

Exercise 1 We consider words over the alphabet $\{a, b\}$ as transition systems $\langle S, E_s, E_r, P_a, P_b \rangle$ where the states S are the positions, the two predicates P_a and P_b label each position with the corresponding letter, and the two edge relations are

$$E_s = \{ \langle i, i+1 \rangle \mid i < n-1 \},$$

$$E_r = \{ \langle i, k \rangle \mid i \leq k < n \}.$$

(where $n = |S|$ is the length of the word). Define the following languages in modal logic.

- (a) All words starting with the letter a .
- (b) All words consisting only of letters a .
- (c) All words ending with the letter a .
- (d) a^*b^*
- (e) All words containing the factor bb .
- (f) All words containing at least two letters b .
- (g) All words containing exactly two letters b .
- (h) $(ab)^*$

Exercise 2 Translate the following formulae into first-order logic.

- (a) $[a]P \rightarrow P$
- (b) $P \rightarrow \langle a \rangle Q$
- (c) $[a](P \wedge \langle b \rangle Q) \rightarrow (\langle a \rangle P \vee \langle b \rangle Q)$

Exercise 3 Prove the following modal formulae using tableaux.

- (a) $\Box(P \leftrightarrow (Q \wedge R)) \rightarrow (\Box P \leftrightarrow (\Box Q \wedge \Box R))$
- (b) $\neg \Box \Box P \rightarrow \Diamond \Diamond \neg P$
- (c) $\Box(P \wedge \neg P) \rightarrow \Box Q$
- (d) $\neg \Diamond P \rightarrow \Box(P \rightarrow Q)$

Exercise 4 Find CTL*-formulae defining the following properties of $\{a, b\}$ -labelled trees.

- (a) There is at least one label b .
- (b) Every path contains some b .
- (c) Every path contains at least two b .
- (d) All paths contain infinitely many b .
- (e) Some path contains infinitely many b .

Exercise 5 We model an elevator in a building with 3 stories.

- (a) Describe the elevator as a transition system.
- (b) Write a specification for the elevator in modal logic and in LTL. Start with the following two statements, then add your own.
 - (i) The elevator never moves when the door is open.
 - (ii) If the button on floor 2 is pressed, the elevator will eventually stop at that floor and open the door.

Check that your system from (a) satisfies these formulae.