

Exercise 1 Consider the following formulae.

- (a) $(A \leftrightarrow B) \rightarrow (\neg A \wedge C)$
- (b) $(A \rightarrow B) \rightarrow C$
- (c) $A \leftrightarrow B$
- (d) $(A \rightarrow B) \leftrightarrow (A \rightarrow C)$
- (e) $(A \vee B) \wedge (A \vee C)$
- (f) $[A \rightarrow (B \vee \neg A)] \rightarrow (B \rightarrow A)$
- (g) $[(A \vee B) \rightarrow (C \rightarrow A)] \leftrightarrow (A \vee B \vee C)$

For each of them

- (1) use a truth table to determine if the formula is valid and/or satisfiable;
- (2) convert the formula into CNF using the truth table;
- (3) convert the formula into CNF using equivalence transformations instead;
- (4) write the formula as a set of clauses.

Exercise 2 Which of the following formulae imply each other?

- (a) $A \wedge B$
- (b) $A \vee B$
- (c) $A \rightarrow B$
- (d) $A \leftrightarrow B$
- (e) $\neg A \wedge \neg B$
- (f) $\neg A$
- (g) $\neg(A \rightarrow B)$

Exercise 3 We can encode n -bit numbers via an n -tuple of propositional variables A_{n-1}, \dots, A_0 .

- (a) Write a formula $\varphi(A_1, A_0, B_1, B_0, C_2, C_1, C_0)$ for the addition of 2-bit numbers ($\bar{A} + \bar{B} = \bar{C}$).
- (b) Write a formula $\varphi(A_{n-1}, \dots, A_0, B_{n-1}, \dots, B_0, C_n, \dots, C_0)$ for the addition of n -bit numbers.

Exercise 4 Use the DPLL algorithm to determine whether the following formulae are satisfiable.

- (a) $\neg[(A \rightarrow B) \leftrightarrow (A \rightarrow C)]$
- (b) $(A \vee B \vee C) \wedge (B \vee D) \wedge (A \rightarrow D) \wedge (B \rightarrow A)$
- (c) $(A \leftrightarrow B) \rightarrow (\neg A \wedge C)$
- (d) $[A \rightarrow (B \vee \neg A)] \rightarrow (B \rightarrow A)$
- (e) $[(A \vee B) \rightarrow (C \rightarrow A)] \leftrightarrow (A \vee B \vee C)$

Exercise 5 Use the resolution method to determine which of the following formulae are valid.

- (a) $(A \wedge \neg B \wedge C) \vee (\neg A \wedge \neg B \wedge \neg C) \vee (B \wedge C) \vee (A \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C)$
- (b) $(A \wedge B) \vee (B \wedge C \wedge D) \vee (\neg A \wedge B) \vee (\neg C \wedge \neg D)$
- (c) $(\neg A \wedge B \wedge \neg C) \vee (\neg B \wedge \neg C \wedge D) \vee (\neg C \wedge \neg D) \vee (A \wedge B) \vee (\neg A \wedge B \wedge C) \vee (\neg A \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge D)$

Exercise 6 Given a finite automaton \mathcal{A} and an input word w , write down a formula $\varphi_{\mathcal{A},w}$ that is satisfiable if, and only if, the automaton \mathcal{A} accepts w .

Exercise 7 Construct the game for the following set of Horn-formulae and determine the winning regions.

$$\begin{array}{cccccc}
 B \wedge C \wedge D \rightarrow A & C \wedge F \rightarrow B & A \wedge F \rightarrow C & D \rightarrow B & A \wedge B \rightarrow F & \\
 E \rightarrow A & D \wedge E \rightarrow B & C & D & &
 \end{array}$$