# **Deep Reinforcement Learning**

Tomáš Brázdil

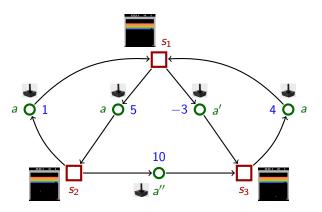


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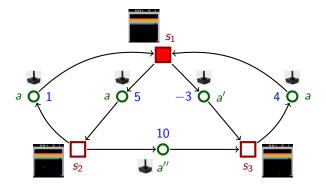
Based on V. Mnih et al, Human-level control through deep reinforcement learning. Nature (2015).

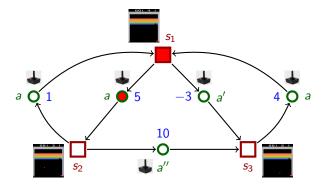
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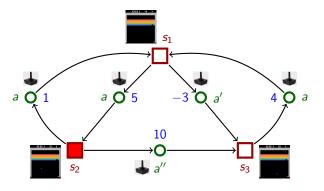


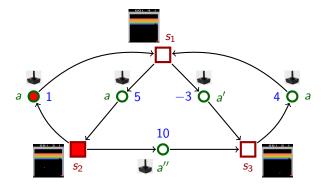
- set of states S,
- set of actions A, each state is assigned a set of enabled actions,
- ▶ transition function  $\delta : S \times A \rightarrow S$ ,
- ▶ reward function  $R: S \times A \rightarrow \mathbb{R}$ .

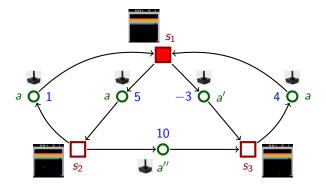
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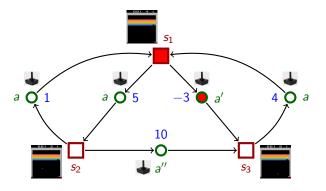


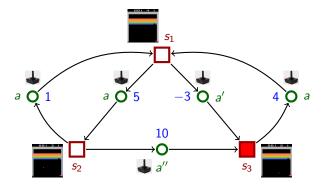




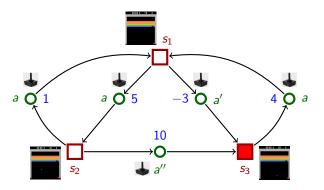








**Policy**  $\pi$  chooses actions based on the current state.



#### Notation:

- $\triangleright$   $S_1, S_2, ...$  where  $S_t$  is the t-th visited state
- $\triangleright$   $A_1, A_2, ...$  where  $A_t$  is the t-th taken action
- $ightharpoonup R_1, R_2, \dots$  where  $R_t$  is the t-th obtained reward

# **Encoding Atari games**



**States** correspond to preprocessed screenshots.

Original screenshots:  $210 \times 160$  in 128 colors.

Preprocessing:

- ▶ Rescale and crop to  $80 \times 80$ ,
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Rewards correspond to changes in the game score.

Squezed into three values: 1 for positive, -1 for negative, 0 for 0.

#### **Definition**

Return G is the total discounted reward  $G = \sum_{k=0}^{\infty} \gamma^k R_{k+1}$ . Here  $0 < \gamma < 1$  is a discount factor.

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#### **Theorem**

Define a policy  $\pi_*$  which in every  $s \in S$  chooses  $a \in A$  so that

$$q_*(s,a) = \max_{a'} q_*(s,a')$$

Then for all  $s \in S$  and  $a \in A$  we have that  $q_{\pi_*}(s, a) = q_*(s, a)$  (i.e.  $\pi_*$  is optimal).

#### Value Iteration

#### Bellman equation (Bellman, 1957):

$$q_*(s,a) = R(s,a) + \gamma \max_{a'} q_*(s',a')$$
 here  $s' = \delta(s,a)$  (1)

The true optimal values  $q_*$  form the unique solution of the above equation.

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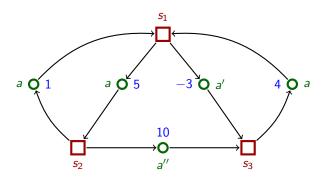
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### Value iteration algorithm:

- ▶ Start with  $q_0(s, a) = 0$  for all s, a.
- ▶ Iteratively apply the right-hand-side of (1):

$$q_{k+1}(s, a) = R(s, a) + \gamma \max_{a'} q_k(s', a')$$
 here  $s' = \delta(s, a)$ 

Then  $q_*(s, a) = \lim_{k \to \infty} q_k(s, a)$ .



$$q_2(s_1, a) = q_1(s_1, a) + \gamma \max\{q_1(s_2, a), q_1(s_2, a'')\} = 5 + \gamma \max\{1, 10\}$$

.

#### Criticism

Minor issue: The value iteration can be used only if the transition relation  $\delta$  is known.

**Major issue**: State/action space is typically huge or infinite:

- ▶ Atari games:  $128^{84 \times 84 \times 4} = 128^{28224}$  possible states!
- ▶ Go: 10<sup>170</sup> states
- Helicopter control: Infinite!

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We solve this problem in two steps:

- Update our approximation of q\* only for "relevant" state-action pairs. (using reinforcement learning)
- 2. Represent our approximation of  $q_*$  succinctly. (using neural networks).

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#### In general:

- ▶ Start with a policy  $\hat{\pi}$  and an estimate Q of  $q_*$ .
- ▶ While (unhappy with the result) do
  - Simulate  $\hat{\pi}$  and update the estimate Q based on "experience".
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#### We need to

- have a good rule for learning from experience (exploit your choice of actions),
- go through important parts of the state-space (explore the state space).

For exploration, consider  $\varepsilon$ -greedy (randomized) policy  $\hat{\pi}$ :

- With probability  $1 \varepsilon$ , choose  $a = \arg \max_{a'} Q(s, a')$ .
- ▶ With probability  $\varepsilon$ , choose an arbitrary action uniformly in random.

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#### Q-learning algorithm:

- ▶ Always follow  $\hat{\pi}$ .
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$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha_t(q_*(S_t, A_t) - Q(S_t, A_t))$$

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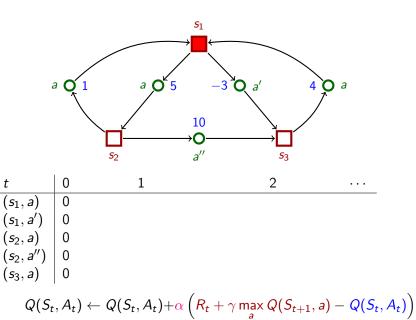
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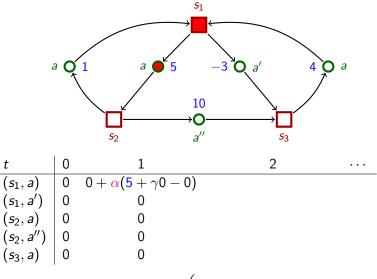
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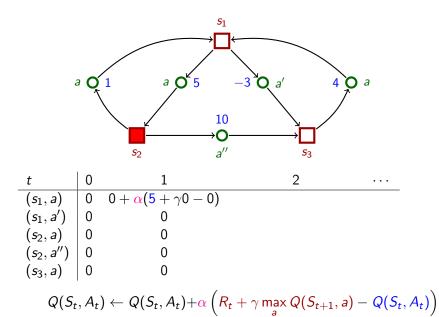
#### Theorem (Watkins & Dayan 1992)

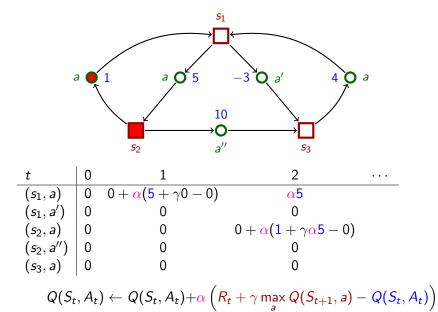
If S is finite and  $\alpha_t = 1/t$ , then each Q(s, a) converges to  $q_*(s, a)$ .

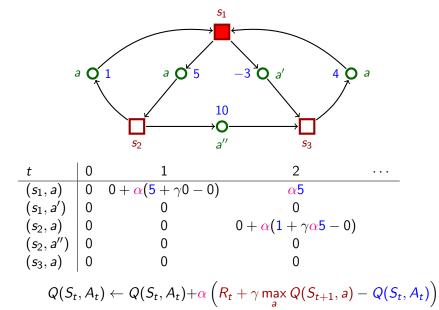




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<sub>11</sub>







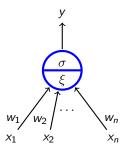
# Q-learning with Function Approximation

The problem: How to represent Q?

- ▶ linear combinations of (manually created) features [typical]
- decision trees
- SVM
- neural networks

### Neural networks

Neural network is a directed graph of interconnected neurons.

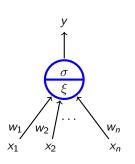


- $w_1, \ldots, w_n \in \mathbb{R}$  are weights
- $ightharpoonup x_1, \ldots, x_n \in \mathbb{R}$  are inputs
- $\blacktriangleright \xi = \sum_{i=1}^n w_i x_i$
- $\mathbf{v} = \sigma(\xi)$

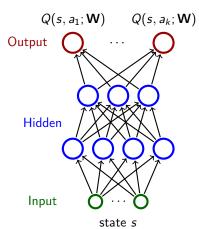
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- ► **W** are weights of all neurons
- ► Q(s, a; W) the Q value in (s, a) represented by the network

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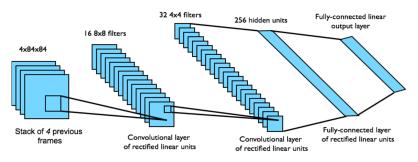
using gradient descent  $\mathbf{W} = \mathbf{W} - \alpha \nabla_{\mathbf{W}} L(\mathbf{W})$  where

$$\nabla_{\mathbf{W}} L(\mathbf{W}) = \nabla_{\mathbf{W}} \left( (1/2)(\tau - Q(S_t, A_t; \mathbf{W}))^2 \right)$$
  
=  $(\tau - Q(S_t, A_t; \mathbf{W}))(-\nabla_{\mathbf{W}} Q(S_t, A_t; \mathbf{W}))$ 

 $\nabla_{\mathbf{W}} Q(S_t, A_t; \mathbf{W})$  can be computed using standard backpropagation.

## Convolutional Networks

In image processing, classical MLP has been superseded by convolutional networks.



First introduced in [LeCun et al., 1989d] for handwritten digits recognition.

Combined with powerful GPU powered computers  $\Rightarrow$  breakthrough in image processing.

Image: D. Silver, UCL Course on RL, http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching.html

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#### Dealyed target values:

► W<sup>-</sup> is several steps old value of weights. (Previously W<sup>-</sup> was the current weight vector.)

Both adjustments considerably improve learning (see results later).

## **Experiments**

### Training:

- ▶ 49 games, the same architecture of network (trained for each game).
- $\triangleright$   $\varepsilon$ -greedy strategy with  $\varepsilon$  annealed linearly from 1.0 to 0.1 over the first million frames.
- ► Trained for 50 million frames (around 38 days of game experience in total).

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#### **Evaluation:**

- ▶ Play each game 30 times for up to 5 min each time with different initial random conditions.
- $\varepsilon$ -greedy policy with  $\varepsilon = 0.05$ .
- ▶ A random agent selecting actions at 10Hz used as baseline.
- Pro human tester under the same emulator, average reward for 20 episodes, max 5 minutes, following around 2h of practice playing each game.

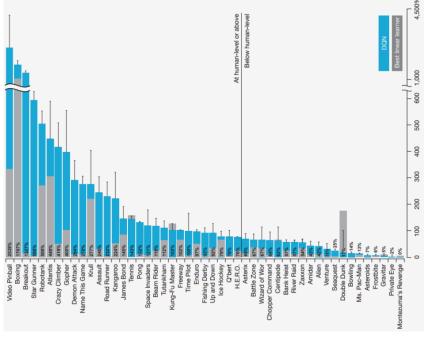


Image: V. Mnih et al, Human-level control through deep reinforcement learning. Nature (2015).

#### Results

	B. Rider	Breakout	Enduro	Pong	Q*bert	Seaquest	S. Invaders
Random	354	1.2	0	-20.4	157	110	179
Sarsa [3]	996	5.2	129	-19	614	665	271
Contingency [4]	1743	6	159	-17	960	723	268
DQN	4092	168	470	20	1952	1705	581
Human	7456	31	368	-3	18900	28010	3690
HNeat Best [8]	3616	52	106	19	1800	920	1720
HNeat Pixel [8]	1332	4	91	-16	1325	800	1145
DQN Best	5184	225	661	21	4500	1740	1075

Sarsa and Contingency are other reinforcement learning methods.

**HNeat Best** and **HNeat Pixel** are methods based on evolutionary policy search.

These methods use a hand-engineered object detector algorithm that outputs the locations and types of objects on the Atari screen.

Image: D. Silver, UCL Course on RL, http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching.html

# Conceptual Limitations (as opposed to humans)

- Prior knowledge:
  - Humans: Huge amount, such as intuitive physics and intuitive psychology.
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  - RL: Brute force, where the correct actions are eventually discovered and internalized into a policy.
- Experience acquisition:
  - ▶ **Humans**: Can figure out what is likely to give rewards without ever actually experiencing the rewarding transition.
  - RL: Has to actually experience a positive reward.

A. Karpathy, Deep Reinforcement Learning: Pong from Pixels,

### **Conclusions**

- Current computers can learn to play games on old computer at (super)human level.
- ▶ The main algorithms used in solution:
  - Reinforcement learning
  - Convolutional networks
- It is a very active area of research, several better solutions than DQN have been recently presented.

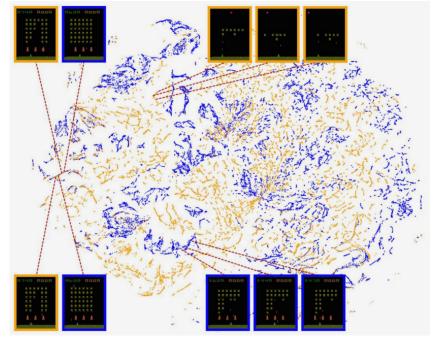


Image: V. Mnih et al, Human-level control through deep reinforcement learning. Nature (2015).