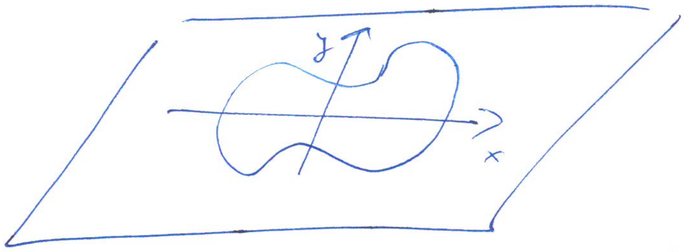
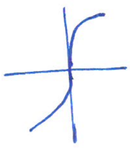
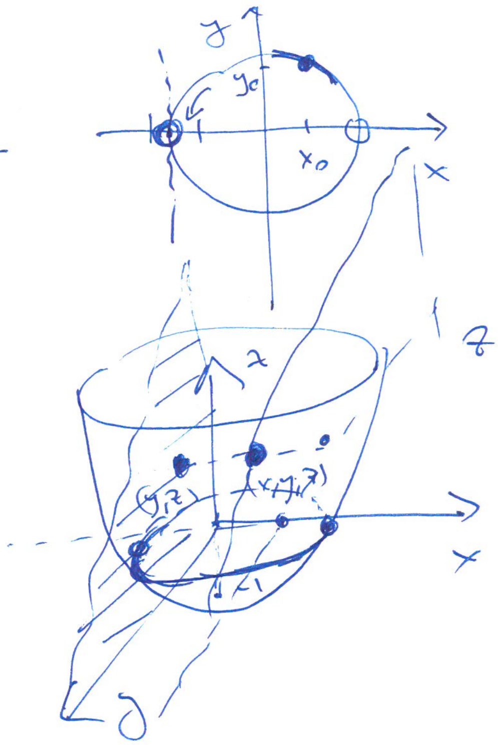


$\mathbb{R}^2$ $\quad F(x,y)=0$

příklad $x^2+y^2-1=0$ je kružnice





$$z = F(x,y) = x^2+y^2-1 = 0$$

$F_y = 2y = 0$ pro $y=0$

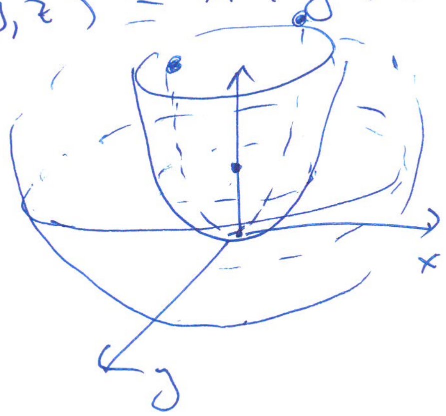
kdyby ex. $y=y(x) \Rightarrow y' = \pm\infty$

$y = y(x) \qquad F(x, y(x)) = 0$

$$0 = \frac{d}{dx} F(x,y(x)) = F_x \cdot 1 + \underset{\neq 0}{F_y} \cdot y' \Rightarrow y' = -\frac{F_x}{F_y}$$

$F_1(x,y,z) = z - x^2 - y^2$ paraboloid

$F_2(x,y,z) = x^2+y^2+(z-1)^2-4 = 0$



$x=0$: ~~$z=1 \dots$~~

$z = y^2 \quad y^2 + (y^2-1)^2 - 4 = 0$

$y^4 - y^2 - 3$

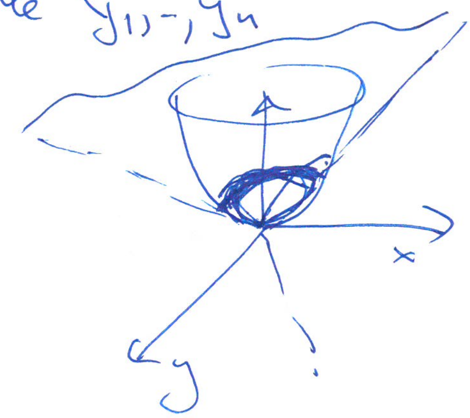
$y^2 = \frac{1 \pm \sqrt{13}}{2}$

$F(x,y) = 0 \qquad x \in \mathbb{R}^m,\ y \in \mathbb{R}^n,\ F: \mathbb{R}^{m+n} \to \mathbb{R}^n$

Hledáme $G: \mathbb{R}^m \to \mathbb{R}^n$ splňující $F(x, G(x)) = 0$

$$D^1F = \Big(\overbrace{\boxed{D^1_x F}}^{m}\ \overbrace{\boxed{D^1_y F}}^{n} \Big) \Big\}\, n$$

vždy derivace podle $x_1, \dots, x_m$ — vždy derivace podle $y_1, \dots, y_n$



$$\underbrace{\begin{matrix} f_1(x,y,z), & f_2(x,y,z) \\ \| & \| \\ z - x^2 - y^2 & z - x - y \end{matrix}}$$

$$D^1F = \left(\boxed{\begin{matrix} -2x \\ -1 \end{matrix}}\ \boxed{\begin{matrix} -2y & 1 \\ -1 & 1 \end{matrix}} \right)$$

$y = F(x) \qquad \tilde{F}(x,y) = -y + F(x) = 0$

$x = G(y)$

$$F_y^{-1} \cdot \begin{pmatrix} F_y & 0 \\ -F_x & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ F_x & F_y \end{pmatrix}^{-1}$$

$$\begin{array}{c|cc} & n & m \\ \hline n & F_y & 0 \\ m & -F_x & E \end{array} \qquad \begin{array}{c|cc} & m & n \\ \hline m & 1 & 0 \\ n & F_x & F_y \end{array}$$

$$\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} = \begin{array}{c} \\ m \\ n \end{array}\!\begin{array}{c} \begin{array}{cc} m & n \end{array} \\ \begin{pmatrix} E & 0 \\ D'_x F & D'_y F \end{pmatrix} \\ \begin{array}{cc} m & n \end{array} \end{array} \cdot \begin{array}{c} \begin{array}{cc} m & n \end{array} \\ \begin{pmatrix} A & B \\ C & D \end{pmatrix} \end{array}\!\begin{array}{c} \\ m \\ n \end{array}$$

$$A = E$$

$$B = 0$$

$$D'_x F + D'_y F \cdot C = 0$$

$$\Rightarrow \boxed{C = -(D'_y F)^{-1} \cdot D'_x F}$$

$$F(x,y) = x^2 + y^2$$

$$M_b = \{(x,y),\ x^2+y^2 = b\} \Rightarrow$$

$M_b = \emptyset$ pri $b<0$

$M_b =$ krožnice s polmerom $\sqrt{b}$ pri $b \geq 0$.