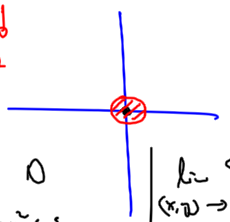
$\lim_{(x,y)\to(0,0)} \boxed{\sin(1/x)}\boxed{\cos y}$ NEMÁ LIMITU

nemá limitu 1

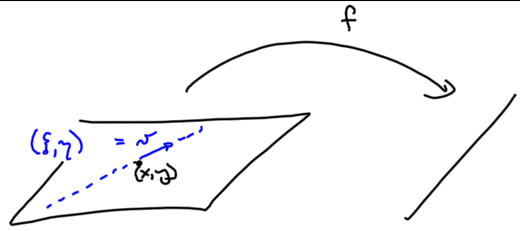


$\lim_{(x,y)\to(0,0)} \sin(1/x)\cdot y^2 = 0$ $x^2+y^2<\varepsilon$

$|\sin(1/x)|\cdot|y^2| \le 1\cdot\varepsilon$ $\in[0,1]$ $\to 0$

$\lim_{(x,y)\to(0,0)} \sin\frac{1}{x^2+y^2}\cdot (x^2+y^2) = \lim_{r\to0}\sin(1/r^2)\cdot r^2 = 0$

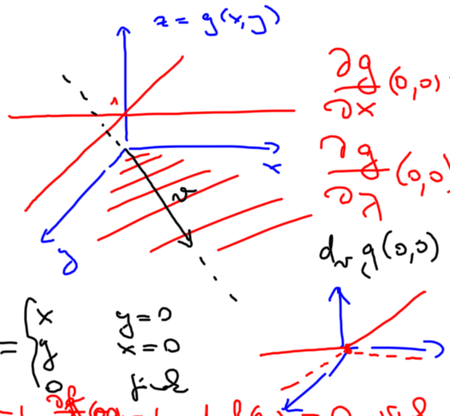
9 27-15:58



$\frac{d}{dt}\Big|_0 f(x+t\xi, y+t\eta) = d_v f(x,y)$

volba $v=(1,0) \Rightarrow \boxed{\frac{\partial f}{\partial x}(x,y) = \frac{d}{dt}\Big|_0 f(x+t,y)}$

9 27-16:10



$\frac{\partial g}{\partial x}(0,0)=0$

$\frac{\partial g}{\partial y}(0,0)=0$

$d_v g(0,0)$ neex.

$f(x,y)=\begin{cases} x & y=0\\ y & x=0\\ 0 & \text{jinak}\end{cases}$

$\frac{\partial f}{\partial x}(0,0)=1$ $\frac{\partial f}{\partial y}(0,0)=1$ $d_v f(0,0)=0$ jinak

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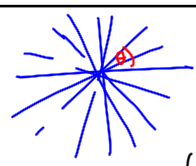
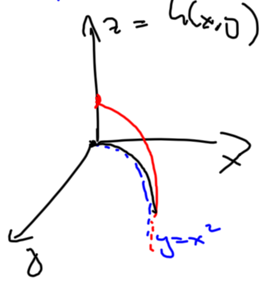


$\varphi(\theta):[0,2\pi]\to\mathbb{R}$

$x=r\cos\varphi$, $y=r\sin\varphi$ $f(r,\varphi)=r\cdot\varphi(\theta)$



$d_v h = 0$ $(v\ (0,0))$

$h(x,y)=\begin{cases}x & \text{kde } y=x^2\neq0\\ 0 & \text{jinak}\end{cases}$

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$\boxed{d_v f(x) = \xi_1\frac{\partial f}{\partial x_1}(x)+\dots+\xi_n\frac{\partial f}{\partial x_n}(x)}$

$v=(\xi_1,\dots,\xi_n)$

$\alpha=(\alpha_1,\dots,\alpha_n)$ $v=\begin{pmatrix}\xi_1\\ \vdots\\ \xi_n\end{pmatrix}$

$\alpha(v) = (\alpha_1 \quad \alpha_n)\begin{pmatrix}\xi_1\\ \\ \xi_n\end{pmatrix}$

$dx_1,\dots,dx_n$

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$f(x,y)$ f_x f_y spojité

$c(t):\mathbb{R}\to\mathbb{R}^2$ $c(t)=(x(t),y(t))$

$F(t)=f\circ c(t)$ $\frac{d}{dt}F(t) = \frac{d}{dt} f(x(t),y(t)) \overset{?}{=}$

$= f_x(x(t),y(t))\cdot\cancel{x(t)} + f_y(x(t),y(t))\cdot y'(t)$

$\cancel{\lim_{t\to0}}\ \frac{1}{t}\big(f(x(t),y(t)) - f(x_0,y_0)\big) =$

$= \boxed{\frac{1}{t}}\big(f(x(t),y(t)) - f(x_0,y(t)) + f(x_0,y(t)) - f(x_0,y_0)\big)$

[illegible] $(x(t)-x_0)\cdot\frac{\partial f}{\partial x}(x(\xi),y(t))$ $(y(t)-y_0)\frac{\partial f}{\partial y}(x_0,y(\eta))$

$= \longrightarrow \boxed{x'(t_0)\cdot f_x(x_0,y_0) + y'(t_0) f_y(x_0,y_0)}$?

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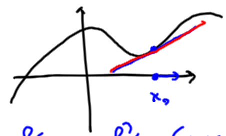
$f(x)=y$



$f(x) = \underbrace{f(x_0) + f'(x_0)\cdot(x-x_0)}_{y=}$

$f(x,y) = x^2y^3$

$f_x = 2xy^3$

$f_{xy} = 2x\,3y^2 = \underline{6xy^2}$

$f_y = 3x^2y^2$

$f_{yx} = \underline{6xy^2}$

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$\underline{f(x,y) \quad f_{xy} = f_{yx}}$

$f(t) = a_0 + a_1 t + a_2 t^2 + \dots$

$f(0) = a_0$

$f'(0) = a_1 \quad (+ a_2 2\cdot 0 + a_3 3\cdot 0^2 + \dots)$

$f''(0) = 2a_2$

$\vdots$

$f^{(k)}(0) = k!\, a_k$

řád 0 lineární přiblížení

$f(t) = f(t_0) + \boxed{f'(t_0)(t-t_0)} + \frac{1}{2!}\boxed{f''(t_0)(t-t_0)^2}$
$+ \dots + \frac{1}{k!}\boxed{f^{(k)}(t_0)(t-t_0)^k} + \dots$

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lineární forma: $\alpha: V \to K$

$\alpha(c_1 v_1 + \dots + c_k v_k) = c_1\alpha(v_1) + \dots + c_k \alpha(v_k)$

$(\alpha_1(e_1) \dots \alpha(e_n))\cdot(x_1 \dots x_n)^T$

bilineární forma: $B: V\times V \to K$

$B(v,-)$, $B(-,v)$ jsou lineární $\forall v \in V$

$v = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$, $w = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \in V$

$g(v) = B(v,v)$ KVADRATICKÉ F.

$B(v,w) = \sum_{i,j=1}^{n} a_{ij} x_i y_j = (x_1 \dots x_n)\, A \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$

$a_{ij} = B(e_i, e_j)$

$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$

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$f(x,y)$

$Hf(x,y) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$

symetrické!

$v = (\xi_1, \dots, \xi_n)$

$D^1 f(x)(v) = d_v f(x)$

$D^2 f(x)(v) = Hf(x)(v)$ $\quad \boxed{v^T \cdot Hf \cdot v}$

$= \sum_{i,j} \frac{\partial^2 f}{\partial x_i \partial x_j} \xi_i \xi_j$ $\quad D^k f(x)(v)$

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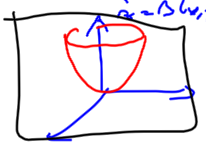
$f: \mathbb{R}^n \to \mathbb{R}$

$f(x+v) = f(x) + \cancel{D^1 f(x)}(v) + \frac{1}{2} Hf(\underline{x+\theta v})(v)$ $\quad = 0$ $\quad a = B(v,v)$

$B(x,y) = \boxed{x^2+y^2}$

$= \boxed{x^2-y^2}$

$= \boxed{-x^2-y^2}$



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