IA168 — Problem set 2

Except for Problem 3, we consider only **pure** strategies.

Problem 1 [3 points]

Give a formal description of rock-paper-scissors as a two-player **imperfect**-information extensive-form game (i.e., according to the definition from the lectures).

Problem 2 [6 points]

Bob has been hired by Mafia to steal some valuable top-secret documents. Now that Bob has done so, he has two options: Either he delivers the documents to Mafia, or he flees to Europe and sells them for twice as much. If Bob flees, Mafia either decide to leave this business and concentrate on other deals, or they start searching for Bob. In the latter case, with probability p, they eventually find Bob and kill him.

We model this scenario as the two-player perfect-information extensive-form game depicted below.



In dependence on the parameter $p, 0 \le p \le 1$, enumerate all:

- a) never-best-response strategies;
- b) maxmin strategies;
- c) Nash equilibria;
- d) subgame-perfect equilibria.

Problem 3 [2 points]

Consider the **one-player** perfect-information extensive-form game depicted below.



In this game, consider a mixed strategy σ given as follows:

$$\sigma(ACEHJLM) = \frac{4}{15}$$

$$\sigma(ACEIKLN) = \frac{2}{5}$$

$$\sigma(BCFHKLM) = \frac{1}{8}$$

$$\sigma(BDFHJLN) = \frac{1}{24}$$

$$\sigma(BDGHJLN) = \frac{1}{6}$$

Find a behavioral strategy β which is equivalent to σ . Is there a unique solution to this task? Justify your answer.

Problem 4 [6 points]

Find a two-player perfect-information extensive-form game where all of the following conditions are satisfied:

- there is a strategy profile whose outcome is for both players better than that of any Nash equilibrium;
- there is a Nash equilibrium whose outcome is for both players better than that of any subgame-perfect equilibrium;
- there are exactly two subgame-perfect equilibria s, s', and the outcome of s is for both players better than that of s'.

Should you fail to find such a game, try your best (for partial points) to find a game which matches the requirements as closely as you can.

Problem 5 [3 points]

Prove or disprove: In every **zero-sum** two-player perfect-information extensive-form game G, all subgameperfect equilibria have the same outcome for player 1.