IA169 System Verification and Assurance

Symbolic Execution and Concolic Testing

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Testing Strategies

Black-box Testing

Black-box

- A product under test is viewed as a **black box**.
- It is analysed through the input-output behaviour.
- Inner details (such as source code) are hidden or not taken into account.



White-box Testing (Glass-box)

- Inner details are taken into account.
- Tests are selected and executed with respect to the inner details of the product, e.g. code coverage.
- Error insertion, modification of the product for the purpose of testing.
- Basically only extends any Black-box approach.

Gray-box Testing

- In between of Black-box and White-box.
- Sometimes the same as White-box, inconsistent terminology.

Testing Techniques

Primary Black-box Strategies

- Domain Testing
- Combinatory Testing
- Scenario Testing
- Risk-based Testing
- Functional Testing
- Fuzz Testing (Mutation Testing)

Primary White-box Extensions

- Model-based Testing
- Unit Testing

Support for Developers

• Regression Testing

Symbolic Execution

Problem

- To detect errors that systematically exhibit only for specific input values is difficult.
- Relates to incompleteness of testing.

Still we would like to ...

- test the program on inputs that make program execute differently from what has already been tested.
- test the program for all inputs.

Idea

• Execute a program so that values of input variables are referred to as to symbols instead of concrete values.

Demo

Program	Selected concrete values	Symbolic representation
read(A)		
A = A * 2	A = 3	$A = \alpha$
	A = 6	$A = \alpha * 2$
A = A + 1	A = 7	$\Lambda = (\alpha + 2) + 1$
output(A)	A = I	$A = (\alpha * 2) + 1$

• Branching in the code put some restrictions on the data depending on the condition of a branching point.

Example

- 1 if (A == 2) $A = (\alpha * 2) + 1$
- 2 then ... $(\alpha * 2) + 1 = 2$
- 3 else ... $(\alpha * 2) + 1 \neq 2$

Path Condition

- Formula over symbols referring to input values.
- Encodes history of computation, i.e. cumulative restrictions implied from all the branching points walked-through up to the curent point of execution.
- Initially set to true.

- The path condition may become unsatisfiable.
- If so, there are no input values that would make the program execute that way.

Example 1

1 i	f (A == B) $A = \alpha, E$	$\beta = \beta$	
2	then	$\alpha = \beta$	
3	if (A == B)		
4	then	$\alpha=\beta\wedge\alpha=\beta$	
5	else	$\alpha=\beta\wedge\alpha\neq\beta$	is UNSAT
6	else	$\alpha \neq \beta$	

Example 2% - operation modulo1 A=A%2 $A = \alpha\%2$ 2 if (A == 3) then ... $\alpha\%2 = 3$ is UNSAT3else ... $\alpha\%2 \neq 3$

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- All possible executions of program may be represented by a tree structure **Symbolic Execution Tree**.
- The tree is obtained by unfolding/unwinding the control flow graph of the program.

Symbolic Execution Tree

• Node of the tree encodes program location, symbolic representation of variables, and a concrete path condition.

location	symbolic valuation	path condition
#12	$A = \alpha + 2, B = \alpha + \beta - 2$	$\alpha = 2 * \beta - 1$

- An edge in the tree corresponds to a symbolic execution of a program instruction on a given location.
- Branching point is reflected as branching in the tree and causes updates of path conditions in individual branches.

Example of Symbolic Execution Tree

Program

- 1 input A,B
- 2 if (B<O) then
- 3 return 0
- 4 else
- 5 while (B > 0)
- 6 { B=B-1
- 7 A=A+B
- 8 }
- 9 return A

Draw Yourself.

Properties of Symbolic Tree Execution

- No nodes are merged, even if they are the same (the structure is a tree).
- A single program location may be contained in (infinitely) many nodes of the tree.
- Tree may contain infinite paths.

Path Explosion Problem

- The number of branches in the symbolic execution tree may be large for non-trivial programs.
- The number of paths may grow exponentially with the number of branching points visited.

Employing Symbolic Execution Tree for Verification

Analysis of the Tree

• Breadth-first strategy, the tree may be infinite.

Deduced Program Properties

- Identification of feasible and unfeasible paths.
- Proof of reachability of a given program location.
- Error detection (division by zero, out-of-array access, assertion violation, etc.).

Synthesis of Test Input Data

- If the formula encoded as a path condition is satisfiable for a symbolic run, the model of the formula gives concrete input values that make the program to follow the symbolic run.
- Excellent for synthesis of tests that increase code coverage.

Automated Test Generation

Principle

- 1 Generate random input values (encode some random path).
- 2 Perform a walk through the Symbolic Execution Tree with the random input values and record the path condition.
- 3 Generate a new path condition from the recorded one by negating one of the restrictions related to a single branching point.
- 4 Find input values satisfying the new path condition.
- 5 Repeat from number 2 until desired coverage is reached.

Practical Notes

- Heuristics for selection of branching point to be negated.
- Augmentation of the code to enable path condition recording.

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Undecidability

- Using complex arithmetic operations on unbounded domains implies general undecidability of the formula satisfaction problem.
- Symbolic Execution Tree is infinite (due to unwinding of cycles with unbound number of iterations).

Computational Complexity

- Path explosion problem.
- Efficiency of algorithms for formula satisfiability on finite domains.

Known Limits

- Symbolic operations on non-numerical variables.
- Not clear how to deal with dynamic data structures.
- Symbolic evaluation of calls to external functions.

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Tools for SAT Solving

Satisfiability Problem – SAT

• Is to decide if there exists a valuation of Boolean variables of propositional logic formula that makes the formula hold true (be valid).

SAT Problem Properties

- Famous NP-complete problem.
- Polynomial algorithm is unlikely to exist.
- Still there are existing SAT solvers that are very efficient and due to a plethora of heuristics can solve surprisingly large instances of the problem.

Tool Z3

ZZZ aka Z3

- Developed by Microsoft Research.
- SAT and SMT Solver.
- WWW interface http://www.rise4fun.com/Z3
- Standardised binary API for use within other verification tools.

Decide using Z3

• Is formula $(a \lor \neg b) \land (\neg a \lor b)$ satisfiable?

Usage of Z3 – SAT

Reformulate into language of Z3 $(a \lor \neg b) \land (\neg a \lor b)$

```
    (declare-const a Bool)

            (declare-const b Bool)
            (assert (and (or a (not b)) (or (not a) b)))
            (check-sat)
            (get-model)
```

Answer of Z3

```
● sat
(model
  (define-fun b () Bool
    false)
   (define-fun a () Bool
    false)
  )
```

Satisfiability Modulo Theory – SMT

- Is to decide satisfiability of first order logic with predicates and function symbols that encode one or more selected theories.
- Typically used theories
 - Arithmetic of integer and floating point numbers.
 - Theories of data structures (lists, arrays, bit-vectors, ...).

Other view (Wikipedia)

• SMT can be thought of as a form of the constraint satisfaction problem and thus a certain formalised approach to constraint programming.

Examples of SMT in Z3

Solve using Z3

http://rise4fun.com/Z3/tutorial/guide

• Are there two integer non-zero numbers x and y such that y=x*(x-y)?

```
(declare-const y Int)
(declare-const x Int)
(assert (= y (* x (- x y))))
(assert (not (= y 0)))
(check-sat)
(get-model)
```

• Are there two integer non-zero numbers x and y such that y=x*(x-(y*y))?

```
(declare-const y Int)
(declare-const x Int)
(assert (= y (* x (- x (* y y)))))
(assert (not (= x 0)))
(check-sat)
```

• A formula is valid if and only if its negation is not satisfiable.

Consequence

• SAT and SMT solvers can be used as theorem provers to show validity of some theorems.

Model Synthesis

- SAT solvers not only decide satisfiability of formulae but in positive case also give concrete valuation of variables for which the formula is valid.
- Unlike general theorem provers they provide a counterexample in case the theorem to be proved is invalid (negation is satisfiable).

Concolic Testing

Motivation

Problem

- Efficient undecidability of path feasibility.
- In practice, unknown result often means unsatisfiability (no witness found).
- However, skipping paths that we only think are unfeasible, may result in undetected errors.
- On the other hand, executing unfeasible path may report unreal errors.

Partial Solution

- Let us use concrete and symbolic values at the same time in order to support decisions that are practically undecidable by a SAT or SMT solver.
- Heuristics.
- An interesting case (correct): UNKNOWN \implies SAT
- Concrete and Symbolic Testing = Concolic Testing

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Hypothetical demo of concolic testing

Program

- 1 input A,B
- 2 if (A==(B*B)%30) then
- 3 ERROR
- 4 else
- 5 return A

Concolic Testing

- 1 A=22, B=7 (random values), test executed, no errors found.
- 2 (22==(7*7)%30) is *False*, path condition: $\alpha \neq (\beta * \beta)%30$
- 3 Synthesis of input data from negation of path condition: $\alpha = (\beta * \beta)\%30 - \text{UNKNOWN}$
- 4 Employ concrete values: $\alpha = (7*7)\%30$ SAT, $\alpha = 19$
- 5 A=19, B=7
- 6 Test detected error location on program line 3.



SAGE Tool

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Systematic Testing for Security: Whitebox Fuzzing

Patrice Godefroid Michael Y. Levin and David Molnar

http://research.microsoft.com/projects/atg/ Microsoft Research



```
Example: Dynamic Test Generation
 void top(char input[4])
 {
                                       input = "good"
    int cnt = 0;
    if (input[0] == 'b') cnt++;
    if (input[1] == 'a') cnt++;
    if (input[2] == 'd') cnt++;
    if (input[3] == '!') cnt++;
    if (cnt > 3) crash();
 }
```

```
Dynamic Test Generation
void top(char input[4])
{
                                      input = "good"
   int cnt = 0;
   if (input[0] == 'b') cnt++;
                                    Path constraint:
   if (input[1] == 'a') cnt++;
   if (input[2] == 'd') cnt++;
                                  I_0 != b'
   if (input[3] == '!') cnt++;
                                    I_1 != a'
   if (cnt > 3) crash();
                                       I_2 != 'd'
}
                                       I_3 != !!'
              Negate a condition in path constraint
              Solve new constraint \rightarrow new input
```
































- Since 1st internal release in April'07: tens of new security bugs found
- Apps: image processors, media players, file decoders,... Confidential !
- Bugs: Write A/Vs, Read A/Vs, Crashes,... Confidential !
- Many bugs found triaged as "security critical, severity 1, priority 1"

Homework

- Follow Klee tutorials 1 and 2 (http://klee.github.io/tutorials)
- Solve The Wolf, Goat and Cabbage problem with Klee