IA168 — Problem set 1

Throughout this problem set, we consider pure strategies only.

Problem 1 [2 points]

Consider the following table

	Y_1	Y_2
X_1	$1, 0$	$2, 1$
X_2	$2, 1$	$4, 0$
X_3	$3, 2$	$6, 1$

and a game defined as follows. Player 1 picks a row and Player 2 picks a column of the table independently of each other. The payoff of Player 1 (Player 2) is the first (second) value in the corresponding cell.

Give a formal description of this game as a game in *strategic form*.

Problem 2 [4 points]

Consider a game G given by the following table

	B_1	B_2	B_3	B_4
A_1	$5, 9$	$7, 7$	$1, 6$	$3, 4$
A_2	$6, 3$	$8, 5$	$3, 4$	$5, 2$
A_3	$7, 5$	$6, 4$	$6, 6$	$4, 6$
A_4	$6, 2$	$3, 1$	$7, 3$	$1, 8$

a) Find the games G^k_{DS} for $k = 0, 1, \dots$ and determine the number of *IESDS* equilibria. Is the game IESDS-solvable?

b) Find the games G^k_{Rat} for $k = 0, 1, \dots$ and determine the number of *rationalizable* equilibria. Is the game solvable by rationalizability?

Problem 3 [4 points]

Prove that rationalizability creates no new Nash equilibria in any finite two-player strategic-form game.

Definition. A strategy profile $s \in S$ **Pareto dominates** a strategy profile $s' \in S$ if $u_i(s) \geq u_i(s')$ for all $i \in N$, and $u_i(s) > u_i(s')$ for at least one $i \in N$.

A strategy profile $s \in S$ is **Pareto-optimal** if it is not Pareto dominated by any other strategy profile.

Problem 4 [4 points]

Find a game with exactly 2 Pareto-optimal strategy profiles and exactly 2 Nash equilibria such that:

a) both of the Nash equilibria are Pareto-optimal;

b) exactly one Nash equilibrium is Pareto-optimal;

c) neither of the Nash equilibria is Pareto-optimal.

Problem 5 [6 points]

Consider the following zero-sum game, defined by the payoff table for Player 1:

	A_2	B_2
A_1	1	x
B_1	0	y

where $x, y \in \mathbb{R}$ and the payoffs of Player 2 are the opposite values of those of Player 1 in the table above (e.g. $u_2(A_1, A_2) = -u_1(A_1, A_2) = -1$).

Player 1 and Player 2 will play this game infinitely many times. For $i \in \{1, 2\}$ and $j \in \mathbb{N}$, we denote by $s_{i,j} \in \{A_i, B_i\}$ the strategy chosen by Player i in the j-th iteration, and by s_j we denote the strategy profile $(s_{1,j}, s_{2,j})$.

For both $i \in \{1, 2\}$, let $s_{i,1} = A_i$ and for $j \geq 2$ we have $s_{i,j} = A_i$ iff A_i is a best response to $s_{3-i,j-1}$ (i.e. a best response to the strategy of the other player in the iteration before), $s_{i,j} = B_i$ otherwise.

In dependence on the parameters x and y, determine the sequence of strategy profiles played by Player 1 and Player 2. Explain your reasoning.