Week assignment 06 (Homework vault part)

Deadline: 16. 11. 2020

Upload scan, photograph, or a typesetted pdf of computation to the homework vault *Assignment week 06*. Do not forget to include your name and personal ID number (učo) on the page(s).

Exercise 1. [1pt] Compute $A\,|w\rangle$ where $A = |v\rangle\langle w|$ and

$$|w\rangle = \begin{pmatrix} 3 \\ 2i \\ 1-i \\ 0 \\ 4 \\ i \\ 2+2i \\ 7 \end{pmatrix}, \qquad |v\rangle = \begin{pmatrix} -1 \\ 2 \\ 3i \\ 1-i \\ 2 \\ 0 \\ -3+i \\ 2i \end{pmatrix}.$$

Use the Dirac notation.

Exercise 2. [2pts] Let us have three orthonormal states

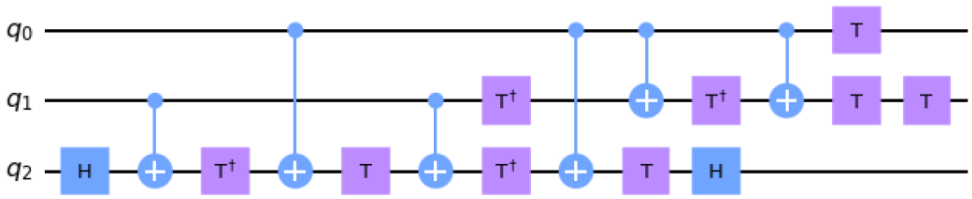
$$|v_1\rangle = \frac{1}{2\sqrt{6}} \begin{pmatrix} 3 \\ -1 \\ -3+i \\ 2i \end{pmatrix}, \qquad |v_2\rangle = \frac{1}{3\sqrt{2}} \begin{pmatrix} 2 \\ 2i \\ 0 \\ -1-3i \end{pmatrix}, \qquad |v_3\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} i \\ 1 \\ i \\ 0 \end{pmatrix}.$$

Compute $\mathrm{Tr}\big(|u\rangle\langle w|\big)$ where

$$|u\rangle = |v_1\rangle + |v_2\rangle \qquad \text{and} \qquad |w\rangle = 2\,|v_1\rangle + |v_2\rangle - |v_3\rangle\,.$$

Use the Dirac notation.

Exercise 3. [3pts] Consider circuit



Show that this circuit performs CCNOT, which is the controlled-controlled not operation with qubits 0 and 1 as controls, that flips the qubit 2 if and only if both control qubits are in state $|1\rangle$. Here

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}.$$

Use the Dirac notation.
Hint: You can show this separately for the four computational basis states of control qubits.