Lesson 4 – Math / Lighting

PV227 – GPU Rendering

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Outline

1. Basics – Repetition
2. Issues with Normals
3. Lighting
   - Light Components: Ambient
   - Light-source type: Directional Light
   - Shading types: Gouraud Shading
   - Light Components: Diffuse
   - Shading types: Flat Shading
   - Light Components: Ambient plus Diffuse
   - Light Components: Specular
   - Light Components: Ambient plus Diffuse plus Specular
   - Shading types: Phong Shading
   - Light-source type: Point Light
   - Light-source type: Spot Light

Points and Vectors

- points (before projection) are quadruples: $(x, y, z, 1.0)$,
  - can be transformed with a 4×4 matrix,
- vectors are also quadruples: $(x, y, z, 0.0)$,
  - can be transformed with a 4×4 or 3×3 matrix.

Transformation

- points are transformed to **eye space** with **modelview matrix**,
- vectors constructed from points (e.g. $P_2 - P_1$) are also transformed with this matrix,
- normals are **not!**

Outline

Normal Transformation Error

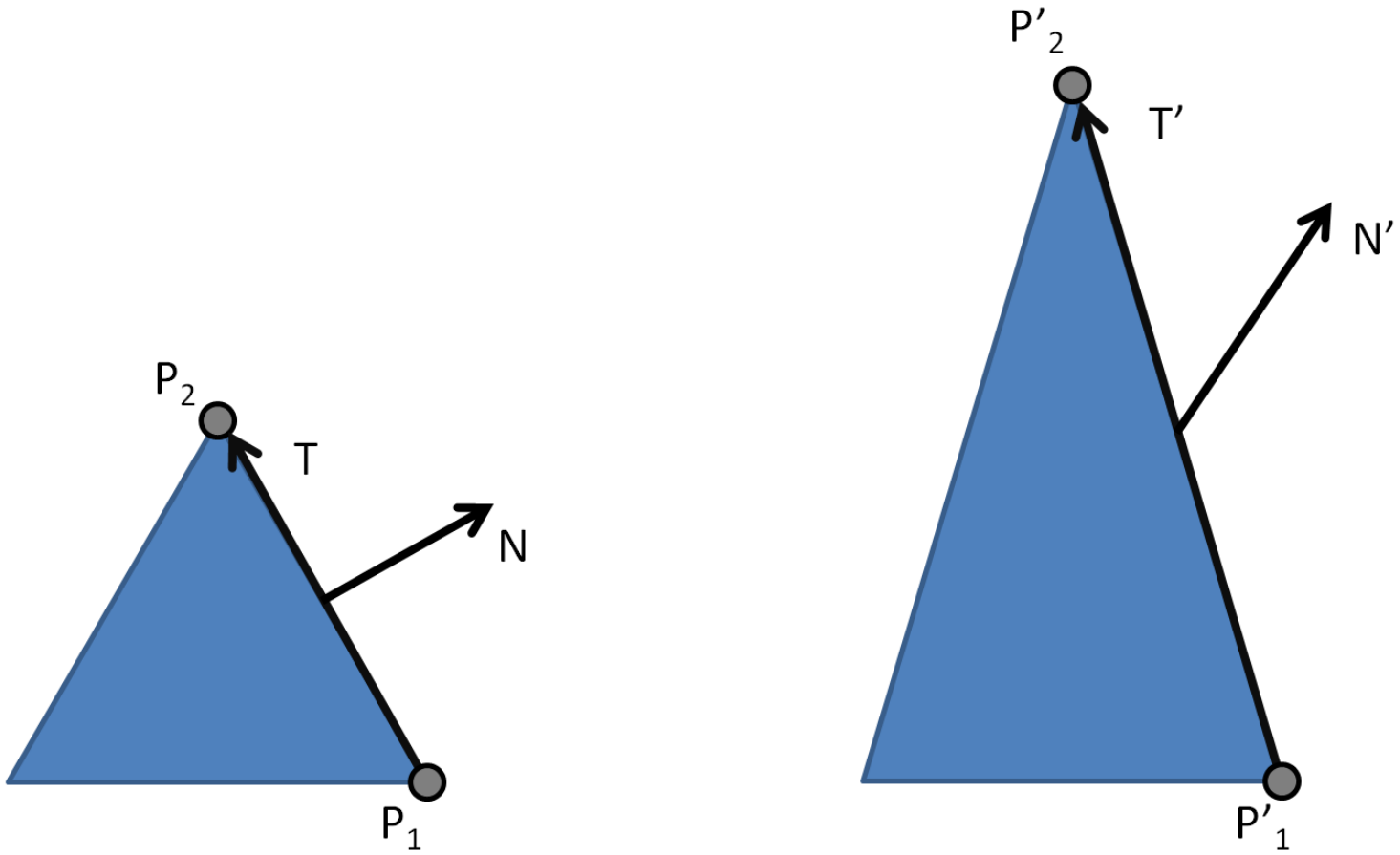


Figure: Taken from lighthouse3d.com

Normal Transformation Solution

- caused by **non-uniform** scale,
- M is the modelview matrix, $\vec{t}$ is tangent vector $(P_2 - P_1)$ and I is identity,
- we need another matrix (N) for transforming normal $\vec{n}$.

$$(M \times \vec{t}) \bullet (N \times \vec{n}) = 0$$
$$(M \times \vec{t})^T \times (N \times \vec{n}) = 0$$
$$\vec{t}^T \times M^T \times N \times \vec{n} = 0$$

Normal Transformation Solution (cont.)

$$\vec{t}^T \times M^T \times N \times \vec{n} = 0$$

$$\boxed{\vec{t} \bullet \vec{n} = 0 \Rightarrow \vec{t}^T \times \vec{n} = 0 \Rightarrow M^T \times N = I}$$

$$\begin{aligned} M^T \times N &= I \\ (M^T)^{-1} \times M^T \times N &= (M^T)^{-1} \\ N &= (M^T)^{-1} \end{aligned}$$

Normal Transformation Result

- N is **inverse transpose** of M (3×3 submatrix of M),
- for orthogonal matrices: $A^T = A^{-1}$ (rotation is orthogonal),
- if M is orthogonal, then: $M = (M^T)^{-1} \Rightarrow N = M$.

Renormalization

- normals must be of unit length,
- can be destroyed by normal transformation $\rightarrow$ must be normalized **in vertex shader**, after the transformation,
- interpolation can also destroy vector length $\rightarrow$ must be normalized **in fragment shader**, before we do anything with it.

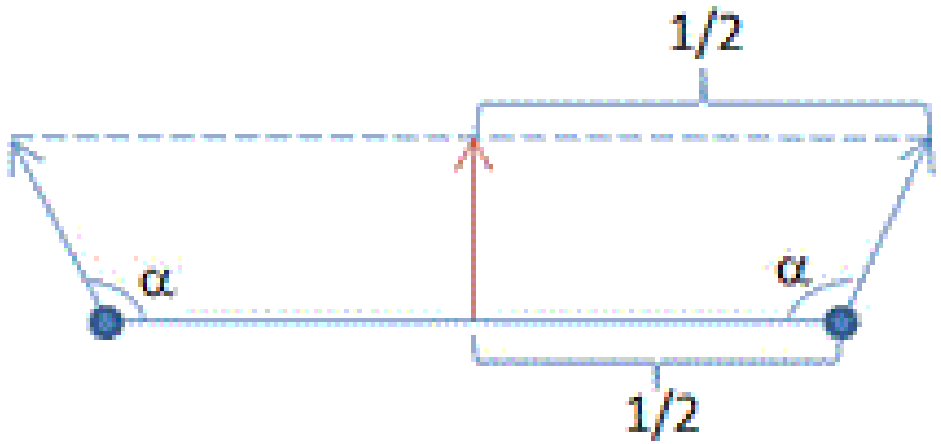


Figure: Taken from lighthouse3d.com

Outline

Lighting

- Computation of light's interaction with surfaces,
- in most cases – huge cheat; major simplification otherwise.

Light:

- light **components**: ambient, diffuse and specular,
- **shading** types: flat, gouraud and phong,
- **light-source** types: directional, point and spot light,
- shadows... no shadow, no bouncing of light.

Outline

Ambient Lighting

- approximates lighting after infinite number of bounces,
- homogeneous,
- prevents black areas that look unnatural,
- usually chosen as fraction of the diffuse (material) color,
- $I = K_a$.

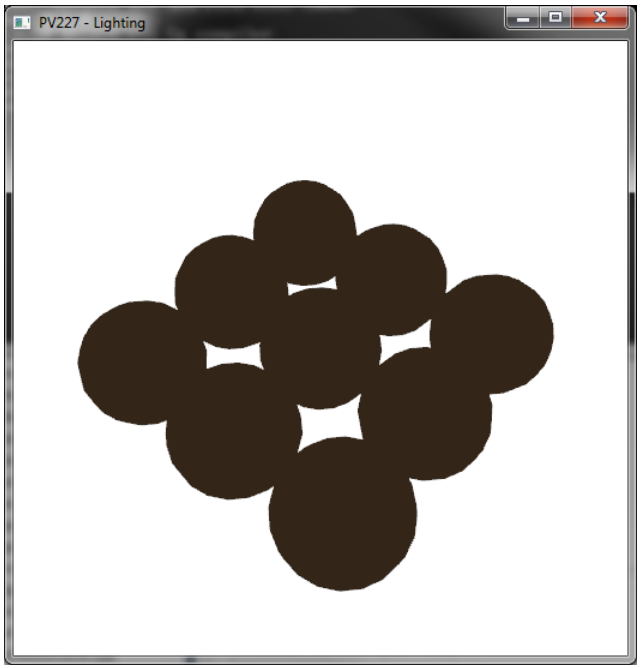


Figure: Ambient spheres

Outline

Light-source type: Directional Light

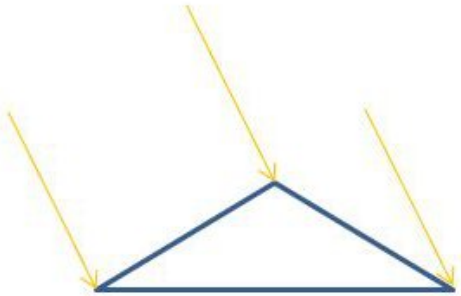


Figure: Taken from lighthouse3d.com

- far away light,
- defined by a direction vector (position is irelevant),
- can represent e.g. the sun.

Outline

Shading types: Gouraud Shading

- per vertex shading,
- interpolation of vertex colors,
- unable to capture lighting details inside polygons.

Outline

Light Components: Diffuse

- simulate light's interaction with perfectly diffuse material,
- light angle dependent,
- significant color component,
- $I = \cos(\alpha) \cdot K_d$.

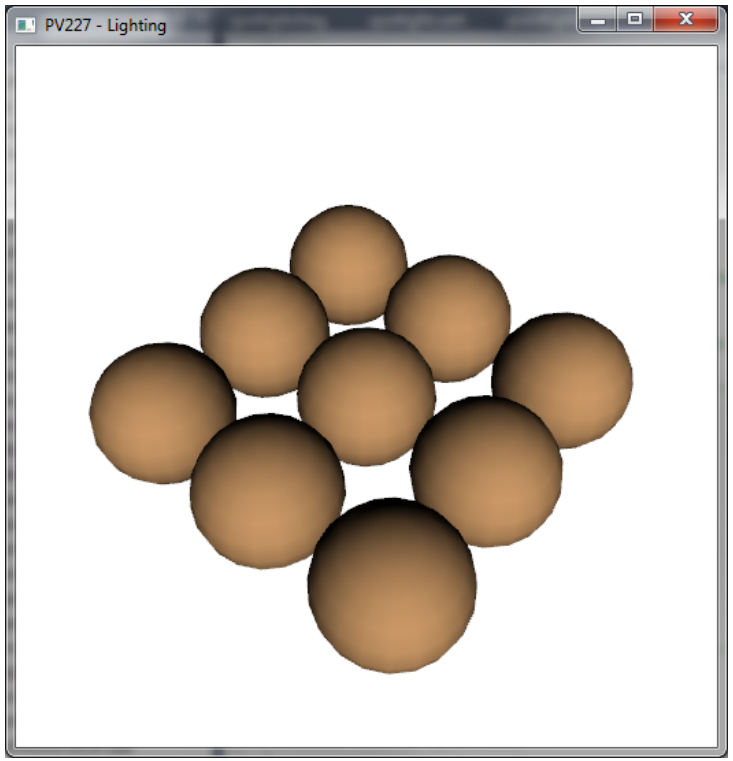


Figure: Diffuse spheres

Light Components: Diffuse (cont.)

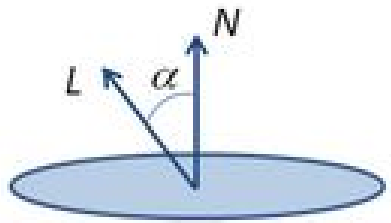


Figure: Taken from lighthouse3d.com

- amount of incoming light diminishes with increasing angle,
- $\cos(\alpha) = \frac{\vec{L} \bullet \vec{N}}{|\vec{L}| \cdot |\vec{N}|}$,
- normalized vectors: $I = (\vec{L} \bullet \vec{N}) \cdot K_d$,
- all vectors must be in same space (usually defined in world space, computation in camera space).

Outline

Flat Shading

- per primitive shading,
- no interpolation,
- unable to capture smooth changes in light intensity.

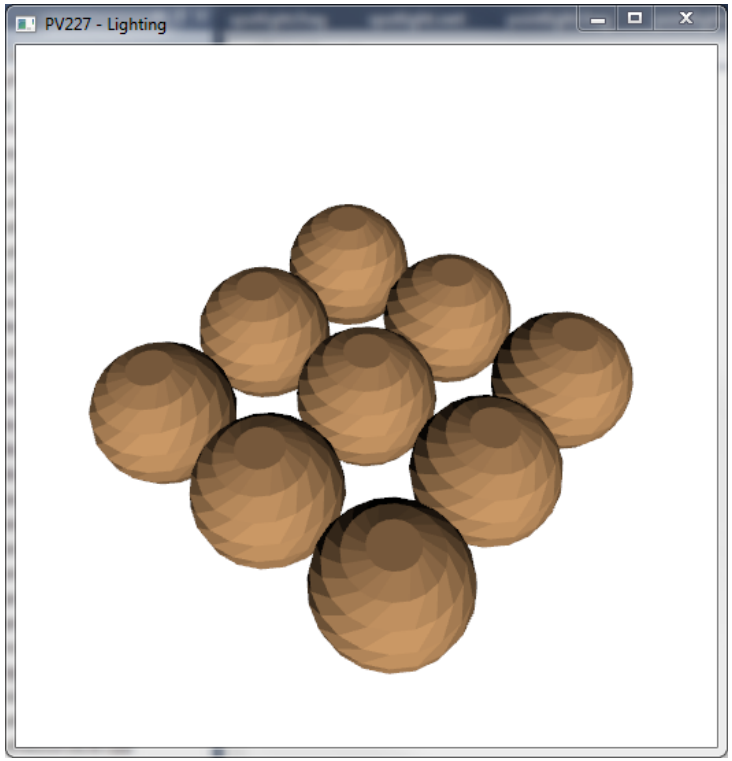


Figure: Flat shading

Outline

Light Components: Ambient plus Diffuse

- light from various sources can be combined (added),
- combination of ambient and diffuse prevents black areas,
- $I = K_a + \cos(\alpha) \cdot K_d$,
- value should not be outside the $[0.0, 1.0]$ range.

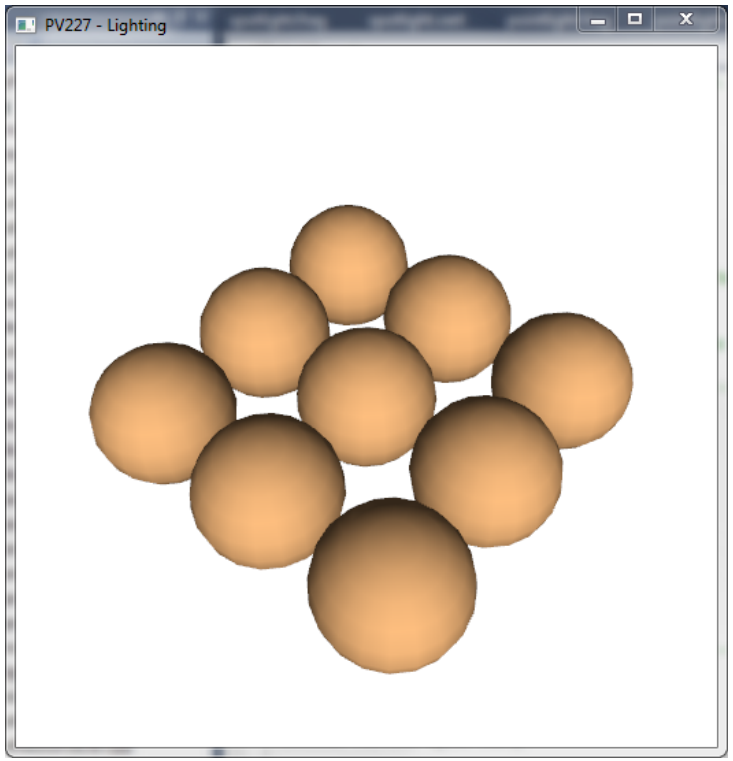


Figure: Ambient + Diffuse spheres

Outline

Light Components: Specular

- simulate light's interaction with reflective material,
- view angle dependent,
- highlight of the light's color, not material color,
- $I = \cos(\beta)^s \cdot K_s$, s controls size of the highlight.

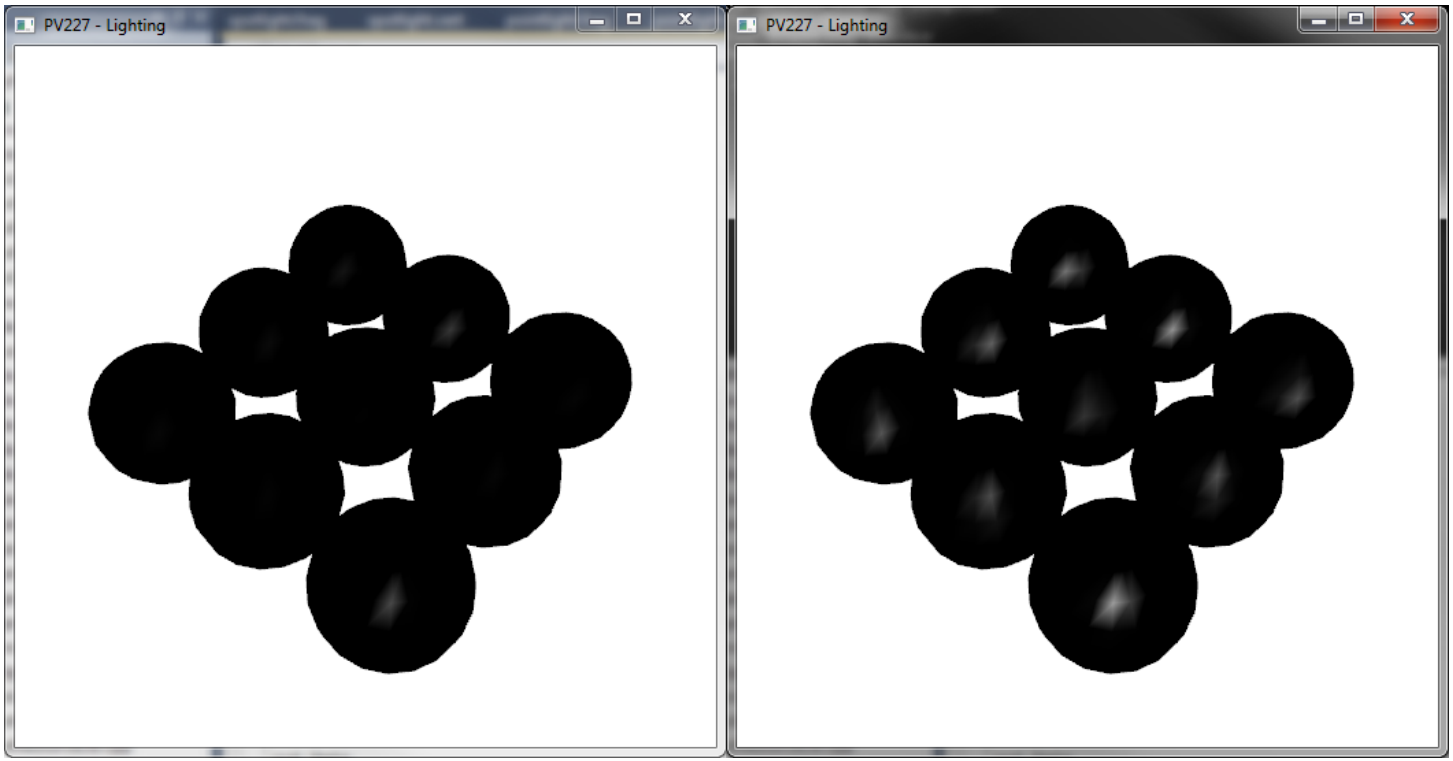


Figure: Specular spheres (Phong *vs* Blinn-Phong)

Phong Lighting

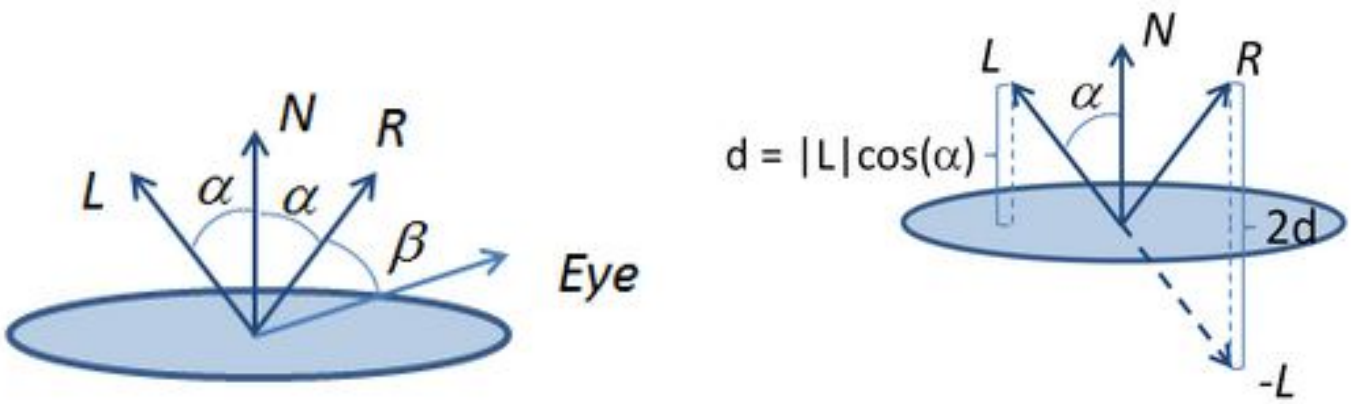


Figure: Taken from lighthouse3d.com

- amount of reflected light diminishes with increasing angle,
- $\vec{R} = -\vec{L} + 2 \cdot (\vec{N} \bullet \vec{L}) \cdot \vec{N}$,
- $\cos(\beta) = \vec{R} \bullet \vec{Eye}$,
- all vectors must be in same space, normalized.

Blinn-Phong Lighting

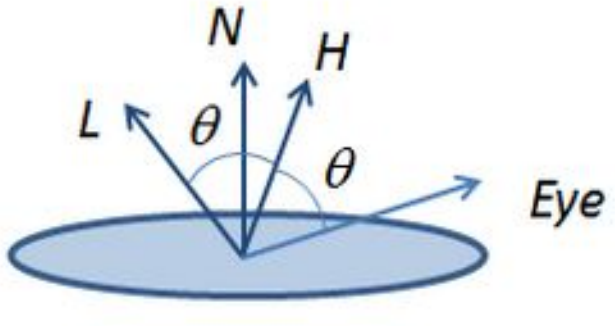


Figure: Taken from lighthouse3d.com

- amount of reflected light diminishes with increasing angle,
- $\vec{H} = \vec{L} + \vec{Eye}$,
- $\cos(\beta) = \vec{H} \bullet \vec{N}$,
- all vectors must be in same space, normalized.

Outline

Basic Lighting

- ambient, diffuse and specular form the baseline lighting,
- $I = K_a + \cos(\alpha) \cdot K_d + \cos(\beta)^s \cdot K_s$,
- light from various sources can be combined (added),
- value should not be outside the $[0.0, 1.0]$ range.

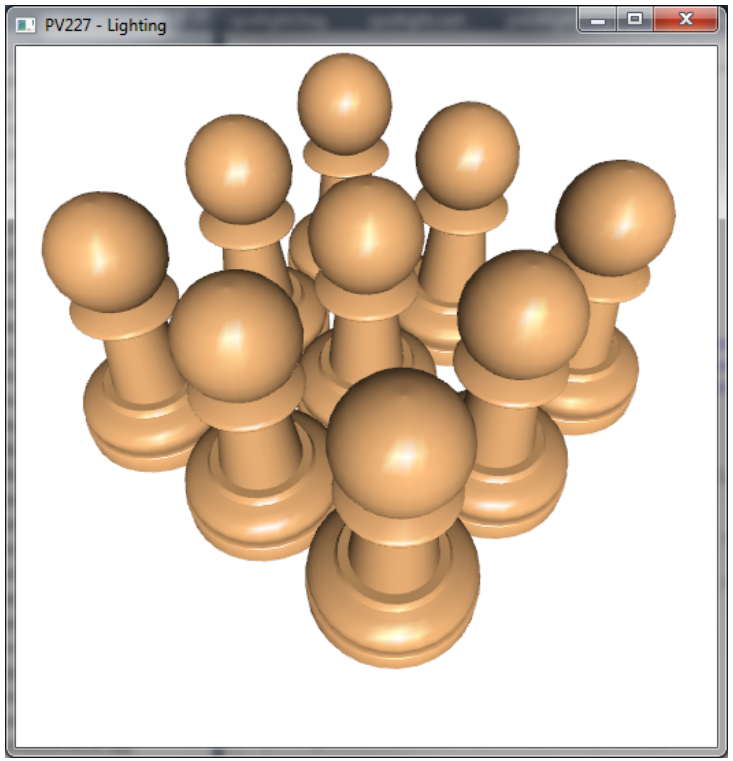


Figure: Ambient + Diffuse + Specular pawns

Outline

Phong Shading

- per pixel shading,
- smooth lighting including details,
- interpolation of vertex attributes (normal, eye, light).

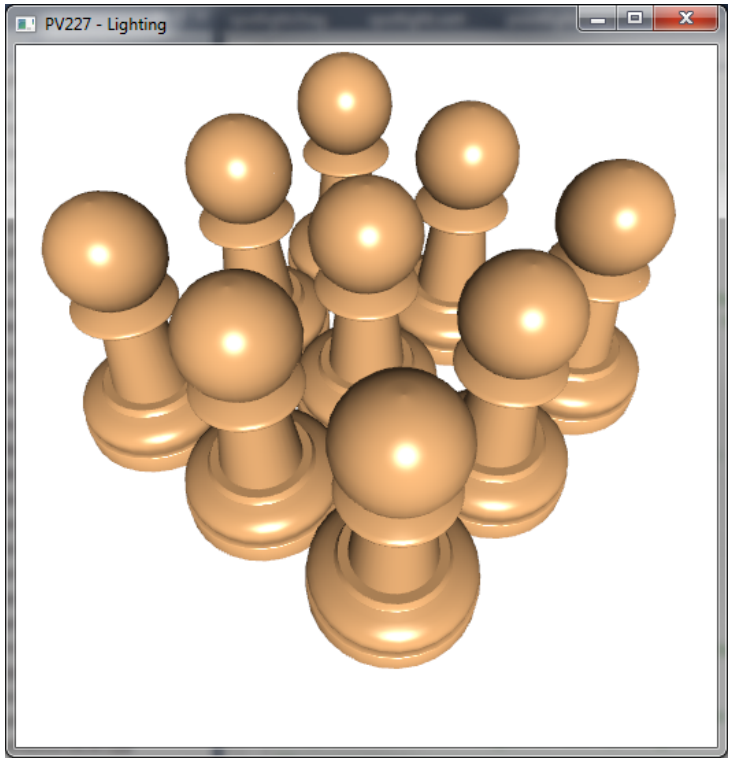
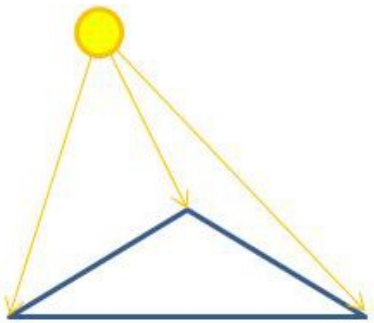


Figure: Per pixel lighting pawns

Outline

Light-source type: Point Light

- light source inside the scene,
- defined by a position vector (all directions),
- can represent e.g. a lightbulb.



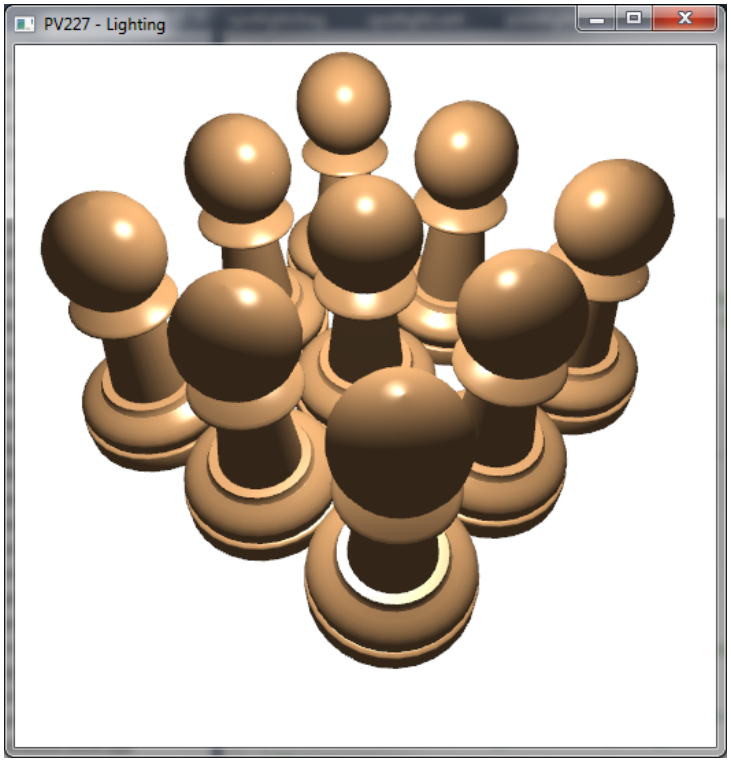
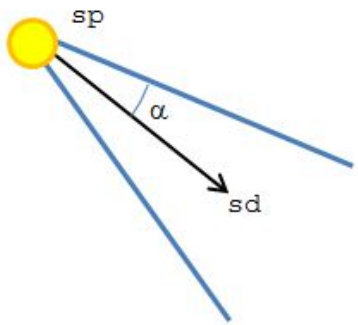


Figure: Taken from lighthouse3d.com Point light pawns.

Outline

Light-source type: Spot Light

- light source inside the scene,
- only a directed cone is illuminated,
- defined by a position vector, direction vector and angle,
- can represent e.g. a flashlight.



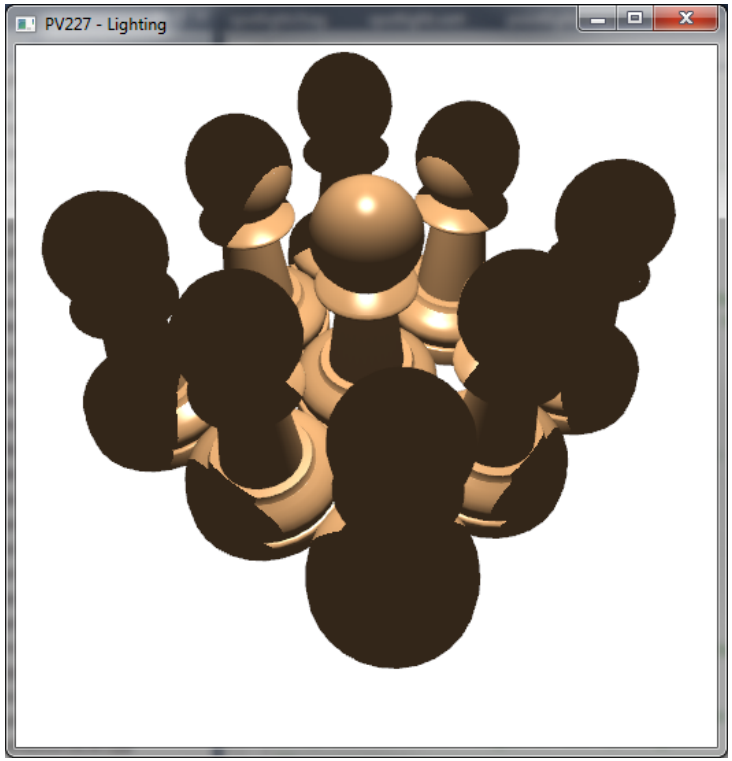


Figure: Spot light pawns