Assignment week 04 (Homework vault part)

Deadline: 9. 11. 2020

Upload scan, photograph, or a typesetted pdf of computation to the homework vault *Assignment week 02.* Do not forget to include your name and personal ID number (učo) on the page(s).

Exercise 1. [2 pts] Let $\{|0\rangle, |1\rangle, |2\rangle\}$ be an orthonormal basis, and

$$|v_1\rangle = |0\rangle + i\,|1\rangle - 2\,|2\rangle\,, \qquad |v_2\rangle = 2\,|0\rangle - 2\,|1\rangle + (1+i)\,|2\rangle\,,$$
$$|v_3\rangle = i\,|0\rangle + |1\rangle\,, \qquad |v_4\rangle = 13\,|0\rangle + (12-5i)\,|1\rangle + (4-6i)\,|2\rangle\,.$$

Out of these four vectors select three such that they form an orthogonal basis and then normalize them to form an orthonormal basis. Express the remaining (unnormalized) vector in this basis. You can do it using either matrix formalism, or bra-ket notation, but try doing them both to get to know them better.

Exercise 2. [2 pts] Let us have state (in the canonical basis)

$$|v\rangle = \frac{1}{5}\begin{pmatrix} 3i \\ 1 \\ 1-i \\ 2+3i \end{pmatrix}.$$

What are the outcome probabilities if we perform measurement in

a) canonical basis $\{|j\rangle\}_{j=1}^4$,

b) basis $\{|u_j\rangle\}_{j=1}^4$, where $|u_1\rangle = |e_1\rangle$, $|u_2\rangle = \frac{1}{\sqrt{2}}(|e_2\rangle + i\,|e_3\rangle)$, $|u_3\rangle = \frac{1}{\sqrt{2}}(|e_2\rangle - i\,|e_3\rangle)$, and $|u_4\rangle = -i\,|e_4\rangle$,

c) basis $\{|w_j\rangle\}_{j=1}^4$, where

$$|w_1\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \qquad |w_2\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \qquad |w_3\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \qquad |w_4\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}.$$