

Dvojný integrál

$$1. \iint_M (x^2 + y^2) dx dy \quad M : x^2 + y^2 \leq 1, y \geq |x| \quad \left[\frac{\pi}{8} \right]$$

$$2. \iint_M \sqrt{x^2 + y^2} dx dy \quad M : x^2 + y^2 - 2y \leq 0 \quad \left[\frac{32}{9} \right]$$

$$3. \iint_M (x^2 + y^2) dx dy \quad M : x^2 - 8x + y^2 \leq 0 \quad [192\pi]$$

$$4. \iint_M (y - x) dx dy \quad M : x \leq y \leq x + 1, 2x - 2 \leq y \leq 2x \quad [1]$$

$$5. \iint_M x^2 dx dy \quad M \text{ je ohraničená křivkami : } y = x, y = \frac{1}{x}, y = 2x, y = \frac{4}{x} \quad \left[\frac{15}{2} \ln 2 \right]$$

$$6. \iint_M e^{-x^2 - y^2} dx dy \quad M : x^2 + y^2 \leq 1, x \geq 0, \quad \left[\frac{\pi}{2} \left(1 - \frac{1}{e} \right) \right]$$

$$7. \iint_M 2xy dx dy \quad M : 0 \leq y \leq x, x^2 + y^2 \leq 9, \quad \left[\frac{81}{8} \right]$$

8. Uprav do tvaru jednoduchého integrálu:

$$\iint_M \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}} dx dy \quad M : x^2 + y^2 \leq 1, x \geq 0, y \geq 0 \quad \left[\frac{\pi}{2} \int_0^1 \rho \sqrt{\frac{1-\rho^2}{1+\rho^2}} d\rho \right]$$

$$9. \iint_M \arctg\left(\frac{y}{x}\right) dx dy \quad M : 1 \leq x^2 + y^2 \leq 3, \frac{x}{\sqrt{3}} \leq y \leq \sqrt{3} \cdot x \quad \left[-\frac{\pi^2}{24} \right]$$

Trojný integrál

$$1. \iiint_M x dx dy dz \quad M : x^2 + y^2 \leq 4, z \geq 0, z + y \leq 2 \quad [0]$$

$$2. \iiint_M x^2 y dx dy dz \quad M : 1 \leq x^2 + y^2 \leq 4, y \geq 0, |z| \leq 2 \quad \left[\frac{248}{15} \right]$$

$$3. \iiint_M (x^2 + y^2) dx dy dz \quad M : A^2 \leq x^2 + y^2 + z^2 \leq B^2, z \leq 0, A < B \quad [0]$$

$$4. \iiint_M x^2 y dx dy dz \quad M : 1 \leq x^2 + y^2 \leq 4, 0 \leq z \leq 3 - y \quad \left[\frac{21}{8} \pi \right]$$

$$5. \iiint_M 24z dx dy dz \quad M : x^2 + y^2 + z^2 \leq 2z, z \leq x^2 + y^2 \quad [2\pi]$$

Nevlastní integrál

$$1. \iiint_M x y e^{-x^2 - y^2} dx dy \quad M : [0, \infty] \times [0, \infty] \quad \left[\frac{1}{4} \right]$$

$$2. \iiint_M e^{-x^2 - y^2} dx dy \quad M : 0 \leq x \leq y \quad \left[\frac{1}{2} \right]$$

$$3. \iint_M \frac{1}{\sqrt{x^2+y^2}} \, dx \, dy \quad M: 0 < x^2 + y^2 \leq 1 \quad [2\pi]$$

$$4. \iint_M e^{\frac{x}{y}} \, dx \, dy \quad M: 0 < y \leq 1, 0 \leq x, \sqrt{x} \leq y \quad \left[\frac{1}{2}\right]$$