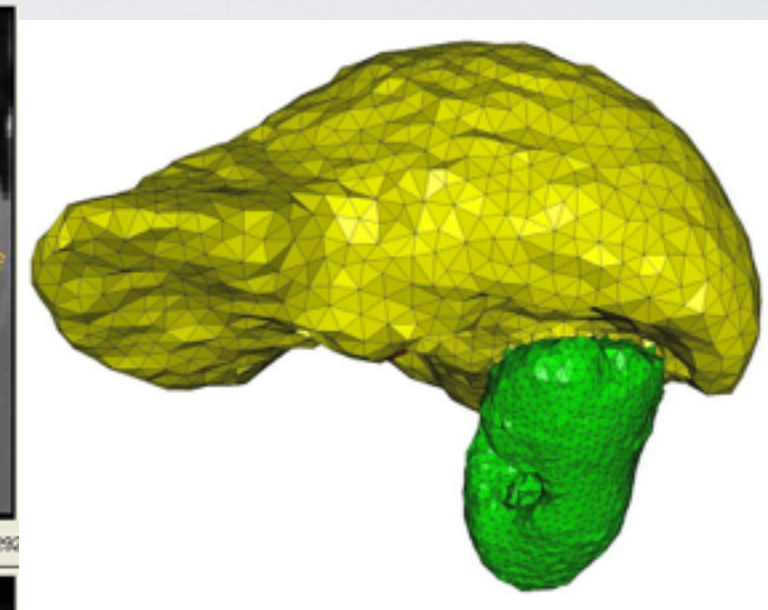
MESHING OF SEGMENTED DOMAIN I

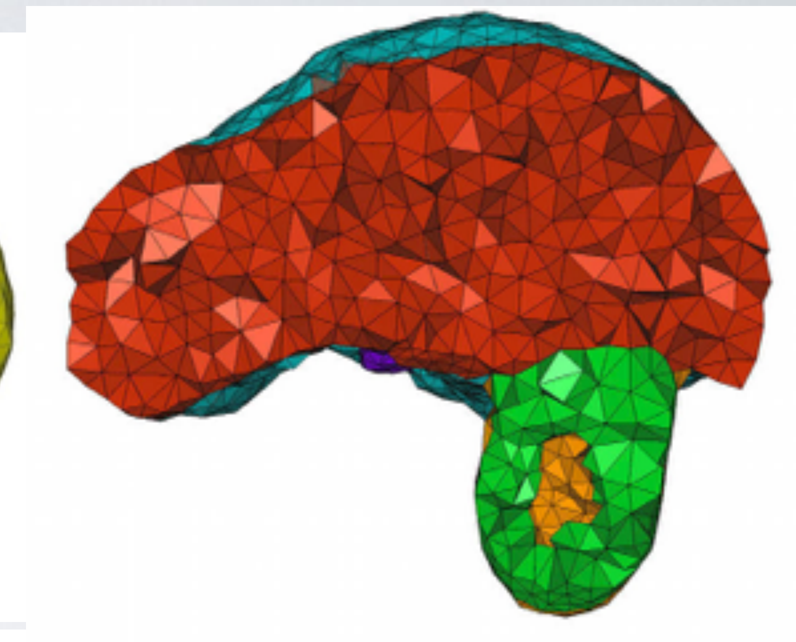
- **two-step approach:**
  - first step: generate surface representation (triangular mesh) of the segmented domain (e.g. marching cubes)
  - second step: generate 3D volume mesh from the surface mesh (e.g. TetGen computing tetrahedral mesh from surface triangular mesh stored in STL)
  - surface meshes can be very dense or with holes: reparation must be performed before the second step (e.g. MeshLab)
- **direct approach:**
  - direct generation of 3D volume mesh from the segmented domain: CGAL.org
  - can be problematic for sharp features (usually not crucial in medical imaging) and correct separation of boundaries (can be a problem, solution exists but is not implemented…)

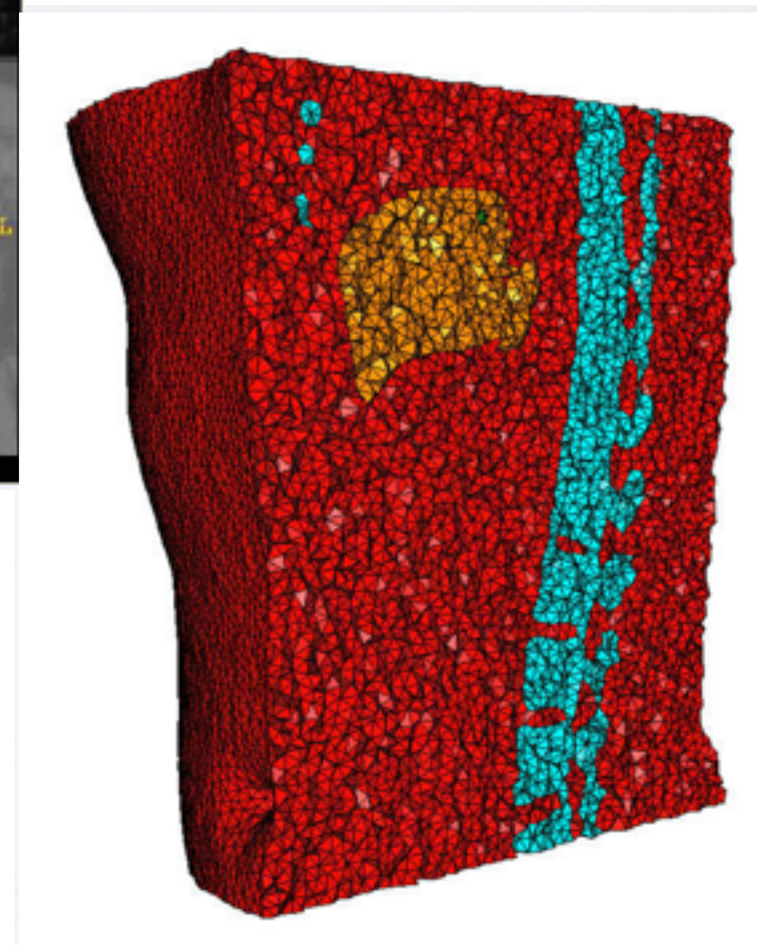
MESHING OF SEGMENTED DOMAIN II

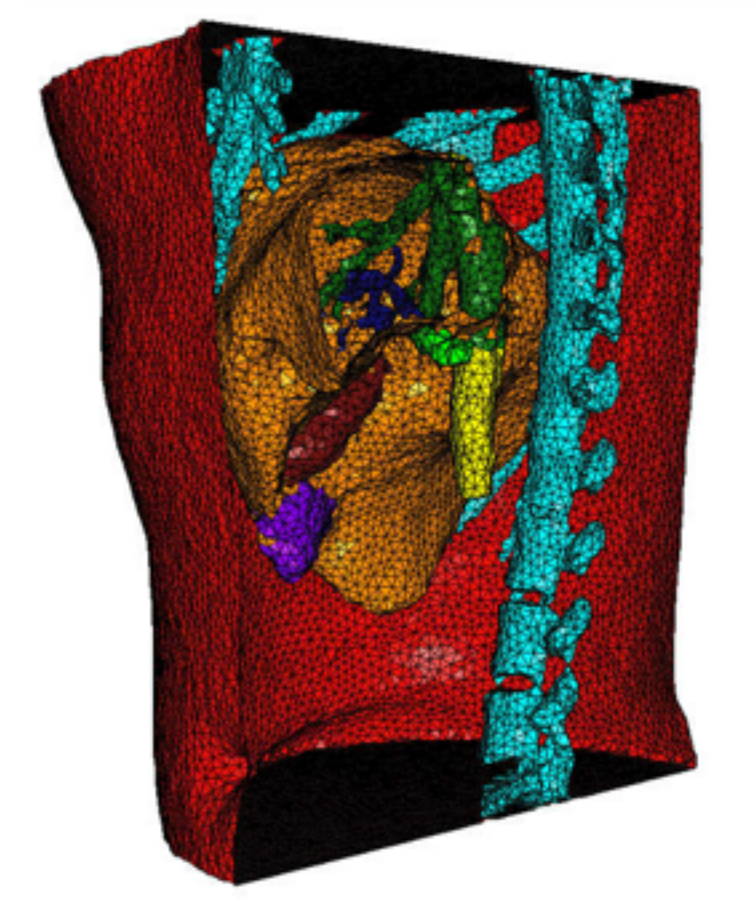






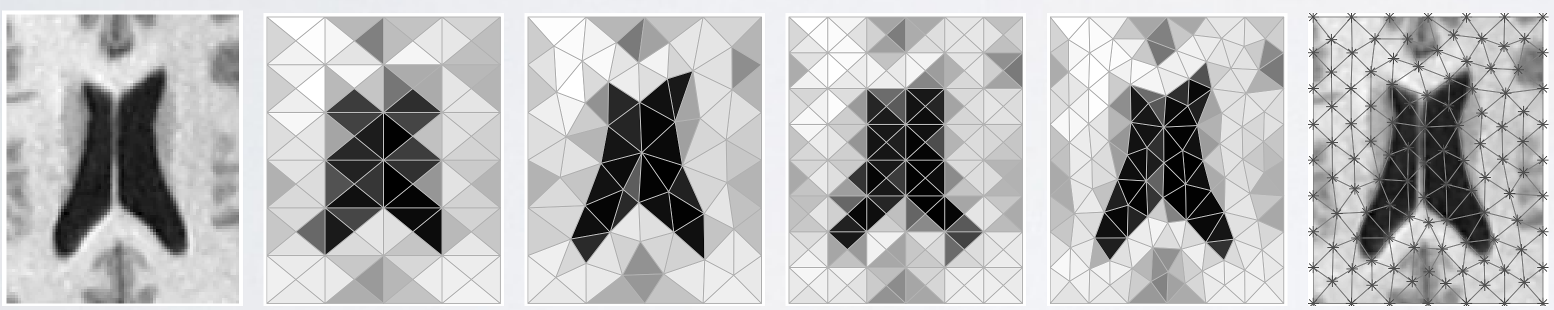






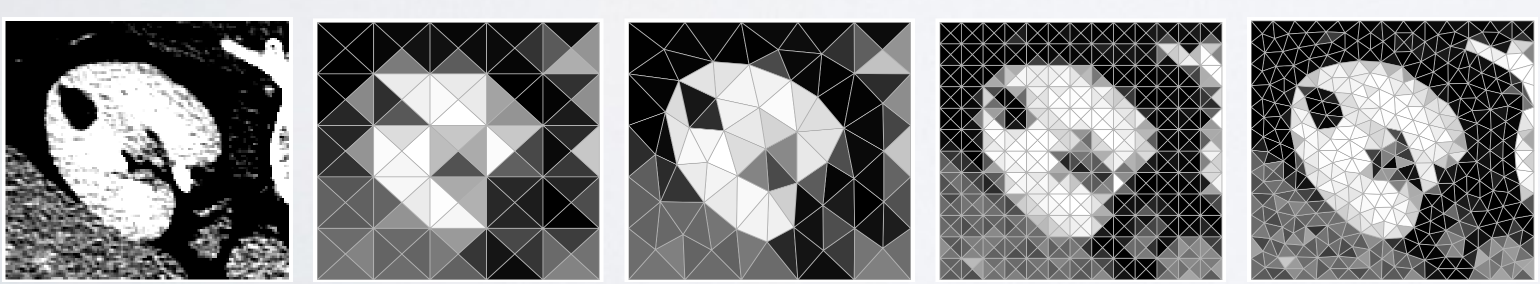
BONUS: VARIATIONAL IMAGE MESHING

- direct generation of meshes from the image
  - no segmentation needed
  - initial regular mesh is adapted to the image
  - works for limited range of intensities



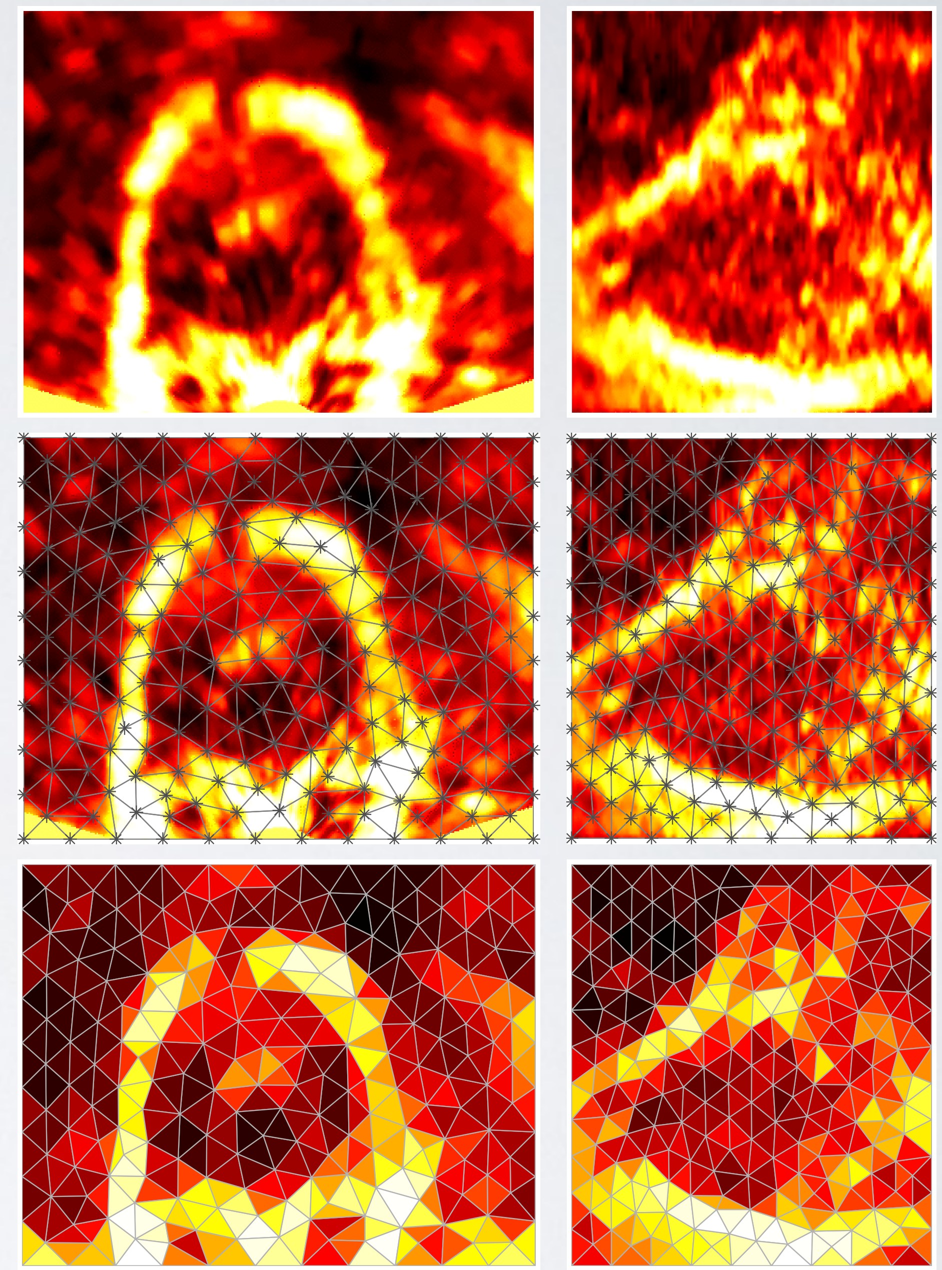
(a) (b) (c) (d) (e) (f)

Fig. 5. Mesh optimization on a 2D MR image slice (a) of brain ventricles. Initial (b) and optimized (c) discretizations with 59 nodes; initial (d) and optimized (e) discretizations with 111 nodes. The finer optimized mesh is seen as overlaid on image (f).



(a) (b) (c) (d) (e)

Fig. 6. Mesh optimization on a 2D CT image slice (a) of the kidney. Initial (b) and optimized (c) discretizations with 61 nodes; and initial (d) and optimized (e) discretizations with 338 nodes.

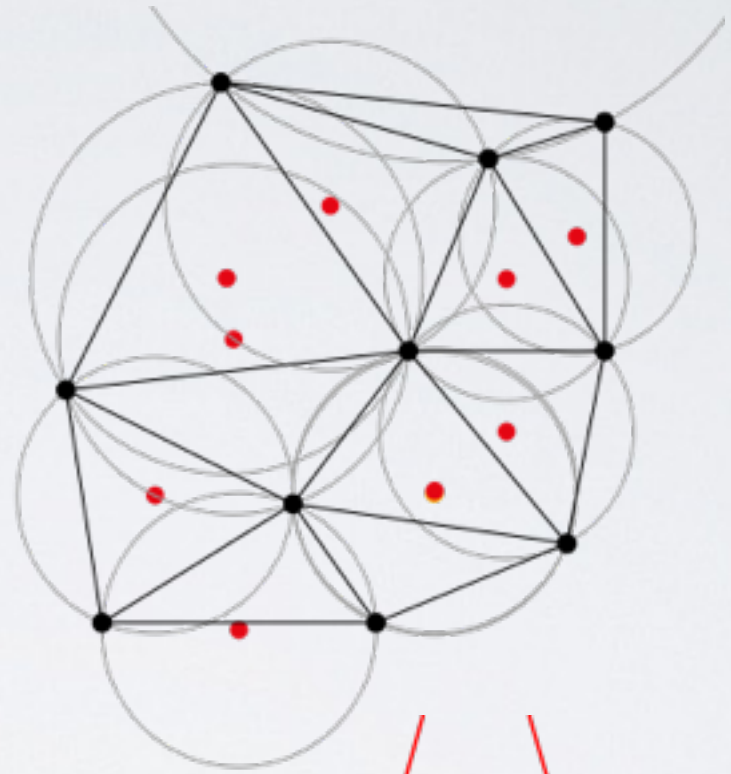


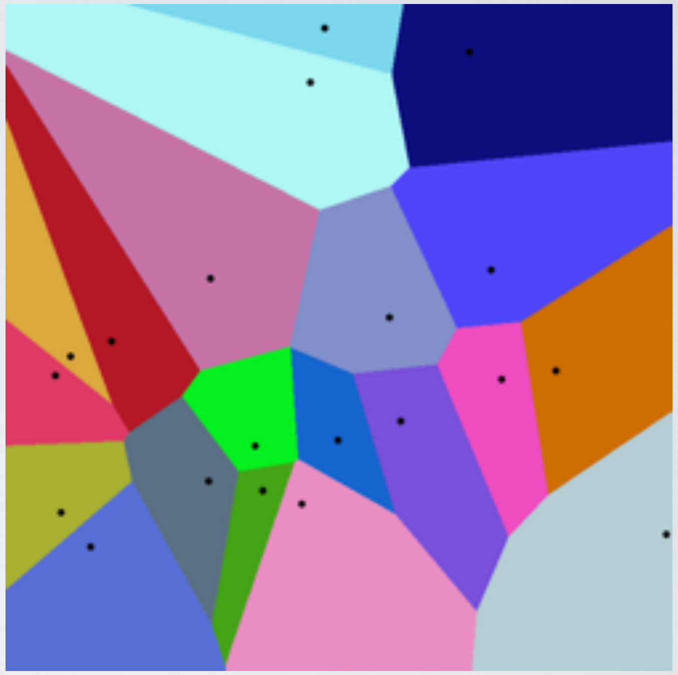
(a) (b)

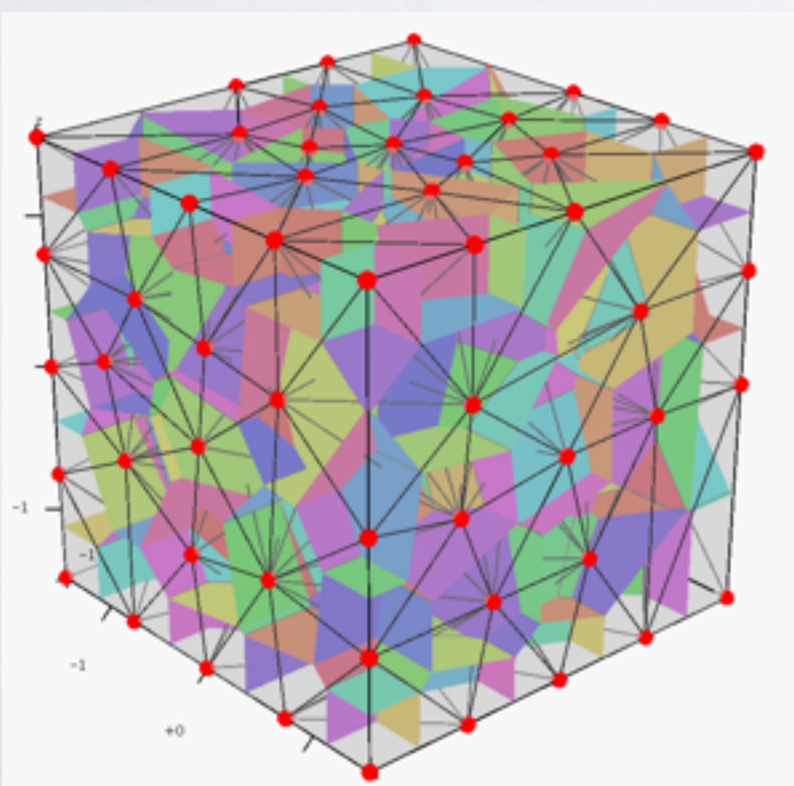
- **Orcun Goksel** and Septimiu E. Salcudean, *"Image-Based Variational Meshing"*, IEEE Trans Medical Imaging 30(1):11-21, Jan 2011.

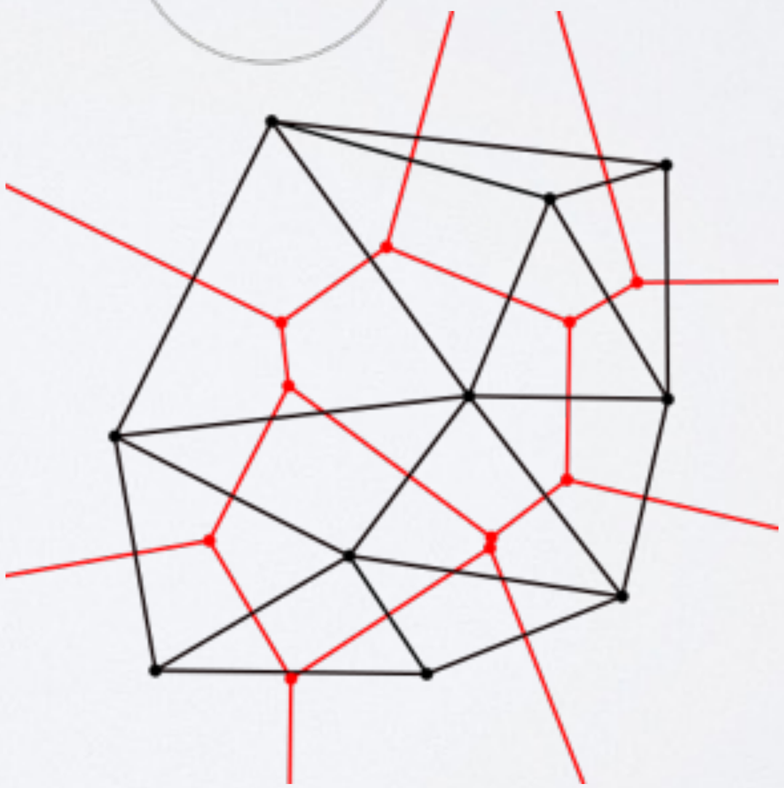
DISCRETISATION METHODS

- wide range of algorithms
  - Tesselations (tiling of a plane)
  - Delaunay triangulations [DT] (no point of the triangulation lies inside any circumcircle of any triangle of the triangulation
  - Voronoi diagrams (dual to DT)









DISCRETIZATION QUALITY

- quality of elements is a crucial in physics-based applications (**vertex Jacobian**)
  - degenerated elements result in numerical instability (**singularity of the Jacobian**)
- various measures of element quality:
  - smallest angle/largest angle (2D)
  - dihedral angle (3D)
  - determinant of vertex Jacobian
  - ratio of inscribed/circumscribed radii
  - others (edge ratio, Frobenius aspect etc)

$$\gamma_K = 4\frac{\sin\hat{a}\sin\hat{b}\sin\hat{c}}{\sin\hat{a}+\sin\hat{b}+\sin\hat{c}}$$

$$\gamma_K = \frac{6\sqrt{6}\,V_k}{\left(\sum_{i=1}^{4} a(f_i)\right)\max_{i=1,\ldots,6} l(e_i)}$$