Deep Reinforcement Learning

Tomáš Brázdil



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Based on V. Mnih et al, Human-level control through deep reinforcement learning. Nature (2015).

Left: http://commons.wikimedia.org/wiki/File:Atari2600a.JPG, Right: http://www.opobotics.com/.



- set of states S,
- set of actions A,

each state is assigned a set of enabled actions,

- transition function $\delta : \mathbf{S} \times \mathbf{A} \rightarrow \mathbf{S}$,
- reward function $R: S \times A \rightarrow \mathbb{R}$.















Policy π chooses actions based on the current state.



Notation:

- S_1, S_2, \dots where S_t is the *t*-th visited state
- A_1, A_2, \dots where A_t is the *t*-th taken action
- $R_1, R_2, ...$ where R_t is the *t*-th obtained reward

Encoding Atari games



States correspond to preprocessed screenshots.

Original screenshots: 210 \times 160 in 128 colors. Preprocessing:

- Rescale and crop to 80×80 ,
- convert to gray-scale,
- use 4 most recent frames in a single state.

The states: Real vectors of dimension $80 \times 80 \times 4$.

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Rewards correspond to changes in the game score. Squezed into three values: 1 for positive, -1 for negative, 0 for 0.

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Theorem

Define a policy π_* which in every $s \in S$ chooses $a \in A$ so that

$$q_*(s,a) = \max_{a'} q_*(s,a')$$

Then for all $s \in S$ and $a \in A$ we have that $q_{\pi_*}(s, a) = q_*(s, a)$ (i.e. π_* is optimal).

Value Iteration

Bellman equation (Bellman, 1957):

$$q_*(s,a)=R(s,a)+\gamma \max_{a'}q_*(s',a') \qquad ext{here } s'=\delta(s,a) \quad (1)$$

The true optimal values q_* form the unique solution of the above equation.

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Value iteration algorithm:

- Start with $q_0(s, a) = 0$ for all s, a.
- Iteratively apply the right-hand-side of (1):

$$q_{k+1}(s,a) = R(s,a) + \gamma \max_{a'} q_k(s',a')$$
 here $s' = \delta(s,a)$

Then $q_*(s, a) = \lim_{k \to \infty} q_k(s, a)$.



 $q_2(s_1, a) = q_1(s_1, a) + \gamma \max\{q_1(s_2, a), q_1(s_2, a'')\} = 5 + \gamma \max\{1, 10\}$

Criticism

Minor issue: The value iteration can be used only if the transition relation δ is known.

Major issue: State/action space is typically huge or infinite:

- Atari games: $128^{84 \times 84 \times 4} = 128^{28224}$ possible states!
- Go: 10¹⁷⁰ states
- Helicopter control: Infinite!

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We solve this problem in two steps:

 Update our approximation of q_{*} only for "relevant" state-action pairs.

(using reinforcement learning)

 Represent our approximation of q_{*} succinctly. (using neural networks).

Reinforcement learning (roughly)

The problem:

How to learn q_* ?



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In general:

- Start with a policy $\hat{\pi}$ and an estimate Q of q_* .
- While (unhappy with the result) do
 - Simulate $\hat{\pi}$ and update the estimate Q based on "experience".
 - Update the policy $\hat{\pi}$ according to Q.

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We need to

- have a good rule for learning from experience (*exploit* your choice of actions),
- go through important parts of the state-space (*explore* the state space).

For exploration, consider ε -greedy (randomized) policy $\hat{\pi}$:

- With probability 1ε , choose $a = \arg \max_{a'} Q(s, a')$.
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Q-learning algorithm:

- Always follow $\hat{\pi}$.
- In every time instant t update Q by

 $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha_t (q_*(S_t, A_t) - Q(S_t, A_t))$

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Theorem (Watkins & Dayan 1992)

If S is finite and $\alpha_t = 1/t$, then each Q(s, a) converges to $q_*(s, a)$.



11





11




Q-learning with Function Approximation

The problem: How to represent Q?

- Inear combinations of (manually created) features [typical]
- decision trees
- SVM
- neural networks
- ...

Neural networks

Neural network is a directed graph of interconnected neurons.



- *w*₁,..., *w*_n ∈ ℝ are weights
 *x*₁,..., *x*_n ∈ ℝ are inputs
 ξ = ∑ⁿ_{i=1} *w*_i*x*_i
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- $\xi = \sum_{i=1}^{n} w_i x_i$ • $y = \sigma(\xi)$



- ► W are weights of all neurons
- Q(s, a; W) the Q value in (s, a) represented by the network

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 - Update weights **W** so that $Q(S_t, A_t; \mathbf{W})$ gets closer to τ

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$$\boldsymbol{L}(\boldsymbol{\mathsf{W}}) = \frac{1}{2}(\tau - \boldsymbol{Q}(\boldsymbol{S}_t, \boldsymbol{A}_t; \boldsymbol{\mathsf{W}}))^2$$

using gradient descent $\mathbf{W} = \mathbf{W} - \alpha \nabla_{\mathbf{W}} L(\mathbf{W})$ where

$$\nabla_{\mathbf{W}} L(\mathbf{W}) = \nabla_{\mathbf{W}} \left((1/2)(\tau - Q(S_t, A_t; \mathbf{W}))^2 \right)$$

= $(\tau - Q(S_t, A_t; \mathbf{W}))(-\nabla_{\mathbf{W}} Q(S_t, A_t; \mathbf{W}))$

 $\nabla_{\mathbf{W}}Q(S_t, A_t; \mathbf{W})$ can be computed using standard backpropagation.

Convolutional Networks

In image processing, classical MLP has been superseded by *convolutional networks*.



First introduced in [LeCun et al., 1989d] for handwritten digits recognition.

Combined with powerful GPU powered computers \Rightarrow breakthrough in image processing.

Image: D. Silver, UCL Course on RL, http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching.html

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Dealyed target values:

▶ W⁻ is several steps old value of weights.

(Previously W^- was the current weight vector.)

Both adjustments considerably improve learning (see results later).

Experiments

Training:

- 49 games, the same architecture of network (trained for each game).
- ▶ ε -greedy strategy with ε annealed linearly from 1.0 to 0.1 over the first million frames.
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Evaluation:

- Play each game 30 times for up to 5 min each time with different initial random conditions.
- ε -greedy policy with $\varepsilon = 0.05$.
- A random agent selecting actions at 10Hz used as baseline.
- Pro human tester under the same emulator, average reward for 20 episodes, max 5 minutes, following around 2h of practice playing each game.



Image: V. Mnih et al, Human-level control through deep reinforcement learning. Nature (2015).

Results

	B. Rider	Breakout	Enduro	Pong	Q*bert	Seaquest	S. Invaders
Random	354	1.2	0	-20.4	157	110	179
Sarsa [3]	996	5.2	129	-19	614	665	271
Contingency [4]	1743	6	159	-17	960	723	268
DQN	4092	168	470	20	1952	1705	581
Human	7456	31	368	-3	18900	28010	3690
HNeat Best [8]	3616	52	106	19	1800	920	1720
HNeat Pixel [8]	1332	4	91	-16	1325	800	1145
DQN Best	5184	225	661	21	4500	1740	1075

Sarsa and Contingency are other reinforcement learning methods.

HNeat Best and **HNeat Pixel** are methods based on evolutionary policy search.

These methods use a hand-engineered object detector algorithm that outputs the locations and types of objects on the Atari screen.

Image: D. Silver, UCL Course on RL, http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching.html

Conceptual Limitations (as opposed to humans)

- Prior knowledge:
 - Humans: Huge amount, such as intuitive physics and intuitive psychology.
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- Abstraction and planning:
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 - RL: Brute force, where the correct actions are eventually discovered and internalized into a policy.
- Experience acquisition:
 - ► **Humans**: Can figure out what is likely to give rewards without ever actually experiencing the rewarding transition.
 - **RL**: Has to actually experience a positive reward.

A. Karpathy, Deep Reinforcement Learning: Pong from Pixels,

http://karpathy.github.io/2016/05/31/rl/.

Conclusions

- Current computers can learn to play games on old computer at (super)human level.
- The main algorithms used in solution:
 - Reinforcement learning
 - Convolutional networks
- It is a very active area of research, several better solutions than DQN have been recently presented.



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