

# IA168 — Problem set 2

Except for Problem 3, we consider only **pure** strategies.

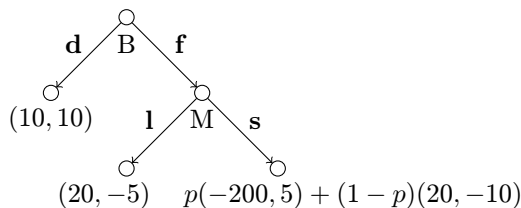
## Problem 1 [3 points]

Give a formal description of rock-paper-scissors as a two-player **imperfect**-information extensive-form game (i.e., according to the definition from the lectures).

## Problem 2 [6 points]

Bob has been hired by Mafia to steal some valuable top-secret documents. Now that Bob has done so, he has two options: Either he **d**elivers the documents to Mafia, or he **f**lees to Europe and sells them for twice as much. If Bob flees, Mafia either decide to leave this business and concentrate on other deals, or they start **s**earching for Bob. In the latter case, with probability  $p$ , they eventually find Bob and kill him.

We model this scenario as the two-player perfect-information extensive-form game depicted below.

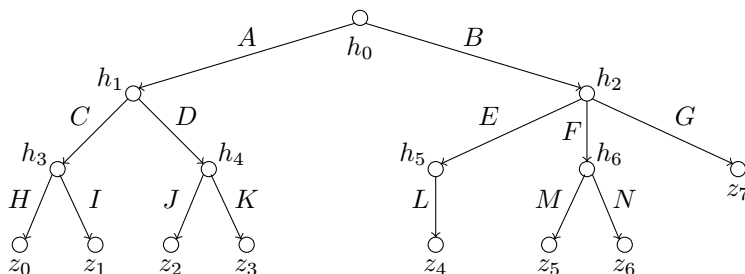


In dependence on the parameter  $p$ ,  $0 \leq p \leq 1$ , enumerate all:

- a) never-best-response strategies;
- b) maxmin strategies;
- c) Nash equilibria;
- d) subgame-perfect equilibria.

## Problem 3 [2 points]

Consider the **one-player** perfect-information extensive-form game depicted below.



In this game, consider a mixed strategy  $\sigma$  given as follows:

$$\begin{aligned}\sigma(ACEHJLM) &= \frac{1}{6} \\ \sigma(ACEIKLN) &= \frac{1}{2} \\ \sigma(BCFHKLM) &= \frac{1}{7} \\ \sigma(BDFHJLN) &= \frac{1}{21} \\ \sigma(BDGHJLN) &= \frac{1}{7}\end{aligned}$$

Find a behavioral strategy  $\beta$  which is equivalent to  $\sigma$ . Is there a unique solution to this task? Justify your answer.

#### Problem 4 [6 points]

Find a two-player perfect-information extensive-form game where all of the following conditions are satisfied:

- there is a strategy profile whose outcome is for both players better than that of any Nash equilibrium;
- there is a Nash equilibrium whose outcome is for both players better than that of any subgame-perfect equilibrium;
- there are exactly two subgame-perfect equilibria  $s, s'$ , and the outcome of  $s$  is for both players better than that of  $s'$ .

Should you fail to find such a game, try your best (for partial points) to find a game which matches the requirements as closely as you can.

#### Problem 5 [3 points]

Prove or disprove: In every **zero-sum** two-player perfect-information extensive-form game  $G$ , all subgame-perfect equilibria have the same outcome for player 1.