Basics in Language and Probability

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Quotes

It must be recognized that the notion "probability of a sentence" is an entirely useless one, under any known interpretation of this term.
Noam Chomsky, 1969

Whenever I fire a linguist our system performance improves.
Frederick Jelinek, 1988

Conflicts?

rationalist	vs.	empiricist
scientist	vs.	engineer
insight	vs.	data analysis
explaining language	vs.	building applications

language

A Naive View of Language

- Language needs to name
  - nouns: objects in the world (dog)
  - verbs: actions (jump)
  - adjectives and adverbs: properties of objects and actions (brown, quickly)
- Relationship between these have to specified
  - word order
  - morphology
  - function words

A Bag of Words

quick

fox

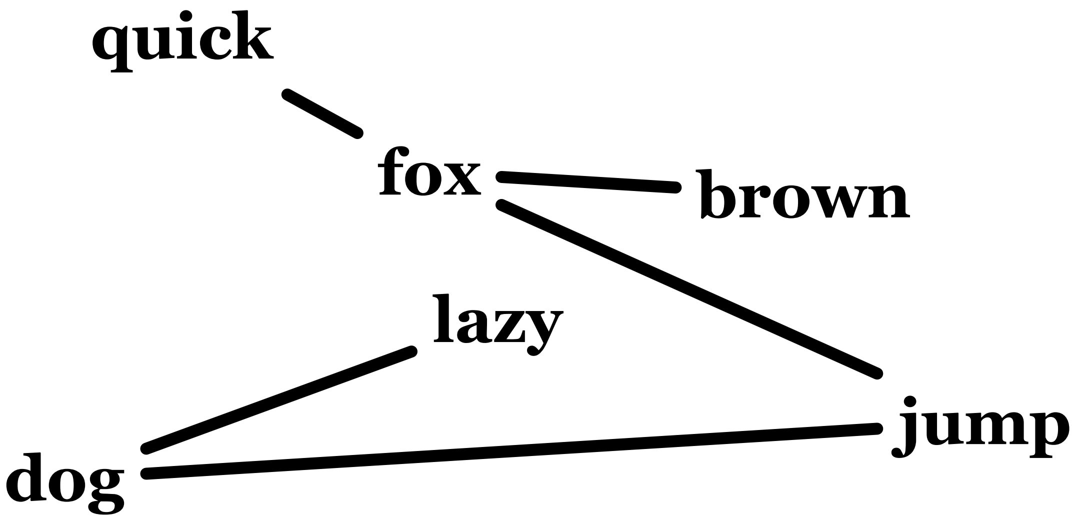
brown

lazy

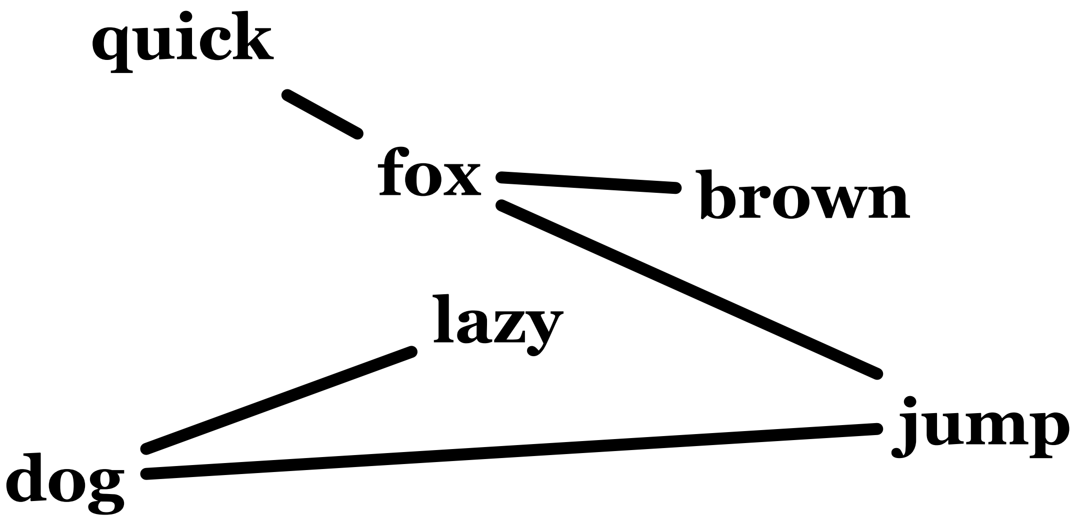
jump

dog

Relationships

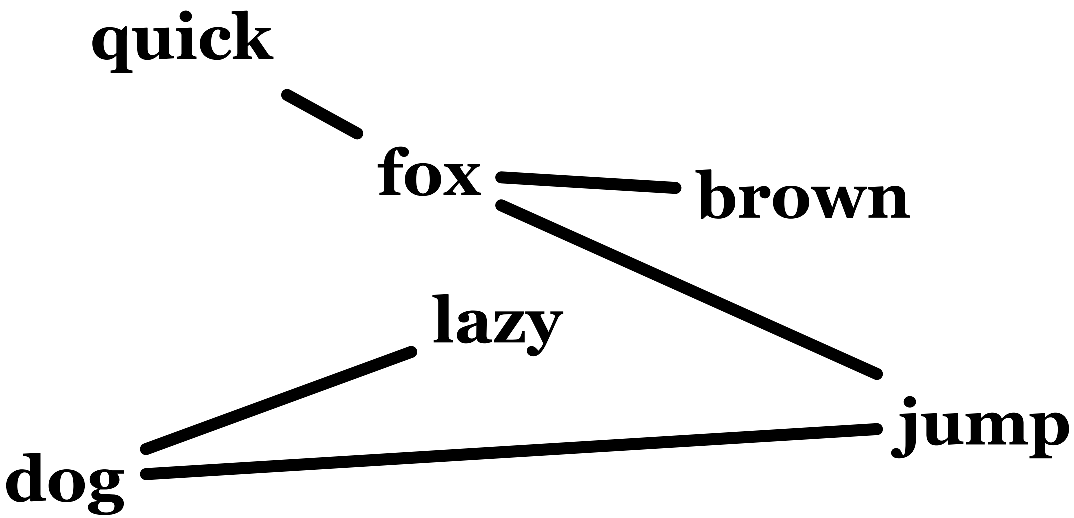


Marking of Relationships: Word Order



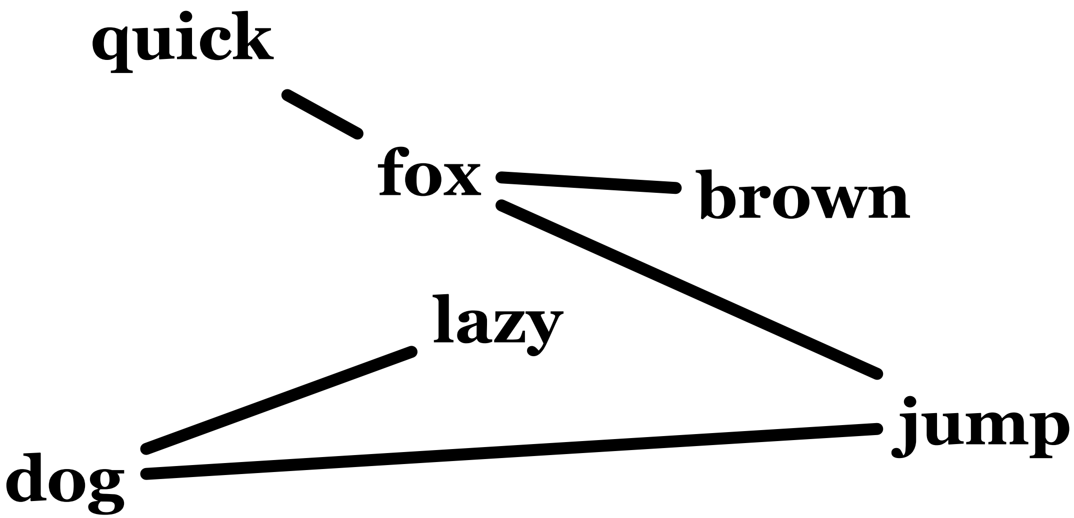
quick brown fox jump lazy dog

Marking of Relationships: Function Words



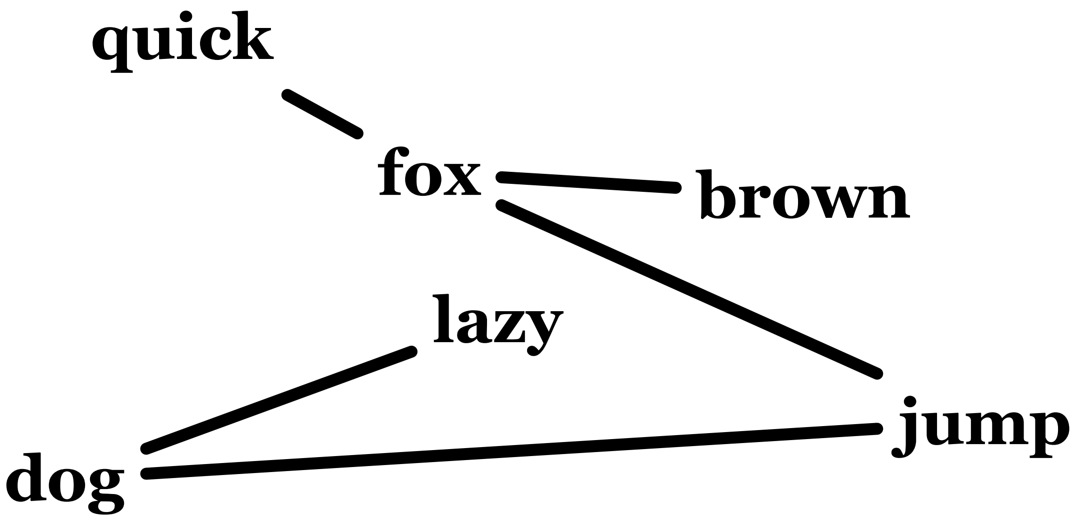
quick brown fox jump over lazy dog

Marking of Relationships: Morphology



quick brown fox jumps over lazy dog

Some Nuance



the quick brown fox jumps over the lazy dog

Marking of Relationships: Agreement

- From Catullus, First Book, first verse (Latin):

Cui	dono	lepidum	novum	libellum	arida	modo	pumice	expolitum	?
Whom	I-present	lovely	new	little-book	dry	manner	pumice	polished	?

(To whom do I present this lovely new little book now polished with a dry pumice?)

- Gender (and case) agreement links adjectives to nouns

Marking of Relationships to Verb: Case

- German:

Die Frau	gibt	dem Mann	den Apfel
The woman	gives	the man	the apple
subject		indirect object	object

Der Frau	gibt	der Mann	den Apfel
The woman	gives	the man	the apple
indirect object		subject	object

- Case inflection indicates role of noun phrases

Case Morphology vs. Prepositions

- Two different word orderings for English:
  - The woman gives the man the apple
  - The woman gives the apple **to** the man

- Japanese:

女性は	男性に	アップルの	を与えます
woman SUBJ	man OBJ	apple OBJ2	gives

- Is there a real difference between prepositions and noun phrase case inflection?

Words

This is a simple sentence **WORDS**

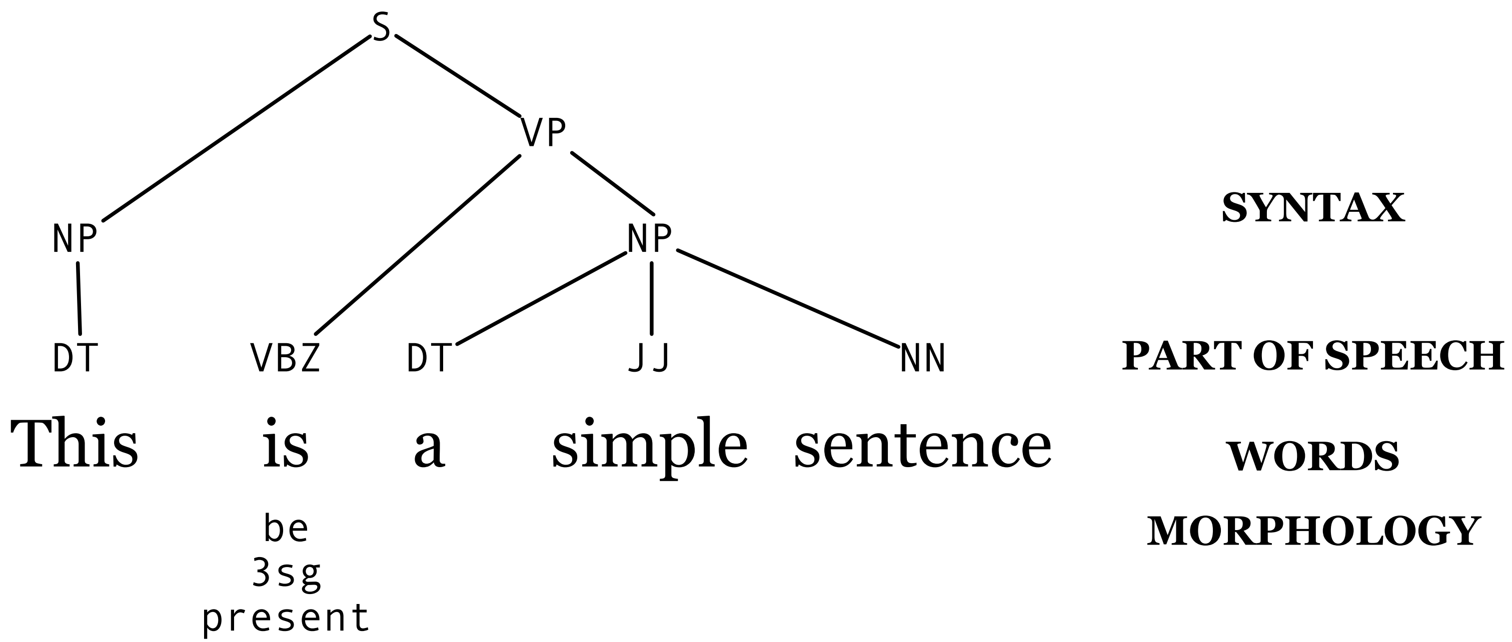
Morphology

This	is	a	simple	sentence	**WORDS**
	be 3sg present				**MORPHOLOGY**

Parts of Speech

DT	VBZ	DT	JJ	NN	**PART OF SPEECH**
This	is	a	simple	sentence	**WORDS**
	be 3sg present				**MORPHOLOGY**

Syntax


S
VP
NP
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DT
VBZ
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JJ
NN
This
is
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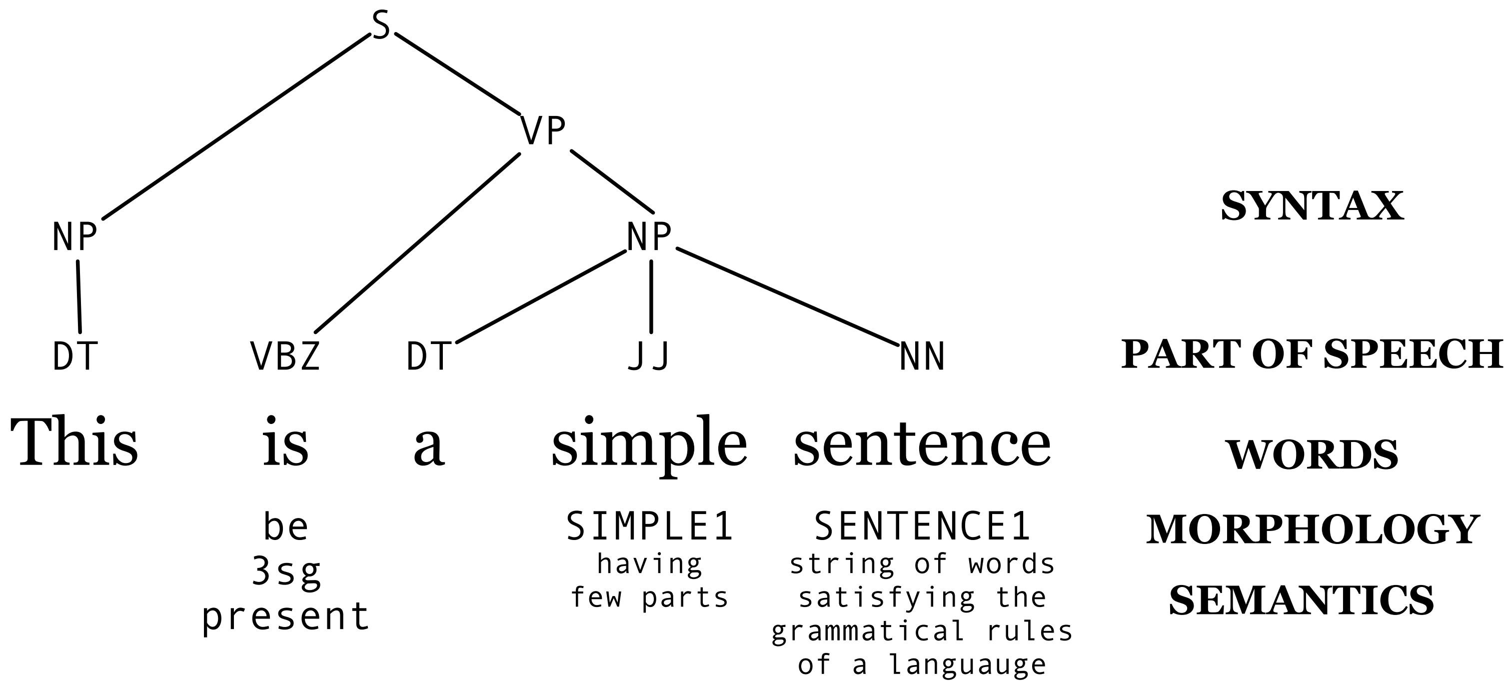
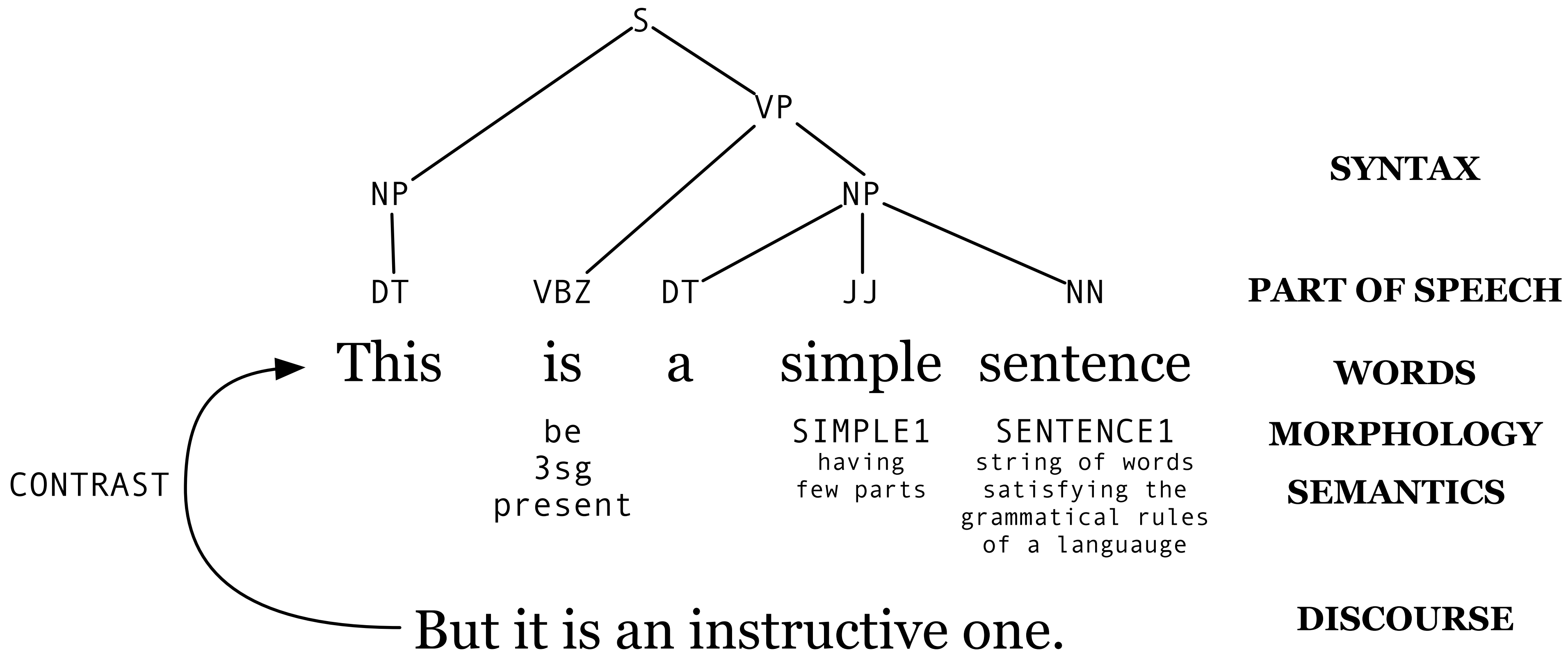
SYNTAX
PART OF SPEECH
WORDS
MORPHOLOGY

Semantics


S
VP
NP
NP
DT
VBZ
DT
JJ
NN
This
is
a
simple
sentence
be
3sg
present
SIMPLE1
having
few parts
SENTENCE1
string of words
satisfying the
grammatical rules
of a languauge
SYNTAX
PART OF SPEECH
WORDS
MORPHOLOGY
SEMANTICS

Discourse


S
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MORPHOLOGY
SEMANTICS
CONTRAST
But it is an instructive one.
DISCOURSE

Why is Language Hard?

- Ambiguities on many levels
- Rules, but many exceptions
- No clear understand how humans process language

- Can we learn everything about language by automatic data analysis?

data

Data: Words

- Definition: strings of letters separated by spaces
- But how about:
  - punctuation: commas, periods, etc. typically separated (tokenization)
  - hyphens: high-risk
  - clitics: Joe's
  - compounds: website, Computerlinguistikvorlesung
- And what if there are no spaces:
  伦敦每日快报指出,两台记载黛安娜王妃一九九七年巴黎死亡车祸调查资料的手提电脑,被从前大都会警察总长的办公室里偷走.

Word Counts

Most frequent words in the English Europarl corpus

any word

Frequency in text	Token
1,929,379	the
1,297,736	,
956,902	.
901,174	of
841,661	to
684,869	and
582,592	in
452,491	that
424,895	is
424,552	a

nouns

Frequency in text	Content word
129,851	European
110,072	Mr
98,073	commission
71,111	president
67,518	parliament
64,620	union
58,506	report
57,490	council
54,079	states
49,965	member

Word Counts

But also:

There is a large tail of words that occur only once.

33,447 words occur once, for instance

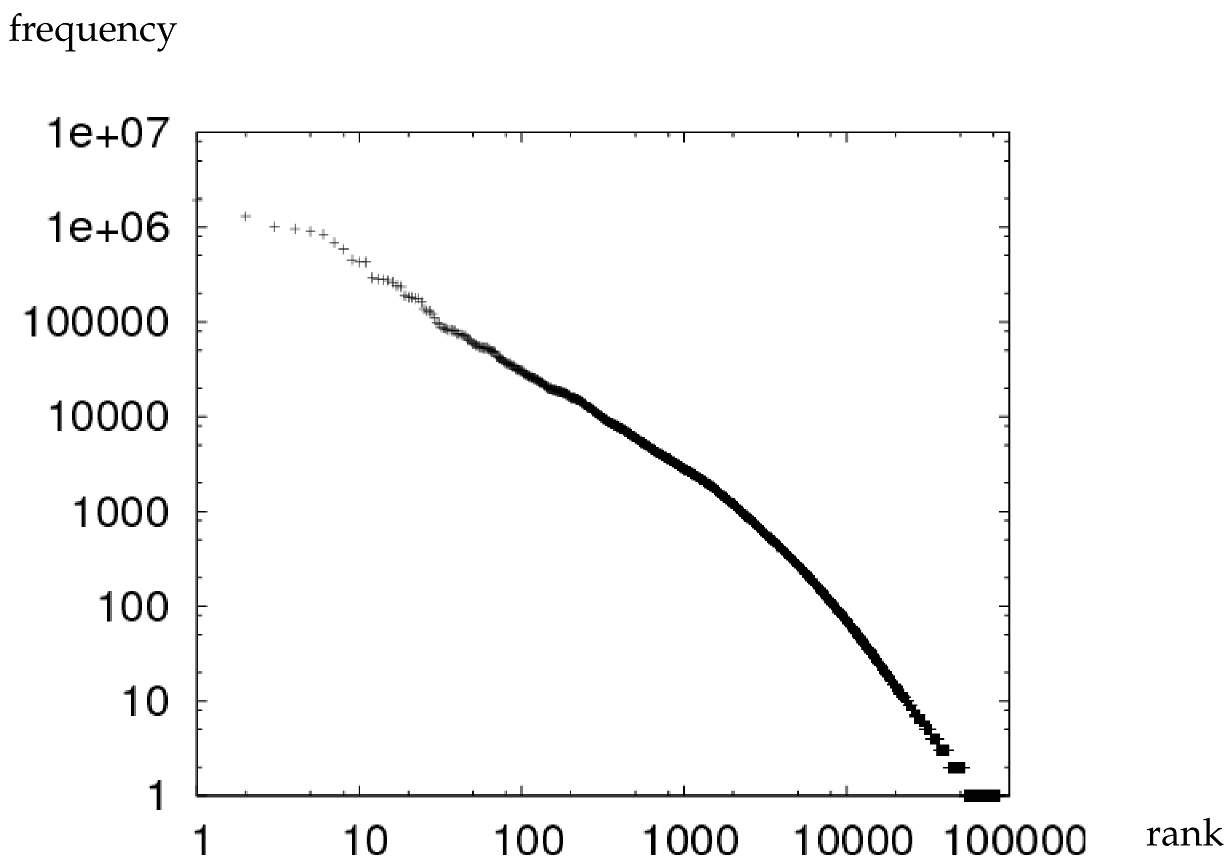
- cornflakes
- mathematicians
- Tazhikhistan

Zipf's law

$$f \times r = k$$

f = frequency of a word
r = rank of a word (if sorted by frequency)
k = a constant

Zipf's law as a graph



Why a line in log-scale?

$$fr = k \;\Rightarrow\; f = \frac{k}{r} \;\Rightarrow\; \log\, f = \log\, k - \log\, r$$

statistics

Probabilities

- Given word counts we can estimate a probability distribution:

  $P(w) = \frac{count(w)}{\sum_{w'} count(w')}$

- This type of estimation is called *maximum likelihood estimation*. Why? We will get to that later.

- Estimating probabilities based on frequencies is called the *frequentist approach* to probability.

- This probability distribution answers the question: If we randomly pick a word out of a text, how likely will it be word w?

A Bit More Formal

- We introduce a **random variable** W.
- We define a **probability distribution** p, that tells us how likely the variable W is the word w:

  $prob(W = w) = p(w)$

Joint Probabilities

- Sometimes, we want to deal with two random variables at the same time.
- Example: Words w_1 and w_2 that occur in sequence (a **bigram**)
  We model this with the distribution: $p(w_1, w_2)$
- If the occurrence of words in bigrams is **independent**, we can reduce this to $p(w_1, w_2) = p(w_1)p(w_2)$. Intuitively, this not the case for word bigrams.
- We can estimate **joint probabilities** over two variables the same way we estimated the probability distribution over a single variable:

  $p(w_1, w_2) = \frac{count(w_1, w_2)}{\sum_{w_{1'}, w_{2'}} count(w_{1'}, w_{2'})}$

Conditional Probabilities

- Another useful concept is **conditional probability**

  $p(w_2|w_1)$

  It answers the question: If the random variable $W_1 = w_1$, how what is the value for the second random variable W_2?

- Mathematically, we can define conditional probability as

  $p(w_2|w_1) = \frac{p(w_1,w_2)}{p(w_1)}$

- If W_1 and W_2 are independent: $p(w_2|w_1) = p(w_2)$

Chain Rule

- A bit of math gives us the chain rule:

  $p(w_2|w_1) = \frac{p(w_1,w_2)}{p(w_1)}$

  $p(w_1)\ p(w_2|w_1) = p(w_1, w_2)$

- What if we want to break down large joint probabilities like $p(w_1, w_2, w_3)$?

  We can repeatedly apply the chain rule:

  $p(w_1, w_2, w_3) = p(w_1)\ p(w_2|w_1)\ p(w_3|w_1, w_2)$

Bayes Rule

- Finally, another important rule: **Bayes rule**

  $p(x|y) = \frac{p(y|x)\ p(x)}{p(y)}$

- It can easily derived from the chain rule:

  $p(x, y) = p(x, y)$

  $p(x|y)\ p(y) = p(y|x)\ p(x)$

  $p(x|y) = \frac{p(y|x)\ p(x)}{p(y)}$

Expectation

- We introduced the concept of a random variable X

  $prob(X = x) = p(x)$

- Example: Roll of a dice. There is a $\frac{1}{6}$ chance that it will be 1, 2, 3, 4, 5, or 6.

- We define the **expectation** $E(X)$ of a random variable as:

  $E(X) = \sum_x p(x)\ x$

- Roll of a dice:

  $E(X) = \frac{1}{6} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 + \frac{1}{6} \times 4 + \frac{1}{6} \times 5 + \frac{1}{6} \times 6 = 3.5$

Variance

- **Variance** is defined as

  $Var(X) = E((X - E(X))^2) = E(X^2) - E^2(X)$
  $Var(X) = \sum_x p(x)\ (x - E(X))^2$

- Intuitively, this is a measure how far events diverge from the mean (expectation)

- Related to this is **standard deviation**, denoted as σ.

  $Var(X) = \sigma^2$
  $E(X) = \mu$

Variance

- Roll of a dice:

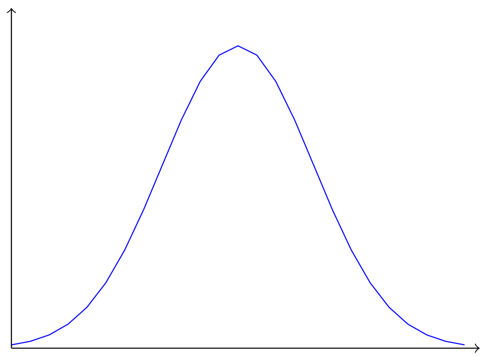
$$
\begin{aligned}
Var(X) &= \frac{1}{6}(1-3.5)^2 + \frac{1}{6}(2-3.5)^2 + \frac{1}{6}(3-3.5)^2 \\
&\quad + \frac{1}{6}(4-3.5)^2 + \frac{1}{6}(5-3.5)^2 + \frac{1}{6}(6-3.5)^2 \\
&= \frac{1}{6}((-2.5)^2 + (-1.5)^2 + (-0.5)^2 + 0.5^2 + 1.5^2 + 2.5^2) \\
&= \frac{1}{6}(6.25 + 2.25 + 0.25 + 0.25 + 2.25 + 6.25) \\
&= 2.917
\end{aligned}
$$

Standard Distributions

- **Uniform**: all events equally likely
  - $\forall x, y : p(x) = p(y)$
  - example: roll of one dice
- **Binomial**: a serious of trials with only only two outcomes
  - probability p for each trial, occurrence r out of n times:
    $b(r; n, p) = \binom{n}{r} p^r (1 - p)^{n-r}$
  - a number of coin tosses

Standard Distributions

- **Normal**: common distribution for continuous values
  - value in the range $[-\inf, x]$, given expectation μ and standard deviation σ: $n(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\mu} e^{-(x-\mu)^2/(2\sigma^2)}$
  - also called **Bell curve**, or **Gaussian**
  - examples: heights of people, IQ of people, tree heights, ...



Estimation Revisited

- We introduced last lecture an estimation of probabilities based on frequencies:

  $P(w) = \frac{count(w)}{\sum_{w'} count(w')}$

- Alternative view: Bayesian: what is the most likely model given the data

  $p(M|D)$

- Model and data are viewed as random variables
  - model M as random variable
  - data D as random variable

Bayesian Estimation

- Reformulation of $p(M|D)$ using Bayes rule:

$$p(M|D) = \frac{p(D|M)\ p(M)}{p(D)}$$

$$argmax_M\ p(M|D) = argmax_M\ p(D|M)\ p(M)$$

- $p(M|D)$ answers the question: What is the most likely model given the data

- $p(M)$ is a prior that prefers certain models (e.g. simple models)

- The frequentist estimation of word probabilities $p(w)$ is the same as Bayesian estimation with a uniform prior (no bias towards a specific model), hence it is also called the **maximum likelihood estimation**

Entropy

- An important concept is **entropy**:

  $H(X) = \sum_x -p(x)\ \log_2 p(x)$

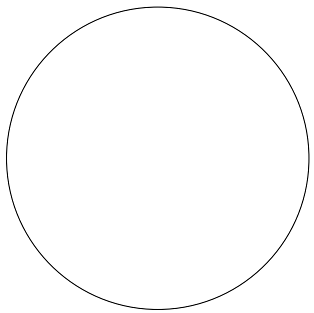
- A measure for the degree of disorder

Entropy Example

One event

$$p(a) = 1$$

$$\begin{aligned} H(X) &= -1 \log_2 1 \\ &= 0 \end{aligned}$$

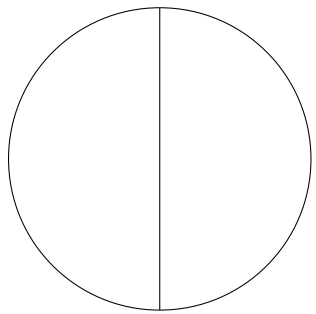


Entropy Example

2 equally likely events:

$$p(a) = 0.5$$
$$p(b) = 0.5$$

$$\begin{aligned} H(X) &= -0.5\log_2 0.5 - 0.5\log_2 0.5 \\ &= -\log_2 0.5 \\ &= 1 \end{aligned}$$

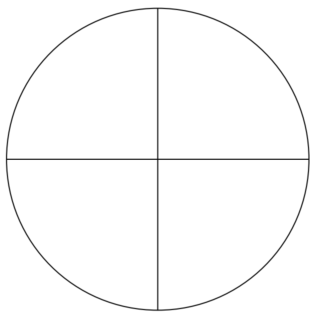


Entropy Example

4 equally likely events:

$$p(a) = 0.25$$
$$p(b) = 0.25$$
$$p(c) = 0.25$$
$$p(d) = 0.25$$

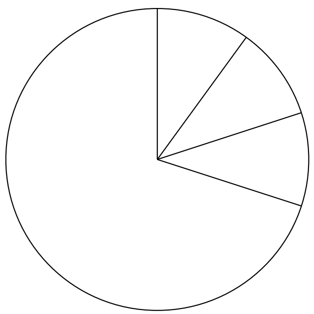
$$\begin{aligned} H(X) = & -0.25 \log_2 0.25 - 0.25 \log_2 0.25 \\ & -0.25 \log_2 0.25 - 0.25 \log_2 0.25 \\ = & -\log_2 0.25 \\ = & \; 2 \end{aligned}$$



Entropy Example

4 equally likely events, one more likely than the others:

$$p(a) = 0.7$$
$$p(b) = 0.1$$
$$p(c) = 0.1$$
$$p(d) = 0.1$$

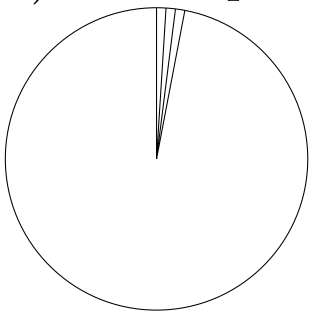


$$\begin{aligned} H(X) = & -0.7\log_2 0.7 - 0.1\log_2 0.1 \\ & -0.1\log_2 0.1 - 0.1\log_2 0.1 \\ = & -0.7\log_2 0.7 - 0.3\log_2 0.1 \\ = & -0.7 \times -0.5146 - 0.3 \times -3.3219 \\ = & \, 0.36020 + 0.99658 \\ = & \, 1.35678 \end{aligned}$$

Entropy Example

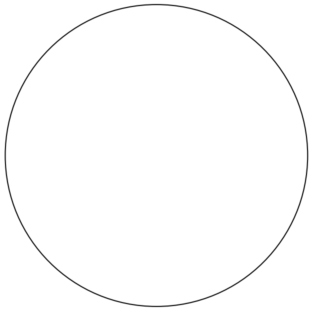
4 equally likely events, one much more likely than the others:

$p(a) = 0.97$
$p(b) = 0.01$
$p(c) = 0.01$
$p(d) = 0.01[1cm]$

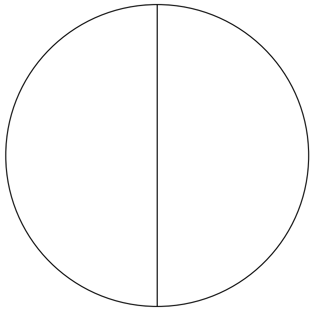


$$\begin{aligned}
H(X) = & -0.97 \log_2 0.97 - 0.01 \log_2 0.01 \\
& -0.01 \log_2 0.01 - 0.01 \log_2 0.01 \\
= & -0.97 \log_2 0.97 - 0.03 \log_2 0.01 \\
= & -0.97 \times -0.04394 - 0.03 \times -6.6439 \\
= & \, 0.04262 + 0.19932 \\
= & \, 0.24194
\end{aligned}$$

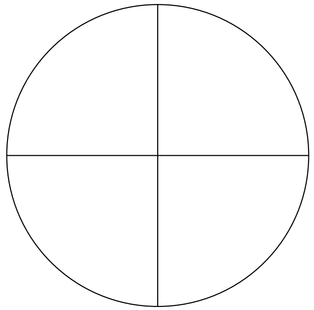
Examples



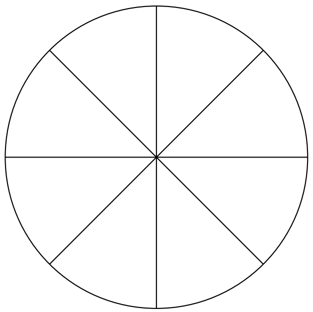
$H(X) = 0$



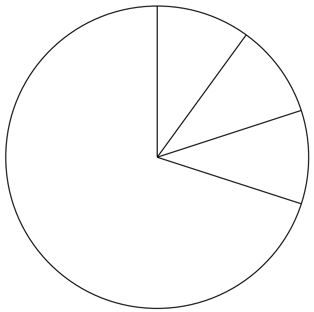
$H(X) = 1$



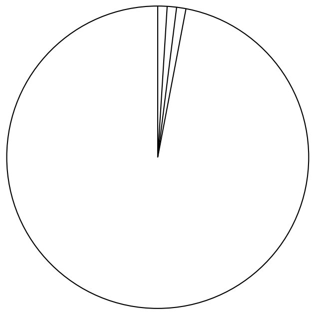
$H(X) = 2$



$H(X) = 3$



$H(X) = 1.35678$



$H(X) = 0.24194$

Intuition Behind Entropy

- A good model has low entropy

$\rightarrow$ it is more certain about outcomes

- For instance a translation table

e	f	$p(e\|f)$
the	der	0.8
that	der	0.2

is better than

e	f	$p(e\|f)$
the	der	0.02
that	der	0.01
...	...	...

- A lot of statistical estimation is about reducing entropy

Information Theory and Entropy

- Assume that we want to encode a sequence of events X
- Each event is encoded by a sequence of bits
- For example
  - Coin flip: heads = 0, tails = 1
  - 4 equally likely events: a = 00, b = 01, c = 10, d = 11
  - 3 events, one more likely than others: a = 0, b = 10, c = 11
  - Morse code: e has shorter code than q
- Average number of bits needed to encode $X \geq$ entropy of X

The Entropy of English

- We already talked about the probability of a word $p(w)$
- But words come in sequence. Given a number of words in a text, can we guess the next word $p(w_n|w_1, ..., w_{n-1})$?
- Assuming a model with a limited window size

Model	Entropy
0th order	4.76
1st order	4.03
2nd order	2.8
human, unlimited	1.3

Next Lecture: Language Models

- Next time, we will expand on the idea of a model of English in the form

$$p(w_n|w_1, ..., w_{n-1}) \tag{1}$$

- Despite its simplicity, a tremendously useful tool for NLP

- Nice machine learning challenge
  - sparse data
  - smoothing
  - back-off and interpolation