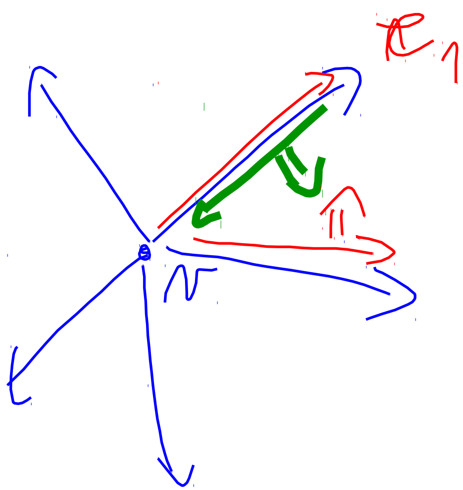
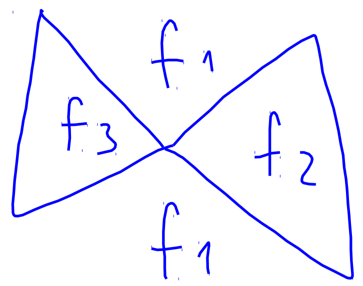
Map overlap

Plane subdivisions — doubly connected edge list

Find all edges coming from given vertex v in the clockwise order.





S_1 ..... red map .... $\mathcal{D}_1$ DCEL

S_2 .... blue map .... $\mathcal{D}_2$ DCEL

$S = \mathcal{O}(S_1, S_2)$ we want to get $\mathcal{D}$ DCEL for the overlap

<u>Algorithm has 2 steps</u>

0 Put tables for vertices and edges of $\mathcal{D}_1$ and $\mathcal{D}_2$ into $\mathcal{D}$ without changes.

1 Using segment intersection algorithm modify the tables for vertices and edges in $\mathcal{D}$ to be correct.

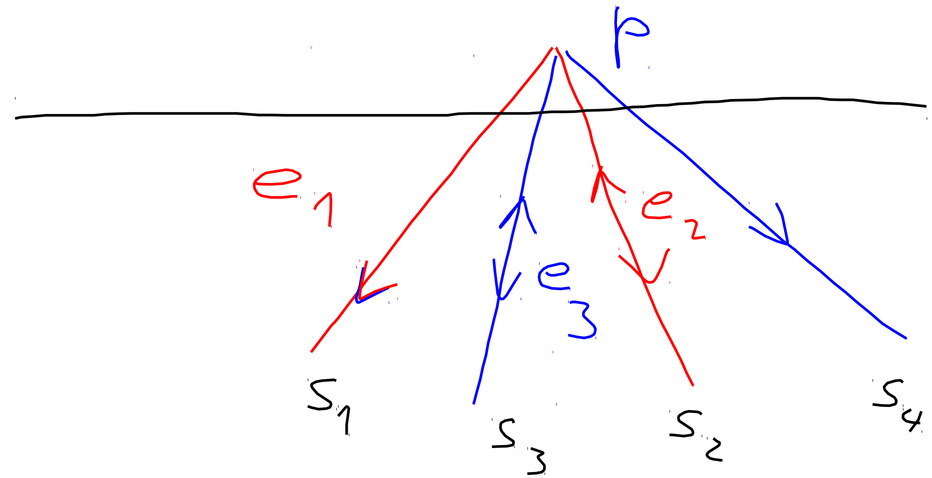
2 Create the table for faces. Moreover, to every face find red and blue faces in which it lies.

Details to be found in the diploma theses of Dominik Janků - see IS.

Step 1 We take blue and red segments that their intersections are
- empty
- one point
- the whole segment.

Now we carry out sweep line method computing only intersections of red and blue segment.
Moreover, we do changes in the tables in $\mathcal{D}$.
p is an event

(A) $C(p) = \emptyset$ in the tables for vertices and edges of $\mathcal{D}$ we will not add another line, we only change records for next and previous edges.

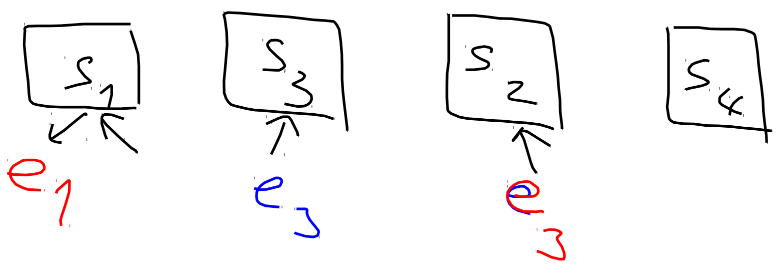


Original table

	prev
e_1	e_2

Modification

	prev
e_1	e_3



Red and blue edge cross in the inner point

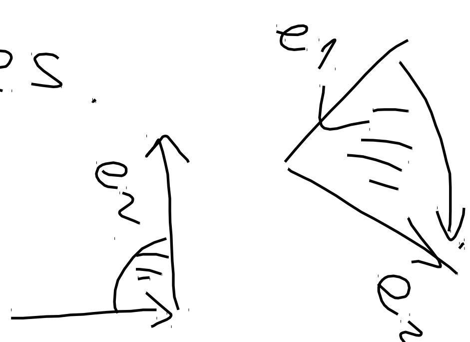
– see e-learning

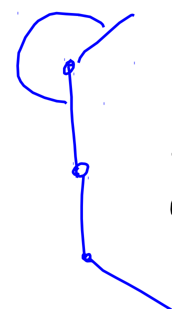
2nd step - finding faces of overlap

Every face (bounded) is determined by its outer boundary which we called oute cycle.

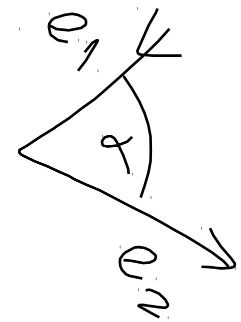
We find all cycles.

We have to distinguish between inner and outer cycles.





decide about the sign of $\det \begin{pmatrix} e_{1x} & e_{2x} \\ e_{1y} & e_{2y} \end{pmatrix}$ >0 $\alpha < 180°$, <0 $\alpha > 180°$



$$\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{e_1} = 1 > 0$$

Now we have outer cycles, inner cycles and one fictional cycle for unbounded face called C_∞.

We need to find which inner cycles belong to which outer cycles.

Picture 3.7

Let us construct an abstract graph $\mathcal{G}$

its nodes (vertices) are cycles

edges are "connections" described by the picture 3.7.

Every component of the graph G (see picture 3.8) contains just one outer cycle and determines just one face in the overlap.

We can make the table for faces and complete the table for edges.

Last task. To find for every new face f faces f_1 and f_2 from the red and the blue map, respectively, in which f lies.

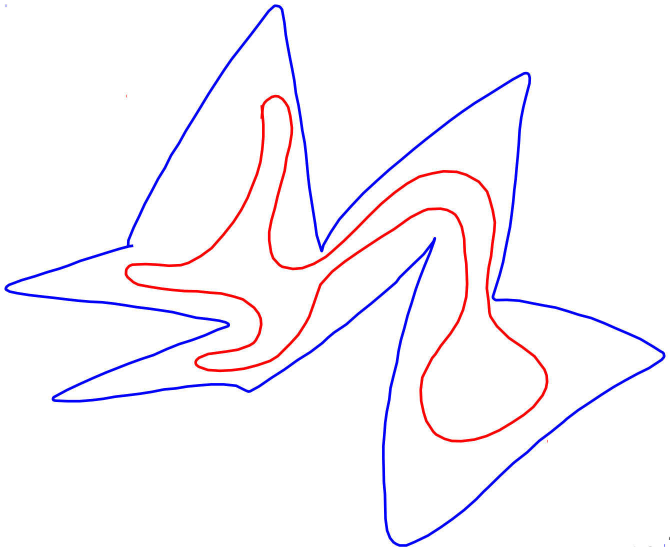
Two cases

- simple one f has a red and a blue edge in its cycles. Then corresponding adjacent faces are f_1 and f_2.

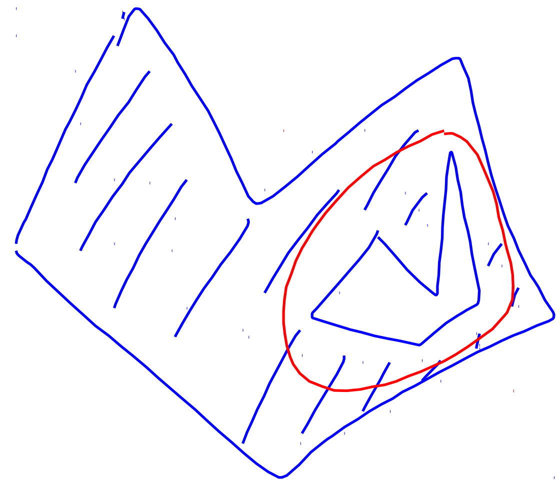
- if f is unbounded then f_1 and f_2 are unbounded faces in the red and the blue map
- if the previous cases do not occur we take the most left point of the outer cycle of f and proceed according to the picture 3.10

Application To find intersections and unions of polygons (non convex)

We take polygons without "holes" - they are called simply connected. It means that every closed curve can be shrunk into point



simply connected



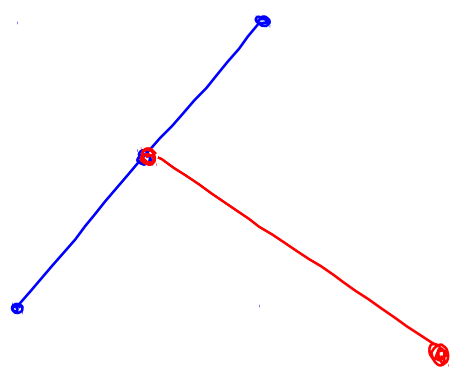
not simply connected

Simply connected polygon

– planar subdivision with only two faces
bounded f_1 and unbounded f_2

Intersection of two polygons is the union of faces of the overlap which appear in bounded faces of both polygons.

Union of two polygons is the union of faces which lie either in the red bounded face or the blue bounded face = complement of the unbounded face.



<u>Theorem</u> The complexity of the algorithm is

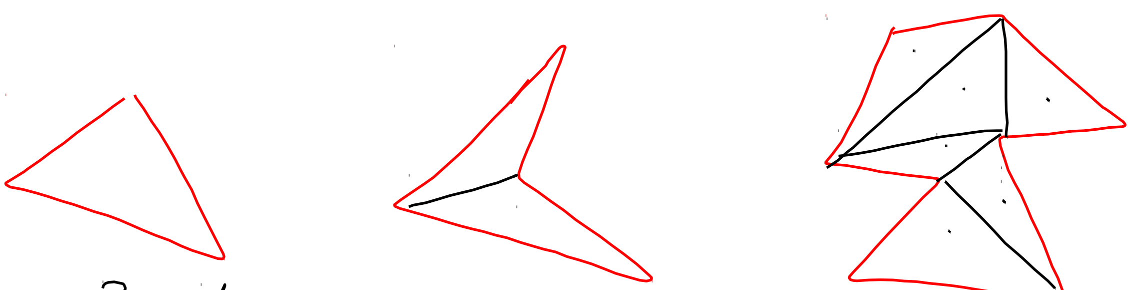
$$O\big((n+k)\log n\big)$$

where $n = n_1 + n_2$, n_1 complexity of S_1, k complexity of the overlap.

Triangulations of polygons

We consider a simply connected generally non convex polygon. We want to divide it into triangles with vertices in the vertices of the polygon.

Theorem: Every polygon has a triangulation. If the polygon has n edges, then the number of triangles is $n-2$.



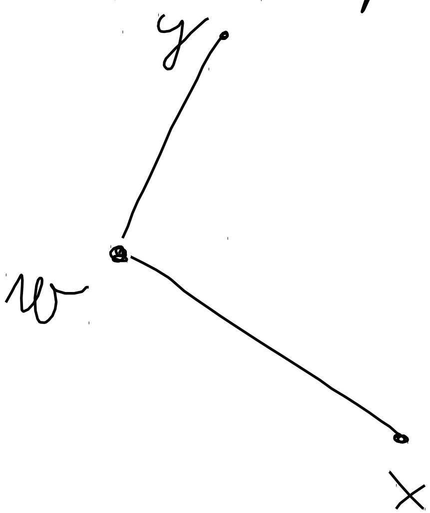
$n = 3$ triangles 1 $\quad$ $n = 4$ 2 $\quad$ $n = 8$ 6

<u>Proof</u>: $n = 3$ clear

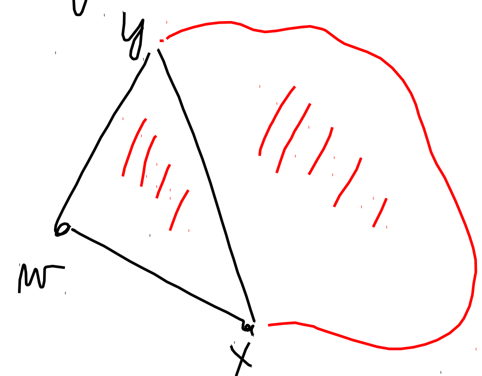
Suppose that the statement is true for all $k < n$
$3 \le k$

Consider the polygon with n edges.

Consider most left vertex w



1) xy lies in the polygon



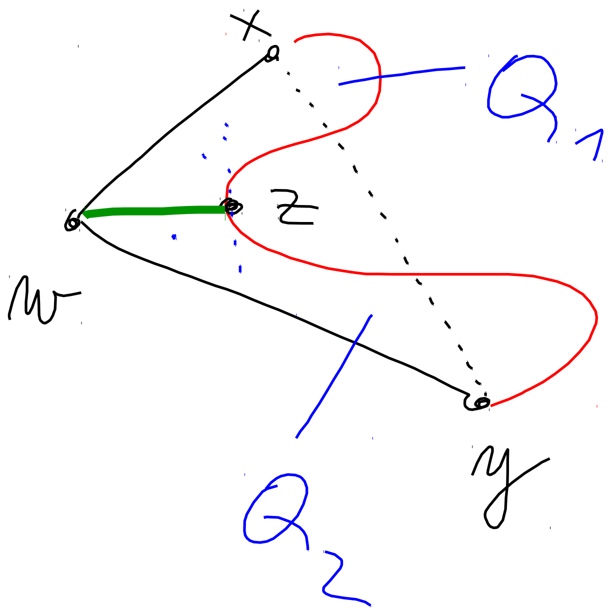
Without w we get $(n-1)$-polygon.

The statement for it is true.

number of triangles for n-polygon is

$$\underbrace{\{(n-1)-2\}}_{\text{without } w} + 1 = n-2$$

If not the case



On the red boundary take the vertex z which is the closest to w.

I assert that wz lies in the interior of the polygon.

wz divides the n-polygon into two polygons with n_1 and n_2 vertices

$$n_1 + n_2 = n + 2$$

Now we know that triangulation of Q_1 has $n_1 - 2$ triangles
of Q_2 has $n_2 - 2$ triangles

All together we have
$$n_1 - 2 + n_2 - 2 = n_1 + n_2 - 4 = n - 2$$