Exercise 1 Consider the following formulae.

(a) $(A \leftrightarrow B) \rightarrow (\neg A \wedge C)$

(b) $(A \rightarrow B) \rightarrow C$

(c) $A \leftrightarrow B$

(d) $(A \rightarrow B) \leftrightarrow (A \rightarrow C)$

(e) $(A \vee B) \wedge (A \vee C)$

(f) $[A \rightarrow (B \vee \neg A)] \rightarrow (B \rightarrow A)$

(g) $[(A \vee B) \rightarrow (C \rightarrow A)] \leftrightarrow (A \vee B \vee C)$

For each of them

(1) use a truth table to determine if the formula is valid and/or satisfiable;

(2) convert the formula into CNF using the truth table;

(3) convert the formula into CNF using equivalence transformations instead;

(4) write the formula as a set of clauses.

Exercise 2 Which of the following formulae imply each other?

(a) $A \wedge B$

(b) $A \vee B$

(c) $A \rightarrow B$

(d) $A \leftrightarrow B$

(e) $\neg A \wedge \neg B$

(f) $\neg A$

(g) $\neg(A \rightarrow B)$

Exercise 3 We can encode n-bit numbers via an n-tuple of propositional variables $A_{n-1}, \dots, A_0$.

(a) Write a formula $\varphi(A_1, A_0, B_1, B_0, C_2, C_1, C_0)$ for the addition of 2-bit numbers ($\bar{A} + \bar{B} = \bar{C}$).

(b) Write a formula $\varphi(A_{n-1}, \dots, A_0, B_{n-1}, \dots, B_0, C_n, \dots, C_0)$ for the addition of n-bit numbers.

Exercise 4 Use the DPLL algorithm to determine whether the following formulae are satisfiable.

(a) $\neg[(A \to B) \leftrightarrow (A \to C)]$

(b) $(A \lor B \lor C) \land (B \lor D) \land (A \to D) \land (B \to A)$

(c) $(A \leftrightarrow B) \to (\neg A \land C)$

(d) $[A \to (B \lor \neg A)] \to (B \to A)$

(e) $[(A \lor B) \to (C \to A)] \leftrightarrow (A \lor B \lor C)$

Exercise 5 Use the resolution method to determine which of the following formulae are valid.

(a) $(A \land \neg B \land C) \lor (\neg A \land \neg B \land \neg C) \lor (B \land C) \lor (A \land \neg C) \lor (\neg A \land B \land \neg C) \lor (\neg A \land \neg B \land C)$

(b) $(A \land B) \lor (B \land C \land D) \lor (\neg A \land B) \lor (\neg C \land \neg D)$

(c) $(\neg A \land B \land \neg C) \lor (\neg B \land \neg C \land D) \lor (\neg C \land \neg D) \lor (A \land B) \lor (\neg A \land B \land C) \lor (\neg A \land C)$
$\lor (A \land \neg B \land C) \lor (A \land \neg B \land D)$

Exercise 6 Given a finite automaton $\mathcal{A}$ and an input word w, write down a formula $\varphi_{\mathcal{A},w}$ that is satisfiable if, and only if, the automaton $\mathcal{A}$ accepts w.

Exercise 7 Construct the game for the following set of Horn-formulae and determine the winning regions.

$$\begin{array}{ccccc} B \land C \land D \to A & C \land F \to B & A \land F \to C & D \to B & A \land B \to F \\ E \to A & D \land E \to B & C & D & \end{array}$$