# IA168 — Problem set 3

## Problem 1 [7 points]

Consider this strategic-form game $G$:

| | $A_2$ | $B_2$ | $C_2$ |
|---|---|---|---|
| $A_1$ | $(x, x)$ | $(26, -1)$ | $(-20, -1)$ |
| $B_1$ | $(-1, 26)$ | $(13, 13)$ | $(-20, 26)$ |
| $C_1$ | $(-1, -20)$ | $(26, -20)$ | $(2x - 10, 2x - 10)$ |

a) For $x = 10$, state (as explicitly as possible) the number of SPEs in the 10-stage game $G_{10\text{-}rep}$.

b) Find all $x \in \mathbb{R}$, $0 \leq x \leq 10$, for which the maximal outcome for player 1 in an SPE is as large as possible. Formally: find all $x \in [0, 10]$, for which $\sup\{u_1(s) \mid s \in SPE(G_{10\text{-}rep})\}$ is maximal. Explain your solution.

## Problem 2 [5 points]

Consider this strategic-form game $G$:

| | $A_2$ | $B_2$ |
|---|---|---|
| $A_1$ | $(2, 1)$ | $(7, -1)$ |
| $B_1$ | $(-2, 6)$ | $(x, y)$ |

Consider also strategy profile $s'$:

$$s'_i(h) = \begin{cases} B_i & \text{if } h \in \{(B_1, B_2)\}^* \\ A_i & \text{otherwise} \end{cases}$$

Find all the pairs $(x, y) \in \mathbb{R}^2$ for which the minimal discount required for $s'$ to be an SPE is equal to $\frac{3}{5}$. Formally: find all the pairs $(x, y) \in \mathbb{R}^2$ such that $\inf\{\delta \in (0, 1) \mid s' \text{ is SPE in } G_\delta\} = \frac{3}{5}$.

## Problem 3 [8 points]

Consider "$3^{\text{rd}}$ price auction" as a game of incomplete information. The payoff of every player is 0, if their bid was not (strictly) highest, and their type minus the $3^{\text{rd}}$ highest bid, if they were the highest bidder. The bid is a nonnegative real number.

a) Prove or disprove the existence of a weakly dominant strategy for player 1.

b) Prove or disprove the existence of ex-post Nash equilibrium.

Now consider the "$3^{\text{rd}}$ price auction" as a Bayesian game, where the type of every player is uniformly distributed on interval $[0, v_{max}]$.

c) Prove that strategy profile $s$ given by $s_i(t_i) = \frac{n-1}{n-2} t_i$ is a Bayesian Nash equilibrium.