MA010 Tutorial 5

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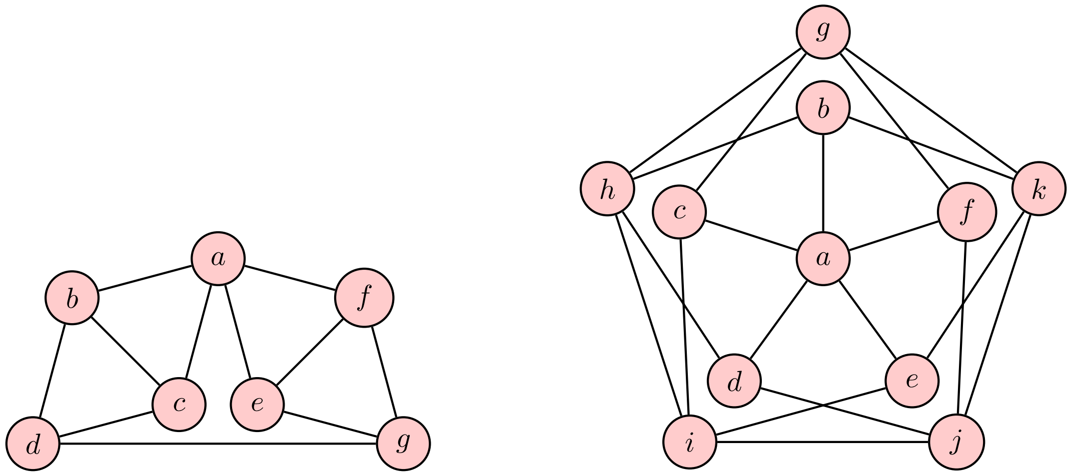
This tutorial covers material from lecture 6.

Problem 1

Prove that the following two graphs are not 3-colourable. Are they 4-colourable?



(a) (b)

Source: https://www.math.ucdavis.edu/~greg/145/hw6sol.pdf

Problem 2

Let G be a graph with chromatic number $\chi(G) \geqslant 11$ and girth $g \geqslant 11$ (the girth is the length of the shortest cycle). Prove that the number of vertices of G is bigger than $10 \cdot 9^4$. *(Hint: Remember Lemma 6.10 from class: every graph G has a subgraph of minimum degree $\geqslant \chi(G) - 1$.)*

Source: www.tau.ac.il/~nogaa/graphs134h.pdf

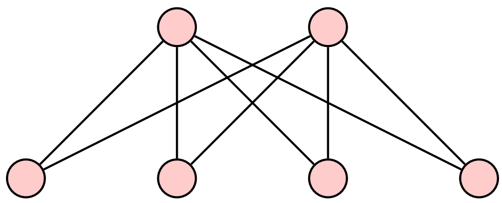
Problem 3

Let $G = (V, E_1 \cup E_2)$ be a graph, where E_1 and E_2 are nonempty matchings. Show that the chromatic number of G is 2.

Source: www.tau.ac.il/~nogaa/graphs134h.pdf

Problem 4

Show that the complete bipartite graph $K_{2,4}$ is not 2-choosable (i.e. 2-list-colourable). That is, assign a list of two colours to every vertex such that no proper colouring of the vertices by colours from their list exists.



$K_{2,4}$