

## v sieti: posielanie správ (sokety)

### server

```
int main(int argc, char *argv[]) {
    int sockfd, newsockfd, portno;
    socklen_t clilen;
    char buffer[256];
    struct sockaddr_in serv_addr, cli_addr;
    int n;

    sockfd = socket(AF_INET, SOCK_STREAM, 0);
    bzero((char *) &serv_addr, sizeof(serv_addr));
    portno = atoi(argv[1]);

    serv_addr.sin_family = AF_INET;
    serv_addr.sin_addr.s_addr = INADDR_ANY;
    serv_addr.sin_port = htons(portno);

    bind(sockfd, (struct sockaddr *) &serv_addr,
        sizeof(serv_addr));
    listen(sockfd,5);
    clilen = sizeof(cli_addr);
    newsockfd = accept(sockfd,
        (struct sockaddr *) &cli_addr, &clilen);
    bzero(buffer,256);

    n = read(newsockfd,buffer,255);
    printf("Here is the message: %s\n",buffer);

    n = write(newsockfd,"I got your message",18);
    close(newsockfd);
    close(sockfd);
}
```

### klient

```
int main(int argc, char *argv[]) {
    int sockfd, portno, n;
    struct sockaddr_in serv_addr;
    struct hostent *server;
    char buffer[256];

    portno = atoi(argv[2]);
    sockfd = socket(AF_INET, SOCK_STREAM, 0);
    server = gethostbyname(argv[1]);

    bzero((char *) &serv_addr, sizeof(serv_addr));
    serv_addr.sin_family = AF_INET;
    bcopy((char *)server->h_addr,
        (char *)&serv_addr.sin_addr.s_addr,
        server->h_length);
    serv_addr.sin_port = htons(portno);

    connect(sockfd,(struct sockaddr *) &serv_addr,
        sizeof(serv_addr));

    printf("Please enter the message: ");
    bzero(buffer,256);
    fgets(buffer,255,stdin);
    n = write(sockfd,buffer,strlen(buffer));
    bzero(buffer,256);
    n = read(sockfd,buffer,255);
    printf("%s\n",buffer);
    close(sockfd);
    return 0;
}
```

## v sieti: posielanie správ (MPI)

```
int main(int argc, char *argv[]) {
    const int tag = 47;
    ...
    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &ntasks);
    MPI_Comm_rank(MPI_COMM_WORLD, &id);

    if (id == 0) {
        MPI_Get_processor_name(msg, &length);
        printf("Hello World from process %d running on %s\n", id, msg);
        for (i=1; i<ntasks; i++) {
            MPI_Recv(msg, 80, MPI_CHAR, MPI_ANY_SOURCE,
                    tag, MPI_COMM_WORLD, &status);
            source_id = status.MPI_SOURCE;
            printf("Hello World from process %d running on %s\n", source_id, msg);
        }
    }
    else {
        MPI_Get_processor_name(msg, &length);
        MPI_Send(msg, length, MPI_CHAR, 0, tag, MPI_COMM_WORLD);
    }
    MPI_Finalize();
    if (id==0) printf("Ready\n");
}
```

## abstraktný sieťový model

- ▶ nezávislé zariadenia (procesy, procesory, uzly)
- ▶ posielanie správ
- ▶ lokálny pohľad (číslovanie portov)
- ▶ asynchrónne, spoľahlivé správy
- ▶ topológia siete
- ▶ wakeup/terminácia

zložitosť: v závislosti od počtu procesorov

- ▶ počet správ / bitov
- ▶ čas (beh normovaný na max. dĺžku 1)

## abstraktný sieťový model

horný odhad pre problém

**Existuje** algoritmus, ktorý **pre všetky** topológie, vstupy, časovania, ....  
... pracuje správne a vymení najviac  $f(n)$  správ

dolný odhad pre problém

**Pre každý** algoritmus, **existuje kombinácia** topológie, vstupu, časovania,  
....  
... že buď nepracuje správne alebo vymení aspoň  $f(n)$  správ

## prehľadávane

- ▶ na začiatku je jeden iniciátor
- ▶ treba informovať všetkých

### shout-and-echo

- ▶ iniciátor pošle **shout**
- ▶ ak príde **shout** do nového vrchola
  - ▶ označí hranu
  - ▶ pošle po neoznačených **shout**
  - ▶ čaká na všetky **echo**
  - ▶ pošle **echo**
- ▶ ak príde **shout** do navštíveného – okamžite **echo**

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### zložitosť

- ▶  $4m$  správ
- ▶  $2diam(G)$  čas

## traverzovanie

- ▶ na začiatku je jeden iniciátor
- ▶ treba informovať všetkých
- ▶ v každom okamihu je len jeden *token*

### $f(x)$ -traverzovanie

Na objavenie  $x$  vrcholov treba  $\max\{f(x), n\}$  správ.

## prehľadávanie do hĺbky

```
var  $used_p[q]$  : boolean    init false for each  $q \in Neigh_p$  ;  
                                (* Indicates whether  $p$  has already sent to  $q$  *)  
     $father_p$  : process    init undef ;
```

For the initiator only, execute once:

```
begin  $father_p := p$  ; choose  $q \in Neigh_p$  ;  
         $used_p[q] := true$  ; send  $\langle \mathbf{tok} \rangle$  to  $q$   
end
```

For each process, upon receipt of  $\langle \mathbf{tok} \rangle$  from  $q_0$ :

```
begin if  $father_p = undef$  then  $father_p := q_0$  ;  
        if  $\forall q \in Neigh_p : used_p[q]$   
            then decide  
        else if  $\exists q \in Neigh_p : (q \neq father_p \wedge \neg used_p[q])$   
            then begin if  $father_p \neq q_0 \wedge \neg used_p[q_0]$   
                then  $q := q_0$   
                    else choose  $q \in Neigh_p \setminus \{father_p\}$   
                        with  $\neg used_p[q]$  ;  
                 $used_p[q] := true$  ; send  $\langle \mathbf{tok} \rangle$  to  $q$   
            end  
        else begin  $used_p[father_p] := true$  ;  
            send  $\langle \mathbf{tok} \rangle$  to  $father_p$   
        end  
end
```

- ▶ zložitosť  $2m$  (čas aj správy)
- ▶ ako ušetriť čas?



## Awerbuch: $4m$ správ, $4n - 2$ čas

```
var  $used_p[q]$  : boolean   init false for each  $q \in Neigh_p$  ;  
                                (* Indicates whether  $p$  has already sent to  $q$  *)  
    $father_p$  : process   init  $undef$  ;
```

For the initiator only, execute once:

```
begin  $father_p := p$  ; choose  $q \in Neigh_p$  ;  
   forall  $r \in Neigh_p$  do send  $\langle vis \rangle$  to  $r$  ;  
   forall  $r \in Neigh_p$  do receive  $\langle ack \rangle$  from  $r$  ;  
    $used_p[q] := true$  ; send  $\langle tok \rangle$  to  $q$   
end
```

For each process, upon receipt of  $\langle tok \rangle$  from  $q_0$ :

```
begin if  $father_p = undef$  then  
   begin  $father_p := q_0$  ;  
     forall  $r \in Neigh_p \setminus \{father_p\}$  do send  $\langle vis \rangle$  to  $r$  ;  
     forall  $r \in Neigh_p \setminus \{father_p\}$  do receive  $\langle ack \rangle$  from  $r$   
   end ;  
   if  $p$  is the initiator and  $\forall q \in Neigh_p : used_p[q]$   
     then decide  
   else if  $\exists q \in Neigh_p : (q \neq father_p \wedge \neg used_p[q])$   
     then begin if  $father_p \neq q_0 \wedge \neg used_p[q_0]$   
       then  $q := q_0$   
       else choose  $q \in Neigh_p \setminus \{father_p\}$   
         with  $\neg used_p[q]$  ;  
        $used_p[q] := true$  ; send  $\langle tok \rangle$  to  $q$   
     end  
   else begin  $used_p[father_p] := true$  ;  
     send  $\langle tok \rangle$  to  $father_p$   
   end  
end
```

For each process, upon receipt of  $\langle vis \rangle$  from  $q_0$ :

```
begin  $used_p[q_0] := true$  ; send  $\langle ack \rangle$  to  $q_0$  end
```

► otázka: treba čakať na ack?

## prehľadávanie do šírky

### Cheung'83: kubická komunikácia, lineárny čas

- ▶ každá správa obsahuje počítadlo hopov
- ▶ na začiatku iniciátor: všetkým susedom pošle 1
- ▶ každý vrchol  $p$ : lokálnu vzdialenosť  $dist_p := \infty$
- ▶ on recv  $i$ :  $dist_p := \min\{i, dist_p\}$ , ak sa zmenila, pošli všetkým

### Cheung,Zhu'87: kvadratická komunikácia aj čas

- ▶ buduje kostru po vrstvách

kombinácia?

## voľba šéfa: úplné grafy

**const:**  $N$  : integer  
 $ID$  : integer  
 $Neigh$  :  $[1..N-1]$  link  
**var:**  $leader$  : boolean  
 $count$  : integer  
 $i$  : integer

Init:

$count := 0$   
 $leader := \mathbf{false}$

Code:

**for**  $i = 1$  **to**  $N - 1$  **do**  
    **send**  $\langle \mathbf{elect}, ID \rangle$  **to**  $Neigh[i]$   
**while**  $count < N - 1$  **wait**  
**for**  $i = 1$  **to**  $N - 1$  **do**  
    **send**  $\langle \mathbf{leader}, ID \rangle$  **to**  $Neigh[i]$   
 $leader := \mathbf{true}$

On receipt  $\langle \mathbf{elect}, id_i \rangle$  from  $Neigh[i]$ :

**if**  $id_i > ID$  **send**  $\langle \mathbf{accept} \rangle$  **to**  $Neigh[i]$

On receipt  $\langle \mathbf{accept} \rangle$  from  $Neigh[i]$ :

$count ++$

On receipt  $\langle \mathbf{leader}, id_i \rangle$  from  $Neigh[i]$ :

*Skonči algoritmus*

## voľba šéfa: úplné grafy II

```
const:  N      : integer
        ID     : integer
        Neigh  : [1..N-1] link
var:    leader : boolean
        state  : {active, captured, killed}
        level  : integer
        parent : link
        msg    : {victory, defeat}
        i      : integer
```

### Init:

```
state := active
level := 0
leader := false
```

### Code:

```
for i = 1 to N - 1 do
  send <capture, [level, ID]> to Neigh[i]
  receive <accept> from Neigh[i]
  level ++
leader := true
for i = 1 to N - 1 do
  send <leader, ID> to Neigh[i]
```

### Dead:

```
loop forever
```

On receipt <capture, [level<sub>i</sub>, id<sub>i</sub>]> from Neigh[i]:

```
if state ∈ {active, killed} and [leveli, idi] > [level, ID]
  state := captured
  parent := Neigh[i]
  send <accept> to parent
  goto Dead
else if state = captured
  send <help, [leveli, idi]> to parent
  receive msg from parent
  if msg = defeat
    send <accept> to Neigh[i]
    parent := Neigh[i]
```

On receipt <help, [level<sub>i</sub>, id<sub>i</sub>]> from Neigh[i]:

```
if [leveli, idi] < [level, ID]
  send <victory> to Neigh[i]
else
  send <defeat> to Neigh[i]
  if state = active
    state := killed
  goto Dead
```

On receipt <leader, id<sub>i</sub>> from Neigh[i]:

*Skonči algoritmus*

## voľba šéfa: úplné grafy II - analýza

### Lema 1

V ľubovoľnom výpočte existuje pre každý level  $l = 0, \dots, N - 1$  aspoň jeden proces, ktorý bol počas výpočtu na leveli  $l$ .

### Lema 2

Nech  $v$  je aktívny proces (*state = active*) s levelom  $l$ . Potom existuje  $l$  zajatých procesov ktoré patria  $v$  (t.j. ich premenná *parent* ukazuje na  $v$ ).

⇒ práve jeden proces je šéf

### Lemma 3

Ľubovoľnom výpočte je najviac  $N/(l + 1)$  procesov, ktoré niekedy dosiahli level  $l$ .

⇒ maximálne  $\sum_{l=1}^{N-1} \frac{N}{l+1} = N(\mathbf{H}_N - 1) \approx N \log N$  postupov o level (=správ)

## dolný odhad

Treba  $\Omega(n \log n)$  správ

- ▶ “globálny” algoritmus
- ▶ graf indukovaný novými správami
- ▶ udržujem výpočet:
  - ▶ jeden súvislý komponent, ostatné vrcholy izolované
  - ▶  $e(k)$  – koľko viem vynútiť na komponente  $k$
  - ▶  $e(2k + 1) = 2e(k) + k + 1$

## jednosmerné kruhy

### Chang, Roberts

**const:**  $ID$  : integer  
 $l_{in}, l_{out}$  : link  
**var:**  $leader$  : integer

Init:

$leader := NULL$

Code:

**send**  $\langle ID \rangle$   
**wait** until  $leader \neq NULL$

On receipt  $\langle i \rangle$ :

**if**  $i < ID$  **then send**  $\langle i \rangle$   
**if**  $i = ID$  **then**  
     $leader := ID$   
    **send**  $\langle leader, ID \rangle$

On receipt  $\langle leader, x \rangle$ :

$leader := x$   
**send**  $\langle leader, ID \rangle$

- ▶  $\Omega(n^2)$  správ v najhoršom prípade
- ▶  $O(n \log n)$  v priemernom

## Chang Roberts - analýza

Koľkokrát sa pohla správa  $i$ ?

$$P(i, k) = \frac{\binom{n-i}{k-1} \cdot (i-1)}{\binom{n-1}{k-1} \cdot (n-k)}$$

$$E[X_i] = \sum_{k=1}^{n-i+1} k \cdot P(i, k)$$

$$E[X] = n + \sum_{i=2}^n E[X_i] = n + \sum_{i=2}^n \sum_{k=1}^{n-i+1} k \cdot P(i, k)$$



## Chang Roberts - analýza

lema

$$\sum_{j=k}^{n-1} \frac{j!}{(j-k)!} = k! \binom{n}{k+1}$$

dôkaz

$$\sum_{j=k}^{n-1} \frac{j!}{(j-k)!} = \sum_{j=k}^{n-1} \frac{k!j!}{k!(j-k)!} = k! \sum_{j=k}^{n-1} \binom{j}{k} = k! \binom{n}{k+1}$$

$$\begin{aligned}
n + \sum_{i=2}^n \sum_{k=1}^{n-i+1} k \cdot \frac{\binom{n-i}{k-1} \cdot (i-1)}{\binom{n-1}{k-1} \cdot (n-k)} &= n + \sum_{j=1}^{n-1} \sum_{k=1}^j k \cdot \frac{\binom{j-1}{k-1} \cdot (n-j)}{\binom{n-1}{k-1} \cdot (n-k)} = \\
&= n + \sum_{j=1}^{n-1} \sum_{k=1}^j \frac{k(j-1)!(n-j)(k-1)!(n-k)!}{(k-1)!(j-k)!(n-1)!(n-k)} = \\
&= n + \sum_{k=1}^{n-1} \left[ \frac{k(n-k-1)!}{(n-1)!} \sum_{j=k}^{n-1} \frac{(j-1)!(n-j)}{(j-k)!} \right] = \\
&= n + \sum_{k=1}^{n-1} \left[ \frac{k(n-k-1)!}{(n-1)!} \left( \sum_{j=k}^{n-1} \frac{n(j-1)!}{(j-k)!} - \sum_{j=k}^{n-1} \frac{j!}{(j-k)!} \right) \right] = \\
&\sum_{j=k}^{n-1} \frac{(j-1)!}{(j-k)!} = (k-1)! \binom{n-1}{k} \quad \text{a} \quad \sum_{j=k}^{n-1} \frac{j!}{(j-k)!} = k! \binom{n}{k+1} \\
&= n + \sum_{k=1}^{n-1} \frac{k(n-k-1)!}{(n-1)!} \left[ n \cdot (k-1)! \binom{n-1}{k} - k! \binom{n}{k+1} \right] = \\
&= n + \sum_{k=1}^{n-1} \frac{n}{k+1}
\end{aligned}$$

## dvojsmerné kruhy

### Hirschberg-Sinclair

- ▶ level  $l$ : dobýjať územie  $2^l$
- ▶  $\log n$  levelov
- ▶  $n/2^l$  vrcholov na leveli
- ▶ každý vrchol pošle  $2^l$  správ

## dvojsmerné kruhy

### Hirschberg-Sinclair

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### Franklin

- ▶ level  $l$ : poraziť susedov (na rovnakom leveli; "synchronizácia")
- ▶  $\log n$  levelov
- ▶  $n$  správ na level

## dvojsmerné kruhy

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- ▶  $\log n$  levelov
- ▶  $n$  správ na level

### Dolev – simulácia na 1-smernom kruhu

- ▶ idea: presunúť identitu

## dolný odhad

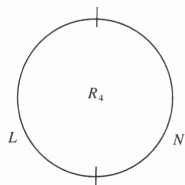
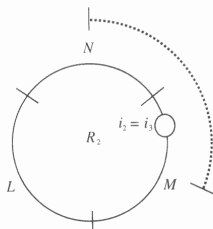
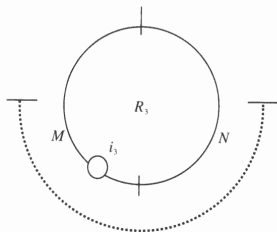
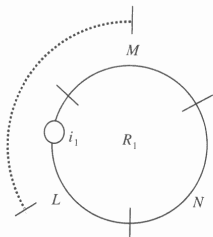
zapojiť do čiary  $L$ , vymení sa  $C(L)$  správ

### lema

Pre každé  $r$  existuje nekonečne veľa čiar dĺžky  $2^r$ , kde  $C(L) \geq r2^{r-2}$

- ▶ indukcia
- ▶ dve vrecia: vyberám trojice, chcem dve spojiť item pri spojení: dĺžka  $2^{r+1}$ , počet správ  $\geq r2^{r-1}$
- ▶ ešte treba  $2^{r-1}$  správ; sporom

# dolný odhad



## GHS

- ▶ ľubovoľná topológia
- ▶ buduje sa kostra
- ▶ “segmenty”
- ▶ spájanie po najlacnejšej odchádzajúcej hrane:
  - A menší sa pripojí k väčšiemu
  - B rovnakí sa spoja
  - C väčší čaká
- ▶ veľkosť  $\Rightarrow$  level



```

var  $state_p$  : (sleep, find, found) ;
       $stach_p[q]$  : (basic, branch, reject) for each  $q \in Neigh_p$  ;
       $name_p, bestwt_p$  : real ;
       $level_p$  : integer ;
       $testch_p, bestch_p, father_p$  :  $Neigh_p$  ;
       $rec_p$  : integer ;

```

- (1) As the first action of each process, the algorithm must be initialized:

```

begin let  $pq$  be the channel of  $p$  with smallest weight ;
       $stach_p[q] := branch$  ;  $level_p := 0$  ;
       $state_p := found$  ;  $rec_p := 0$  ;
      send  $\langle connect, 0 \rangle$  to  $q$ 

```

**end**

- (2) Upon receipt of  $\langle connect, L \rangle$  from  $q$ :

```

begin if  $L < level_p$  then (* Combine with Rule A *)
      begin  $stach_p[q] := branch$  ;
          send  $\langle initiate, level_p, name_p, state_p \rangle$  to  $q$ 
      end
      else if  $stach_p[q] = basic$ 
          then (* Rule C *) process the message later
          else (* Rule B *) send  $\langle initiate, level_p + 1, \omega(pq), find \rangle$  to  $q$ 

```

**end**

- (3) Upon receipt of  $\langle initiate, L, F, S \rangle$  from  $q$ :

```

begin  $level_p := L$  ;  $name_p := F$  ;  $state_p := S$  ;  $father_p := q$  ;
       $bestch_p := undef$  ;  $bestwt_p := \infty$  ;
      forall  $r \in Neigh_p$  :  $stach_p[r] = branch \wedge r \neq q$  do
          send  $\langle initiate, L, F, S \rangle$  to  $r$  ;
      if  $state_p = find$  then begin  $rec_p := 0$  ; test end

```

**end**

# GHS

- ```
(4) procedure test:
  begin if  $\exists q \in Neigh_p : stach_p[q] = basic$  then
    begin  $testch_p := q$  with  $stach_p[q] = basic$  and  $\omega(pq)$  minimal ;
      send  $\langle test, level_p, name_p \rangle$  to  $testch_p$ 
    end
  else begin  $testch_p := undef$  ; report end
end
```
- (5) Upon receipt of  $\langle test, L, F \rangle$  from  $q$ :
- ```
  begin if  $L > level_p$  then (* Answer must wait! *)
    process the message later
  else if  $F = name_p$  then (* internal edge *)
    begin if  $stach_p[q] = basic$  then  $stach_p[q] := reject$  ;
      if  $q \neq testch_p$ 
        then send  $\langle reject \rangle$  to  $q$ 
      else test
    end
  else send  $\langle accept \rangle$  to  $q$ 
end
```
- (6) Upon receipt of  $\langle accept \rangle$  from  $q$ :
- ```
  begin  $testch_p := undef$  ;
    if  $\omega(pq) < bestwt_p$ 
      then begin  $bestwt_p := \omega(pq)$  ;  $bestch_p := q$  end ;
    report
  end
```
- (7) Upon receipt of  $\langle reject \rangle$  from  $q$ :
- ```
  begin if  $stach_p[q] = basic$  then  $stach_p[q] := reject$  ;
    test
  end
```

- 
- (8) **procedure** *report*:
- ```

begin if  $rec_p = \#\{q : stach_p[q] = branch \wedge q \neq father_p\}$ 
      and  $testch_p = undef$  then
        begin  $state_p := found$  ; send  $\langle report, bestwt_p \rangle$  to  $father_p$  end
      end

```
- (9) Upon receipt of  $\langle report, \omega \rangle$  from  $q$ :
- ```

begin if  $q \neq father_p$ 
      then (* reply for initiate message *)
        begin if  $\omega < bestwt_p$  then
          begin  $bestwt_p := \omega$  ;  $bestch_p := q$  end ;
           $rec_p := rec_p + 1$  ; report
        end
      else (*  $pq$  is the core edge *)
        if  $state_p = find$ 
          then process this message later
        else if  $\omega > bestwt_p$ 
          then changeroot
          else if  $\omega = bestwt_p = \infty$  then stop
        end
      end

```
- (10) **procedure** *changeroot*:
- ```

begin if  $stach_p[bestch_p] = branch$ 
      then send  $\langle changeroot \rangle$  to  $bestch_p$ 
      else begin send  $\langle connect, level_p \rangle$  to  $bestch_p$  ;
           $stach_p[bestch_p] := branch$ 
        end
      end

```
- (11) Upon receipt of  $\langle changeroot \rangle$ :
- ```

begin changeroot end

```

## analýza

### správnosť

ukázať, že sa zvolí práve jeden šéf: nenastane deadlock

### počet správ

- ▶ testovacie správy: jeden test po každej hrane
- ▶ kostrové správy: fragment s  $n_i$  vrcholmi pri postupe o level  $O(n_i)$  správ
- ▶ postupy na level  $l$ : dizjunktné vrcholy

## KKM

- ▶  $f(x)$ -traverzovanie
- ▶ tokeny traverzujú/označujú územia
- ▶ levely: keď sa stretnú dva, vznikne nový
- ▶ naháňanie

```

var  $lev_p$       : integer      init -1 ;
       $cat_p, wait_p$  :  $\mathcal{P}$         init undef ;
       $last_p$      :  $Neigh_p$     init undef ;

begin if  $p$  is initiator then
  begin  $lev_p := lev_p + 1$  ;  $last_p := trav(p, lev_p)$  ;
         $cat_p := p$  ; send  $\langle annex, p, lev_p \rangle$  to  $last_p$ 
  end ;
  while ... (* Termination condition, see text *) do
    begin receive token  $(q, l)$  ;
      if token is annexing then  $t := A$  else  $t := C$  ;
      if  $l > lev_p$  then (* Case I *)
        begin  $lev_p := l$  ;  $cat_p := q$  ;
               $wait_p := undef$  ;  $last_p := trav(q, l)$  ;
              send  $\langle annex, q, l \rangle$  to  $last_p$ 
        end
      else if  $l = lev_p$  and  $wait_p \neq undef$  then (* Case II *)
        begin  $wait_p := undef$  ;  $lev_p := lev_p + 1$  ;
               $last_p := trav(p, lev_p)$  ;  $cat_p := p$  ;
              send  $\langle annex, p, lev_p \rangle$  to  $last_p$ 
        end
      else if  $l = lev_p$  and  $last_p = undef$  then (* Case III *)
         $wait_p := q$ 
      else if  $l = lev_p$  and  $t = A$  and  $q = cat_p$  then (* Case IV *)
        begin  $last_p := trav(q, l)$  ;
              if  $last_p = decide$ 
                then  $p$  announces itself leader
                else send  $\langle annex, q, l \rangle$  to  $last_p$ 
              end
        end
      else if  $l = lev_p$  and  $((t = A$  and
         $q > cat_p)$  or  $t = C)$  then (* Case V *)
        begin send  $\langle chase, q, l \rangle$  to  $last_p$  ;  $last_p := undef$  end
      else if  $l = lev_p$  then (* Case VI *)
         $wait_p := q$ 

```

## KKM

### počet správ

- ▶ naháňacie: 1 na vrchol a level, spolu  $n$  za level
- ▶ objavovacie:  $\sum_i f(n_i)$
- ▶ ak  $f$  je konvexná, t.j.  $f(a) + f(b) \leq f(a + b)$ , tak je  $O(\log n(n + f(n)))$  správ