$= n\cdot p \sum_{i=0}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i}$

$= 1$

$P(a \le X < b)$

$P(X = x_i)$

$a \quad b \quad c \quad x_i \quad d \quad \mathbb{R}$

$P(c \le X < d) \overset{?}{=} P(X = x_i)$

$1-p \quad p$

$0 \quad 1$

$\boxed{X \sim A(p)}$

$\boxed{EX} = 0\cdot(1-p) + 1\cdot p = \boxed{p}$

$X \sim Bi(n,p) \quad EX = \sum_{i=0}^{n} i\cdot\binom{n}{i} p^i (1-p)^{n-i} = np$

$= \sum_{i=0}^{n} \frac{n(n-1)\cdots(n-i+1)\cdot i}{i!} \cdot p^i (1-p)^{n-i} = np \sum_{i=1}^{n} \frac{(n-1)\cdots(n-i+1)}{(i-1)!} \cdot p^{i-1}(1-p)^{n-i}$

11 29-16:01

$$EX = \begin{cases} \sum_i x_i f_X(x_i) \leftarrow \text{diskr.} \\ \int_{-\infty}^{\infty} x\, f(x)\, dx \leftarrow \text{spojité} \end{cases}$$

$Y = a + bX \qquad EY = \sum_i (a + bx_i) f_X(x_i)$

$= a \underbrace{\sum_i f_X(x_i)}_{=1} + b \sum_i x_i f_X(x_i)$

$= a + bEX$

11 29-16:23

$Y = \psi(X)$

$$EY = \begin{cases} \sum_i \psi(x_i) f_X(x_i) \\ \int_{-\infty}^{\infty} \psi(x) f(x)\, dx \end{cases}$$

$E(X+Y) = \sum_i \sum_j (x_i + y_j) \overbrace{P(X = x_i, Y = y_j)}^{f_{X,Y}(x_i, y_j)}$

$= \sum_i x_i \underbrace{\sum_j P(X = x_i, Y = y_j)}_{f_X(x_i)} + \sum_j y_j \underbrace{\sum_i P(X = x_i, Y = y_j)}_{f_Y(y_j)}$

$= \sum_i x_i f_X(x_i) + \sum_j y_j f_Y(y_j) = EX + EY$

11 29-16:25

$W = (X_1, \dots, X_n), \quad a \in \mathbb{R}^m \quad B \in \mathrm{Mat}_{m,n}$

$E(a + BW) = a + B(EW)$

$$\begin{pmatrix} \text{———} \\ \\ \\ \end{pmatrix} \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} = \begin{pmatrix} b_{11}X_1 + \dots + b_{1n}X_n \\ \\ \\ \end{pmatrix}$$

$(X, Y) \qquad f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y)$

11 29-16:32

$s_X = \frac{1}{n}\sum_i (x_i - \bar{x})^2$

$\operatorname{var} X = E(X - EX)^2 = E(X^2 - 2XEX + (EX)^2)$

$= EX^2 - (EX)^2$

$\operatorname{var}(a + bX) = E(\cancel{a} + bX - E(\cancel{a} + bX))^2$

$= E(b^2(X - EX)^2) = b^2 \operatorname{var} X$

$\boxed{\operatorname{var} X \ge 0}$

11 29-16:38

$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$

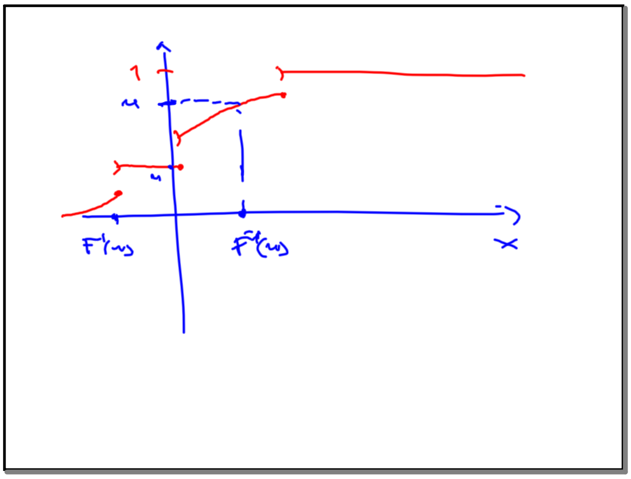
$Y = \mu + \sigma Z$

$\Rightarrow Z = \frac{Y - \mu}{\sigma}$

$?$

$F_Y(y) = \int_{-\infty}^{y} f_Y(x)\, dx$

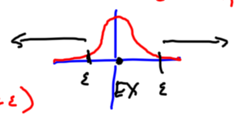
11 29-16:45



11 29-16:53

$$\operatorname{var} X = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx \qquad EX = \mu \qquad Z \sim N(0,1)$$

$$= \int_{|x-\mu|\ge\varepsilon} (x-\mu)^2 f(x)\,dx + \cancel{\int_{|x-\mu|<\varepsilon} (x-\mu)^2 f(x)\,dx}$$



$$\ge \int_{|x-\mu|\ge\varepsilon} \varepsilon^2 f(x)\,dx = \varepsilon^2 P(|X-EX|\ge\varepsilon)$$

$$\varepsilon = k\sigma$$

$$P(|X-EX|\ge k\sigma) \le \frac{\cancel{\sigma^2}}{k^2\cancel{\sigma^2}} = \frac{1}{k^2}$$

$$\operatorname{var} X = E(X-EX)^2$$

$X_1, X_2, \dots \quad X_n \sim Bi(n,p) \quad \boxed{Y_n = \frac{1}{n} X_n}$

$$E\left(\tfrac{1}{n}X_n\right) = \tfrac{1}{n} np = p$$

$$\operatorname{var}\left(\tfrac{1}{n}X_n\right) = \tfrac{1}{n^2} np(1-p) = \frac{p(1-p)}{n}$$

$$P(|Y_n - p| \ge \varepsilon) \le \boxed{\frac{p(1-p)}{n} \cdot \frac{1}{\varepsilon^2}} \xrightarrow[n\to\infty]{} 0$$

$$\lim_{n\to\infty} P(|Y_n - p| \ge \varepsilon) = 0$$

11 29-16:57

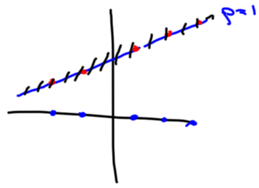
$$\operatorname{cov}(a+bX, c+dY) = E\left(\cancel{a}+bX - E(\cancel{a}+bX)\right)\cdot(\;\cdot/\cdot\;)$$
$$= bd\, E(X-EX)(Y-EY)$$

$$\operatorname{var}(X+Y) = E(X+Y-EX-EY)(X+Y-EX-EY)$$
$$= E(X^2 + 2XY + Y^2 + \dots$$
$$= \operatorname{var} X + \operatorname{var} Y + 2\cdot\operatorname{cov}(X,Y)$$

$X \sim A(p) \quad EX = p \quad E(X-p)^2 = +p^2(1-p) + (1-p)^2 p = -p^2 + p = p(1-p)$

11 29-17:17

$$\rho_{X,X} = \frac{\operatorname{var} X}{\sqrt{\operatorname{var} X}\sqrt{\operatorname{var} X}} = 1$$

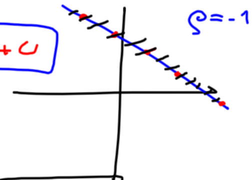


$|\rho_{X,Y}| \le 1$ (?)

$$0 \le \operatorname{var}\left(\frac{Y-EY}{\sqrt{\operatorname{var} Y}} + t\,\frac{X-EX}{\sqrt{\operatorname{var} X}}\right)$$
$$= 1 + t^2 + 2t\,\rho_{X,Y}$$

$D = 4\rho_{XY}^2 - 4 \le 0$ $\quad \boxed{Y = \alpha X + c}$

$\Rightarrow |\rho_{X,Y}| \le 1$ $\quad$ +/−



$D = 0 \Leftrightarrow \rho_{XY} = \pm 1$

pro t_0 = jediný kořen nab

$\frac{Y-EY}{\sqrt{\operatorname{var} Y}} + t_0 \frac{X-EX}{\sqrt{\operatorname{var} X}}$ nulový rozptyl!

$\boxed{\operatorname{var} X = 0}$ $\Rightarrow X = EX$

11 29-17:30

$$\operatorname{var} W = E(W-EW)(W-EW)^T$$

$$i \to \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} (X_1 \;\; \dots \;\; X_n)$$

$\uparrow$
j

$$i,j: \quad E(X_i - EX_i)(X_j - EX_j)$$

11 29-17:45