Decision making methodology and algorithm : AHP (Analytical Hierarchical Process)

Creation of Information Systems on the Basis of Team Work

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Analytical Hierarchical process AHP

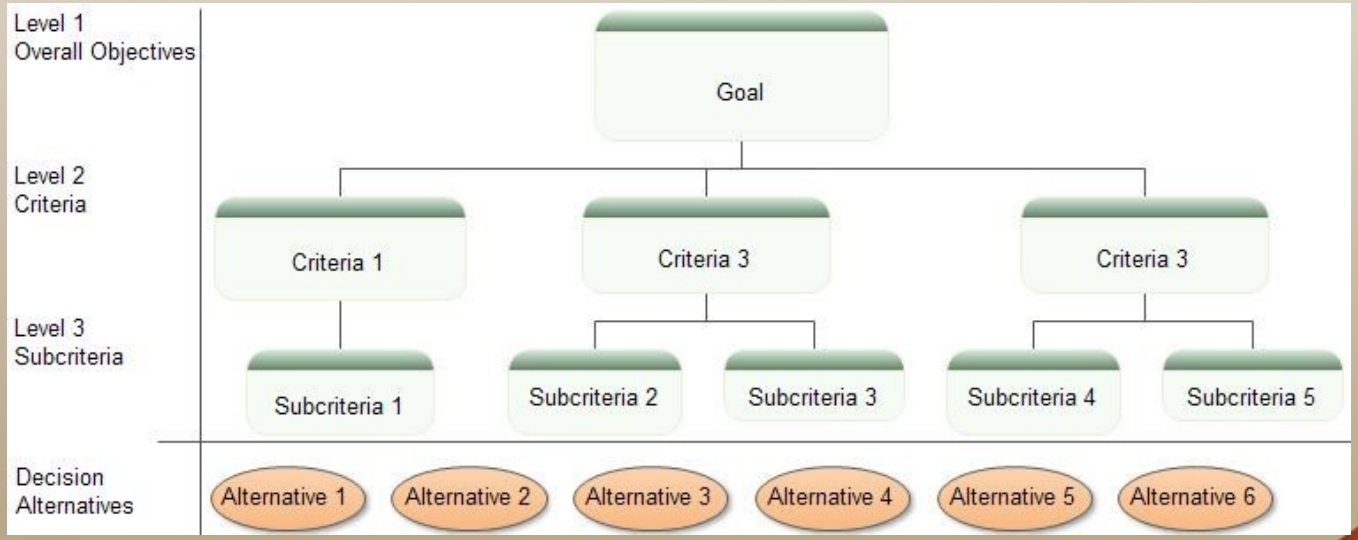
- Author: Saaty 1980.
- Based on expert evaluation
- Allows expert groups
- Application of quantitative and qualitative criteria
- Process of pairwise evaluation (instead of overall assignment of values)
- Consistency check
- Result – ranking by importance, weights

AHP algorithm

- Two stage algorithm, where evaluation is done by **pairwise** comparison of all possible pairs of items:

  Stage 1: Criteria assessment -weight assignment to the criteria, arranged as a hierarchical tree

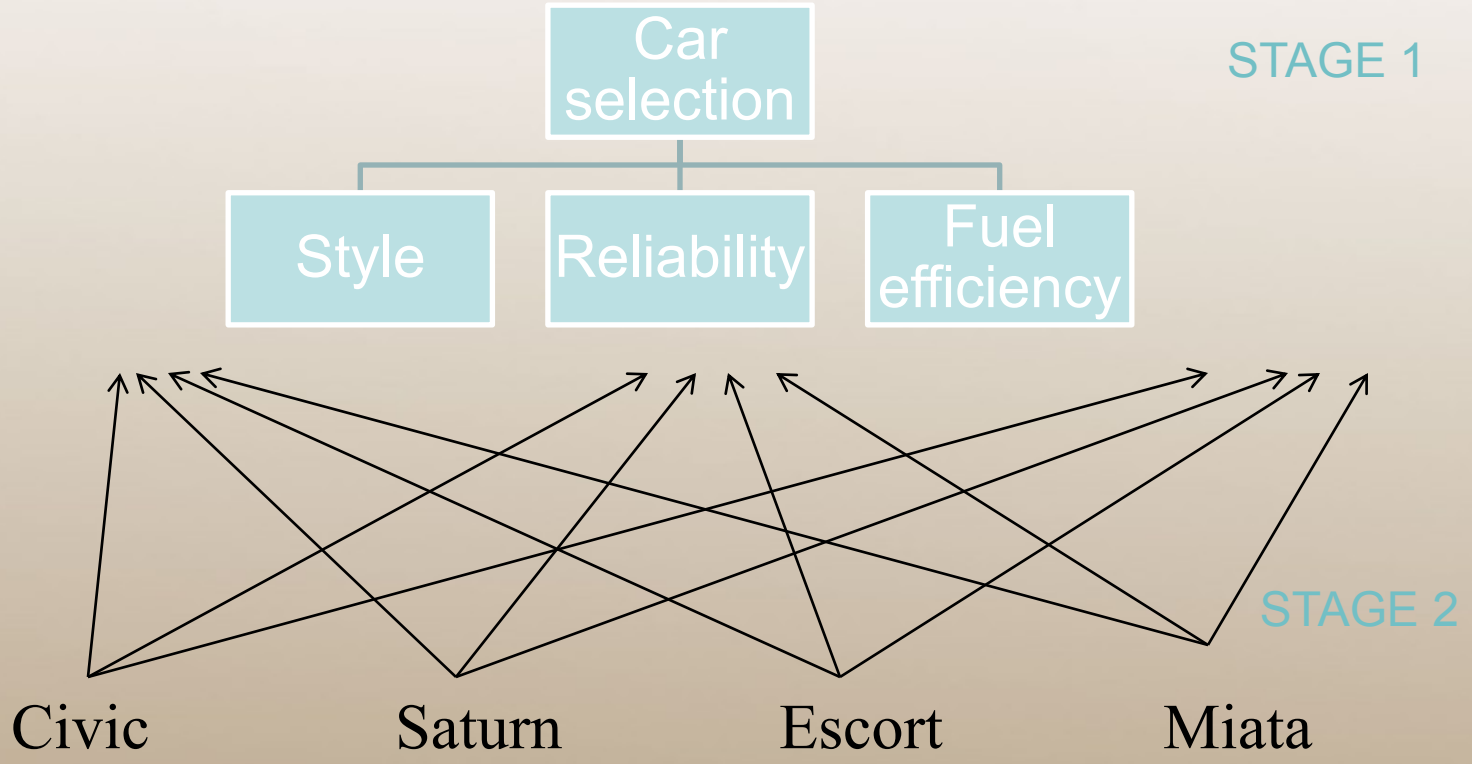
  Stage 2: Assessment of decision **alternatives** (a list of selected objects- weight assignment by each criterion and globally



Example – car selection

- Goal- car
- Criteria – style, reliability, fuel, price
- Alternatives (list of items)
  - Civic Coupe, Saturn Coupe, Ford Escort, Mazda Miata

Stage 1- Criteria hierarchy

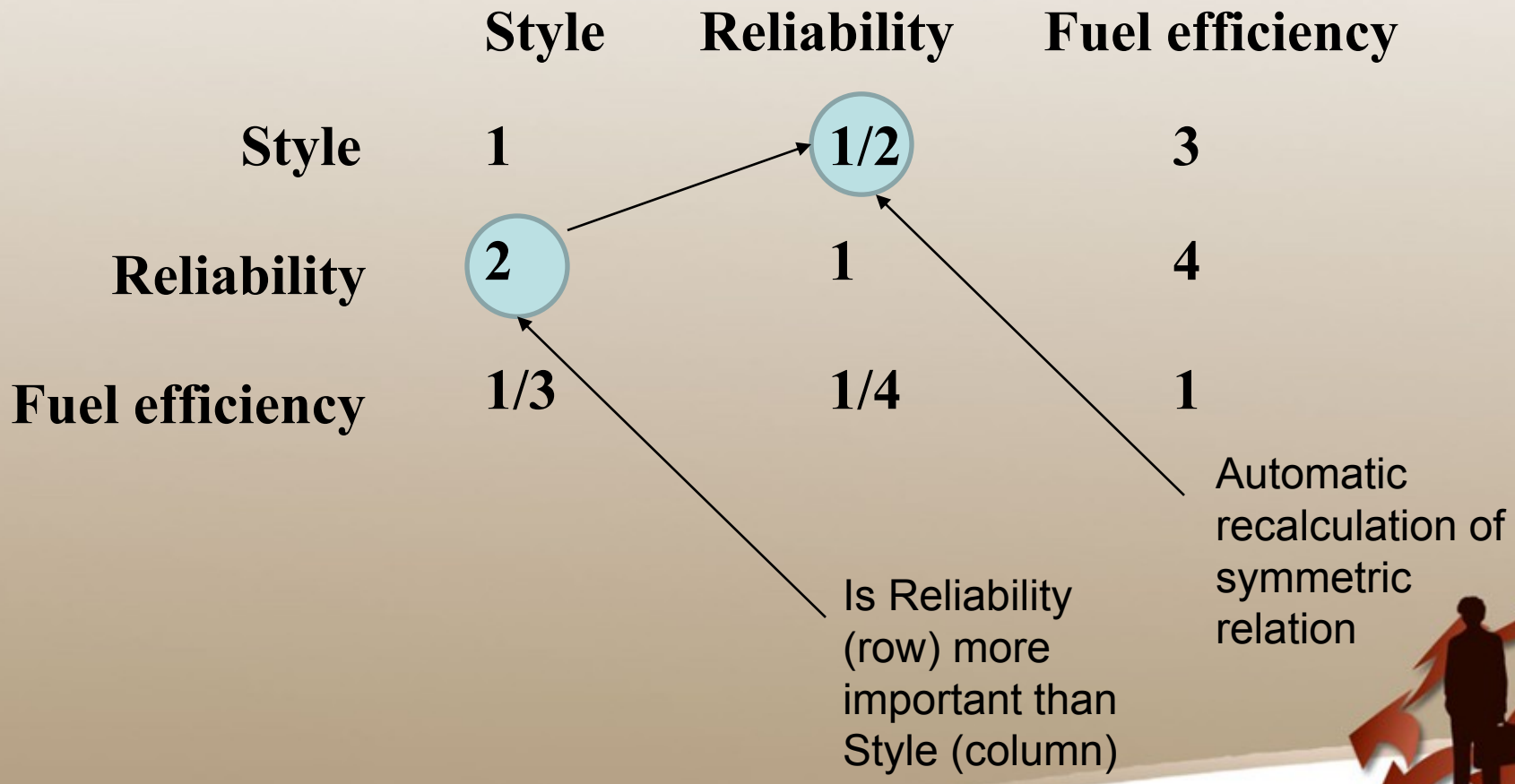


Ranking criteria and alternatives

- Expert ranking: scale 1-9 : symetric relation

Intensyvumas, rangas	Apibrėžimas
1	Equal
3	Slightly more important
5	Definitely More important
7	Much more important
9	Absolutely more important
2,4,6,8	Intermediate values

Evaluation of Criteria: experts assign importance value by analysis of pairs. Is the criterion i (row) more important to criterion j (column) ?



Calculating weights

- We analyse the matric [$Ax = \lambda_{max}x$] where:
  - A – matrix of citeria ranks n n. (n- matrix rank equal to the number of items/criteria)
  - x eigenvalue vector n 1 – vector of output weights.
  - λ_{max} eigenvalue, $\lambda_{max} \in \Re >$ n.

- The output weights are estimated by calculating eigenvalue vector X:

  1) Matrix A comlumns are normalized (the sum columns should become equal to 1) .

  2) Calculate average values by rows.

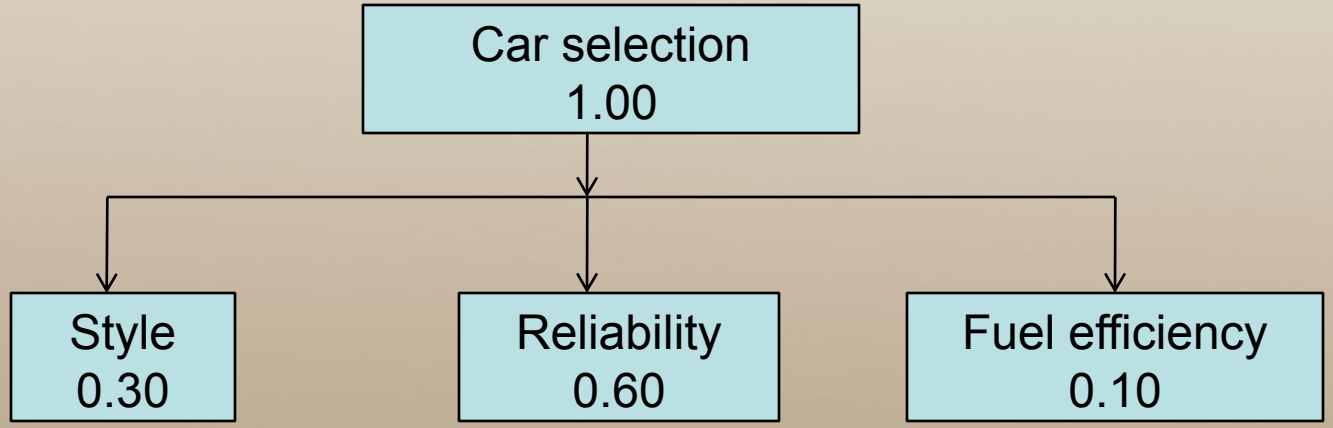
$$A = \begin{bmatrix} 1 & 0.5 & 3 \\ 2 & 1 & 4 \\ 0.33 & 0.25 & 1.0 \end{bmatrix} \xrightarrow{\text{Normalization}} \begin{bmatrix} 0.30 & 0.29 & 0.38 \\ 0.60 & 0.57 & 0.50 \\ 0.10 & 0.14 & 0.13 \end{bmatrix} \xrightarrow{\text{Row average}} X = \begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}$$

Weight vector X

Sums of columns 3.33 1.75 8.00 1.00 1.00 1.00

STAGE 1 output: Criteria weights

- Style 0.30
- Reliability 0.60
- Fuel efficiency 0.10



Consistency check, evaluation of CR (Consistency ratio)

- Consistency Ratio (CR) shows if the expert evaluations were logical, without self-contradiction or random guess.
- AHP relies on the common logics of expert evaluations – if A is more important than B, and criterion B is more important than C, then A should be more important than C.

- If CR is bigger that 0.1 (>10%) the evaluation matrix is not reliable and has high extent of random guess

CR calculation

- In order to find eigenvalue λ_{max}

we analyse the matric expression [$Ax = \lambda_{max} x$] where x is the weight vector (eigenvector), and A is the primary (not normalized) expert evaluation matrix, n is matrix rank (number of rows, columns):

$$\underset{A}{\begin{bmatrix} 1 & 0.5 & 3 \\ 2 & 1 & 4 \\ 0.333 & 0.25 & 1.0 \end{bmatrix}} \underset{x}{\begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}} = \underset{Ax}{\begin{bmatrix} 0.90 \\ 1.60 \\ 0.35 \end{bmatrix}} = \lambda_{max} \underset{x}{\begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}}$$

λmax=average of divided values: {0.90/0.30, 1.60/0.6, 0.35/0.10}=3.06

- Consistency **index** is calculated : CI=(λmax-n)/(n-1)=(3.06-3)/(3-1)= 0.03
- Consistancy **ratio** CR=CI/RI=0.03/0.58=0.05. If CR<0.1 evaluations a logical and not random. If CR > 10% (0,1) , the experts should revise evaluation or be excluded

Matrix rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI-random index	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

STAGE 2: Assessment of decision **alternatives** by EACH criterion

Style	Civic	Saturn	Escort	Miata
Civic	1	1/4	4	1/6
Saturn	4	1	4	1/4
Escort	1/4	1/4	1	1/5
Miata	6	4	5	1

Priority vector

$$\begin{bmatrix} 0.13 \\ 0.24 \\ 0.07 \\ 0.56 \end{bmatrix}$$

Reliability	Civic	Saturn	Escort	Miata
Civic	1	2	5	1
Saturn	1/2	1	3	2
Escort	1/5	1/3	1	1/4
Miata	1	1/2	4	1

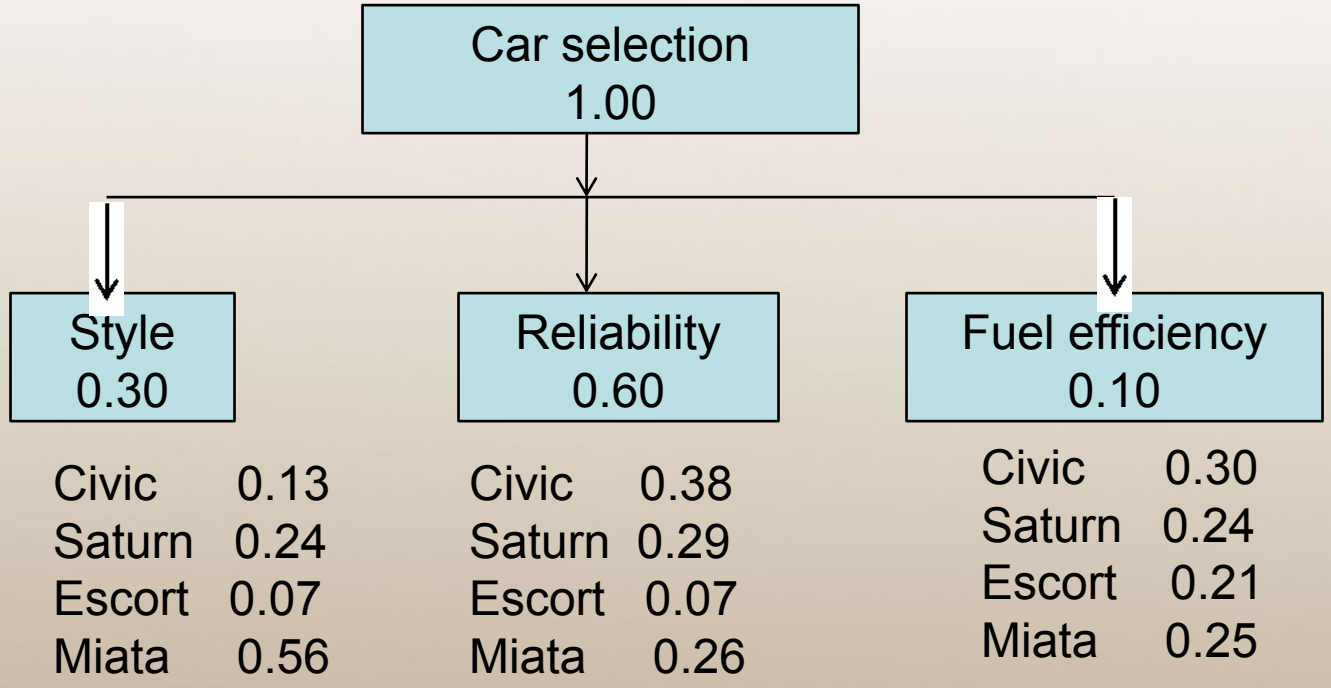
$$\begin{bmatrix} 0.38 \\ 0.29 \\ 0.07 \\ 0.26 \end{bmatrix}$$

Ranking values for including quantitative (not expert) evaluation

		KM/L	Normalized
Fuel efficiency	Civic	34	.30
	Saturn	27	.24
	Escort	24	.21
	Miata	28	.25
		113	1.0

- The indicators of fuel consumption for 100km is recalculated to the indicator “How many km we can drive with 1 liter” (the bigger value is the better should be used for all evaluations – both quantitative and expert)
- If we’d like to reflect difference of fuel efficiency depending on seasons (winter/summer or surroundings (city/highway), the expert ranking can be used again

The result of two-stage evaluation: criteria weights and evaluation of alternatives by all criteria



Global evaluation- weights are calculated by matrix multiplication

	Style	Reliability	Fuel efficiency				
Civic	.13	.38	.30	x	.30	=	.30
Saturn	.24	.29	.24		.60		.27
Escort	.07	.07	.21		.10		.08
Miata	.56	.26	.25				.35

Weight matrix
Stage 2

Criteria weights
Stage 1

Global evaluations

Cost criterion for decision making

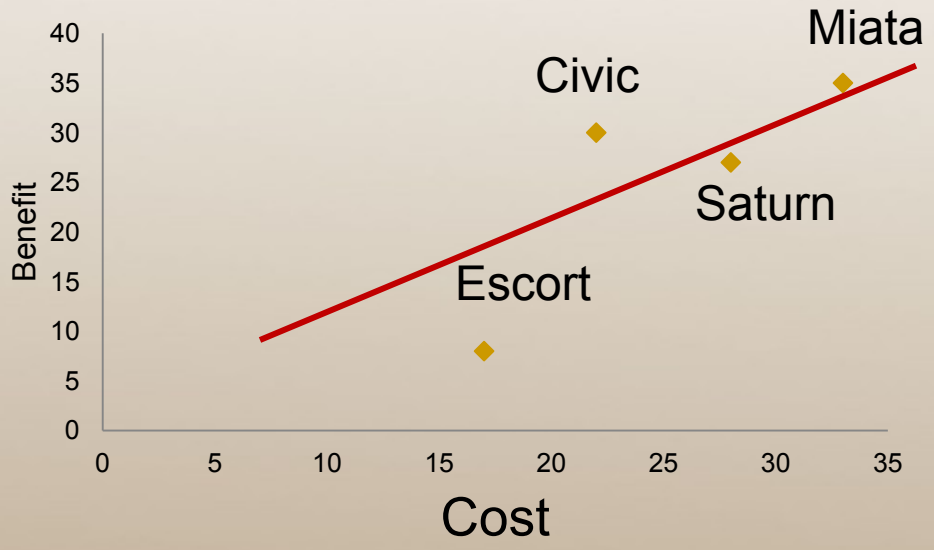
Cost can be included to the lost of all criteria for expert pairwise evaluation/ However, it tends to dominate and can hide importance of some criteria.

Instead, it be can use it for **COST/BENEFIT** analysis . The normalized price is used to estimate COST (price) and is divided by BENEFIT (global evaluation). The smaller value, the more attractive is alternative, as the benefit exceeds cost (CIVIC is most attractive for smallest ratio 0.73, ESCORT is overpriced comparing to its overall benefit – ratio 2.13))

	Price	Normalized price	Benefit (global weights)	Cost/Benefit ratio
• CIVIC	12K	0.22	0.30	0.73
• SATURN	15K	0.28	0.27	1.03
• ESCORT	9K	0.17	0.08	2.13
• MIATA	18K	0.33	0.35	0.92

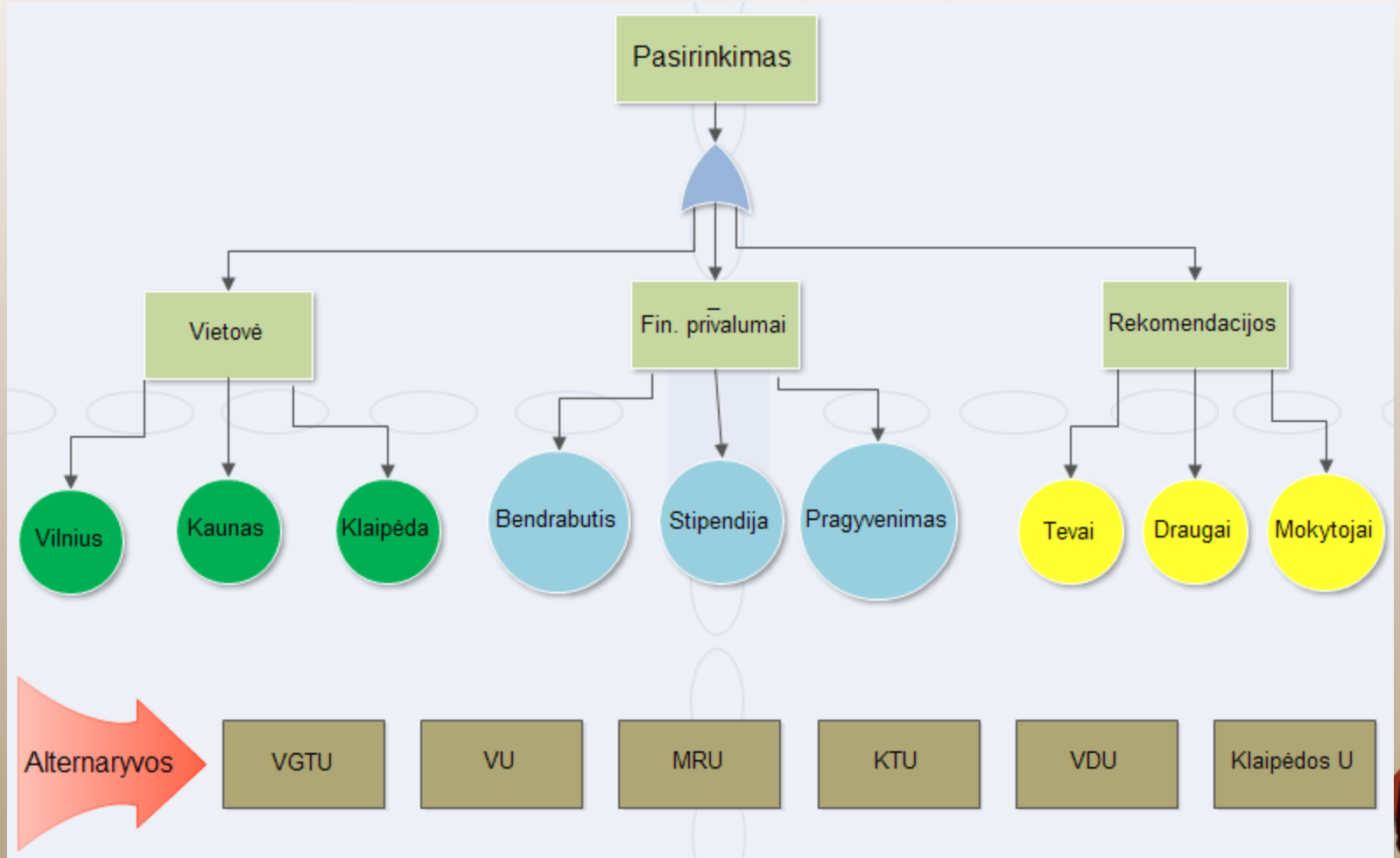
Regression analysis: Graphical visualization of result

- Regression analysis is applied



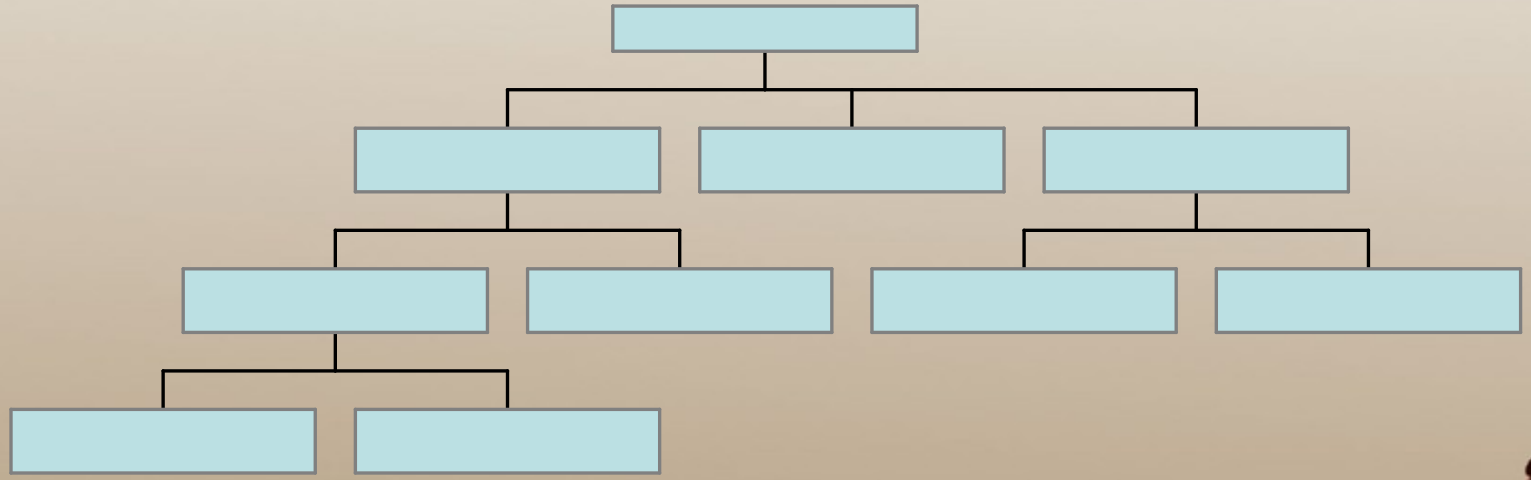
- Benefit exceeds the cost (beneficial for us) for all cases above the trendline

Choosing place to study. Criteria (Location, budget, recommendations). Alternatives (universities)



Complex decisions

•Many levels of criteria and sub-criteria exists for complex problems.



Group Decision Making

The AHP allows group decision making, where group members can use their experience, values and knowledge to break down a problem into a hierarchy and solve. Doing so provides:

- Understand the conflicting ideas in the organization and try to reach a consensus.
- Minimize dominance by a strong member of the group.
- Members of the group may vote for the criteria to form the AHP tree. (Overall priorities are determined by the weighted averages of the priorities obtained from members of the group.)

However;
The GDSS does not replace all the requirements for group decision making. Open meetings with the involvement of all members are still an asset.

More about AHP: Pros and Cons

Pros

- It allows multi criteria decision making.
- It is applicable when it is difficult to formulate criteria evaluations, i.e., it allows qualitative evaluation as well as quantitative evaluation.
- It is applicable for group decision making environments

Cons

- There are hidden assumptions like consistency. Repeating evaluations is cumbersome. → Users should be trained to use AHP methodology.
- Difficult to use when the number of criteria or alternatives is high, i.e., more than 7. → Use GDSS / Use constraints to eliminate some alternatives
- Difficult to add a new criterion or alternative → Use cost/benefit ratio if applicable
- Difficult to take out an existing criterion or alternative, since the best alternative might differ if the worst one is excluded.

Example 1: : Evaluation of Job Offers

Ex: Peter is offered 4 jobs from Acme Manufacturing (A), Bankers Bank (B), Creative Consulting (C), and Dynamic Decision Making (D).
He bases his evaluation on the criteria such as location, salary, job content, and long-term prospects.

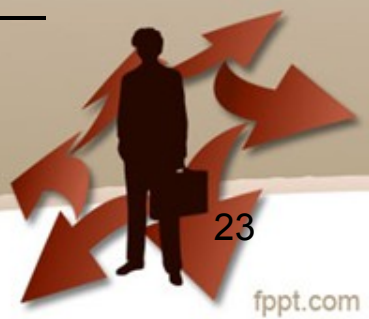
Step 1: Decide upon the relative importance of the selection criteria:

	Location	Salary	Content	Long-term
Location	1	1/5	1/3	1/2
Salary	5	1	2	4
Content	3	1/2	1	3
Long-term	2	1/2	1/3	1

Example 1: Priority Vectors:

1) Normalize the column entries by dividing each entry by the sum of the column.
2) Take the overall row averages

	Location	Salary	Content	Long-term	Average
Location	0.091	0.102	0.091	0.059	0.086
Salary	0.455	0.513	0.545	0.471	0.496
Content	0.273	0.256	0.273	0.353	0.289
Long-term	0.182	0.128	0.091	0.118	0.130
	+				+
	1	1	1	1	1



Example 1: Evaluation of Job Offers

Step 2: Evaluate alternatives w.r.t. each criteria

Location Scores

	A	B	C	D
A	1	1/2	1/3	5
B	2	1	1/2	7
C	3	2	1	9
D	1/5	1/7	1/9	1

Relative Location Scores

	A	B	C	D	Avg.
A	0.161	0.137	0.171	0.227	0.174
B	0.322	0.275	0.257	0.312	0.293
C	0.484	0.549	0.514	0.409	0.489
D	0.032	0.040	0.057	0.045	0.044

Example 1: Calculation of Relative Scores

	Relative Scores for Each Criteria					Relative weights for each criteria		Relative scores for each alternative
	Location	Salary	Content	Long-Term				
A	0.174	0.050	0.210	0.510		0.086		0.164
B	0.293	0.444	0.038	0.012	x	0.496	=	0.256
C	0.489	0.312	0.354	0.290		0.289		0.335
D	0.044	0.194	0.398	0.188		0.130		0.238

Example 2: AHP in project management

Prequalification of contractors aims at the elimination of incompetent contractors from the bidding process.

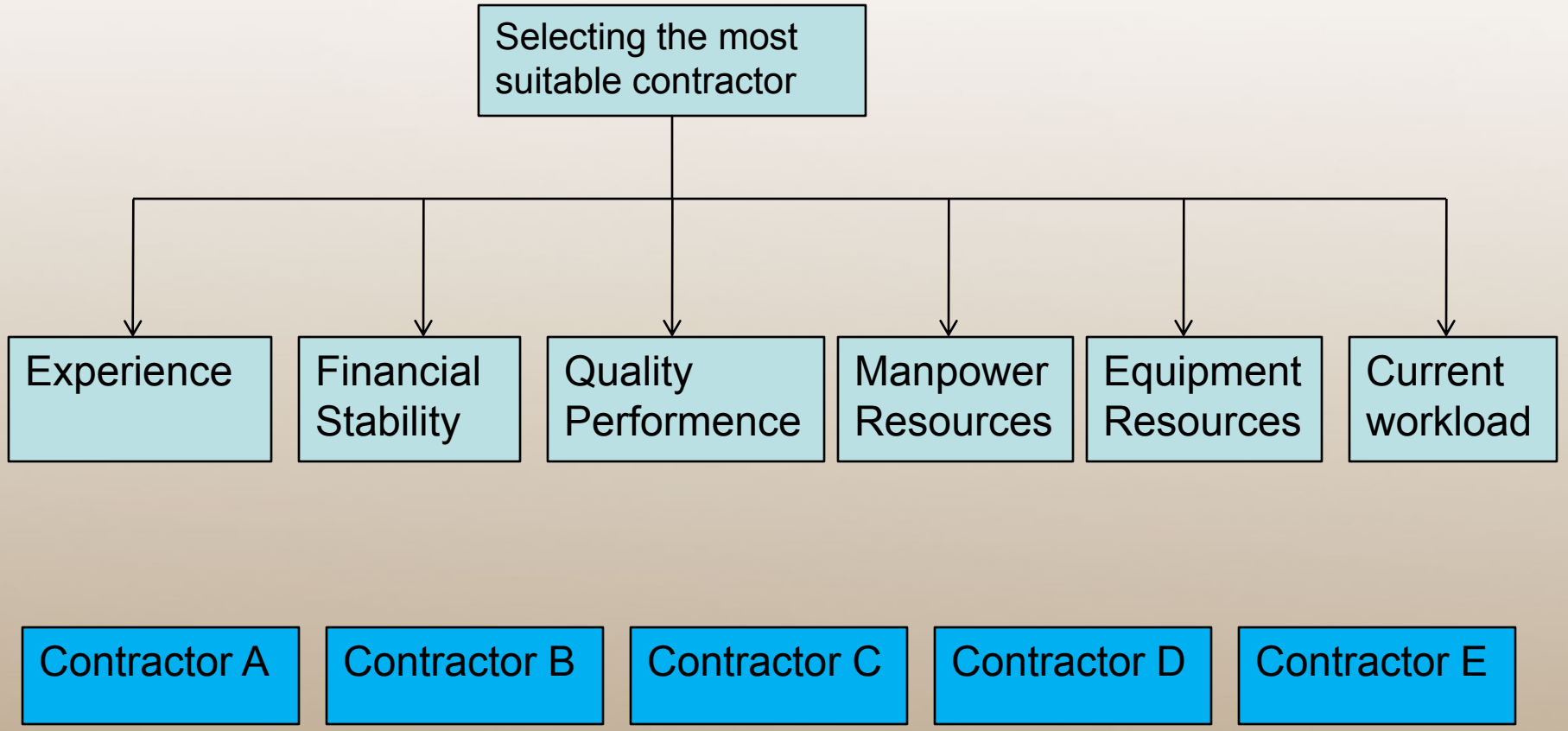
It is the choice of the decision maker to eliminate contractor E from the AHP evalution since it is not “feasible” at all !!

	Contractor A	Contractor B	Contractor C	Contractor D	Contractor E
Experience	5 years experience	7 years experience	8 years experience	10 years experience	15 years experience
	Two similar projects	One similar project	No similar project	Two similar projects	No similar project
		Special procurement experience	1 international project		
Financial stability	$7 M assets	$10 M assets	$14 M assets	$11 M assets	$6 M assets
	High growth rate	$5.5 M liabilities	$6 M liabilities	$4 M liabilities	$1.5 M liabilities
	No liability	Part of a group of companies		Good relation with banks	
Quality performance	Good organization	Average organization	Good organization	Good organization	Bad organization
	C.M. personnel	C.M. personnel	C.M. team	Good reputation	Unethical techniques
	Good reputation	Two delayed projects	Government award	Many certi®cates	One project terminated
	Many certi®cates	Safety program	Good reputation	Cost raised in some projects	Average quality
	Safety program		QA/QC program		
Manpower resources	150 labourers	100 labourers	120 labourers	90 labourers	40 labourers
	10 special skilled labourers	200 by subcontract	Good skilled labors	130 by subcontract	260 by subcontract
		Availability in peaks	25 special skilled labourers		

Example 2 (cont.'d)

	Contractor A	Contractor B	Contractor C	Contractor D	Contractor E
Equipment resources	4 mixer machines	6 mixer machines	1 batching plant	4 mixer machines	2 mixer machines
	1 excavator	1 excavator	2 concrete transferring trucks	1 excavator	10 others
	15 others	1 bulldozer	2 mixer machines	9 others	2000 sf steel formwork
		20 others	1 excavator		6000 sf wooden formwork
		15,000 sf steel formwork	1 bulldozer		
			16 others		
			17,000 sf steel formwork		
Current works load	1 big project ending	2 projects ending (1 big+ 1 medium)	1 medium project started	2 big projects ending	2 small projects started
	2 projects in mid (1 medium +1 small)		2 projects ending (1 big + 1 medium)	1 medium project in mid	3 projects ending (2 small + 1 medium)

Example 2: Hierarchy Tree



Example 2: AHP in project management

Step 1: Evaluation of the weights of the criteria

Pair-wise comparison matrix for the six criteria[a]

	Exp.	FS	QP	MPR	ER	CWL	Priority vector
Exp.	1	2	3	6	6	5	0.372
FS	1/2	1	3	6	6	5	0.293
QP	1/3	1/3	1	4	4	3	0.156
MPR	1/6	1/6	1/4	1	2	1/2	0.053
ER	1/6	1/6	1/4	1/2	1	1/4	0.039
CWL	1/5	1/5	1/3	2	4	1	0.087
							$\sum = 1.00$

[a] $\lambda_{max} = 6.31$, CI = 0.062, RI = 1.24, CR = 0.05 < 0.1 OK.

Step 2: a) Pairwise comparison matrix for experience

Exp.	A	B	C	D	E
A	1	1/3	1/2	1/6	2
B	3	1	2	1/2	4
C	2	1/2	1	1/3	3
D	6	2	3	1	7
E	1/2	1/4	1/3	1/7	1

Exp.	A	B	C	D	E	Priority vector
A	0.08	0.082	0.073	0.078	0.118	0.086
B	0.24	0.245	0.293	0.233	0.235	0.249
C	0.16	0.122	0.146	0.155	0.176	0.152
D	0.48	0.489	0.439	0.466	0.412	0.457
E	0.04	0.061	0.049	0.066	0.059	0.055
						$\sum = 0.999$

[a] $\lambda_{max} = 5.037$, CI = 0.00925, RI = 1.12, CR = 0.0082 < 0.1 OK.

Example 2: AHP in project management

Calculation of priority vector:

	Exp. (0.372)	FS (02.93)	QP (0.156)	MPR (0.053)	ER (0.039)	CWL (0.087)
A	0.086	0.425	0.269	0.151	0.084	0.144
B	0.249	0.088	0.074	0.273	0.264	0.537
C	0.152	0.178	0.461	0.449	0.556	0.173
D	0.457	0.268	0.163	0.081	0.057	0.084
E	0.055	0.039	0.031	0.045	0.038	0.062

$$\times \begin{bmatrix} 0.372 \\ 0.293 \\ 0.156 \\ 0.053 \\ 0.039 \\ 0.087 \end{bmatrix} = \begin{bmatrix} 0.222 \\ 0.201 \\ 0.241 \\ 0.288 \\ 0.046 \end{bmatrix}$$

Probably Contractor-E should have been eliminated. It appears to be the worst.

Note that a DSS supports the decision maker, it can not replace him/her. Thus, an AHP Based DSS should allow the decision maker to make sensitivity analysis of his judgements on the overall priorities !

Multi Criteria Decision Making Models: PROMETHEE

- One of the most efficient and easiest MCDM methodologies.
- Developed by Jean-Pierre Brans and Bertrand Mareschal at the ULB and VUB universities since 1982
- Considers a set of criteria and alternatives. Criteria weights are determined that indicate the relative importance
- Utilizes a function reflecting the degree of advantage of one alternative over the other, along with the degree of disadvantage that the same alternative has with respect to the other alternative.
- In scaling, there are six options allowing the user to express meaningful differences by minimum gaps between observations. When type I is used, only relative advantage matters; type 6 is based on standardization with respect to normal distribution.

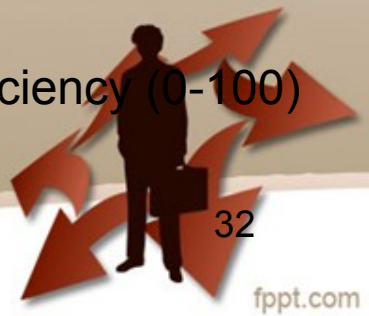
Example 3: Media Selection for a Bicycle Co.

A bicycle manufacturing company is intending to advertise its products.

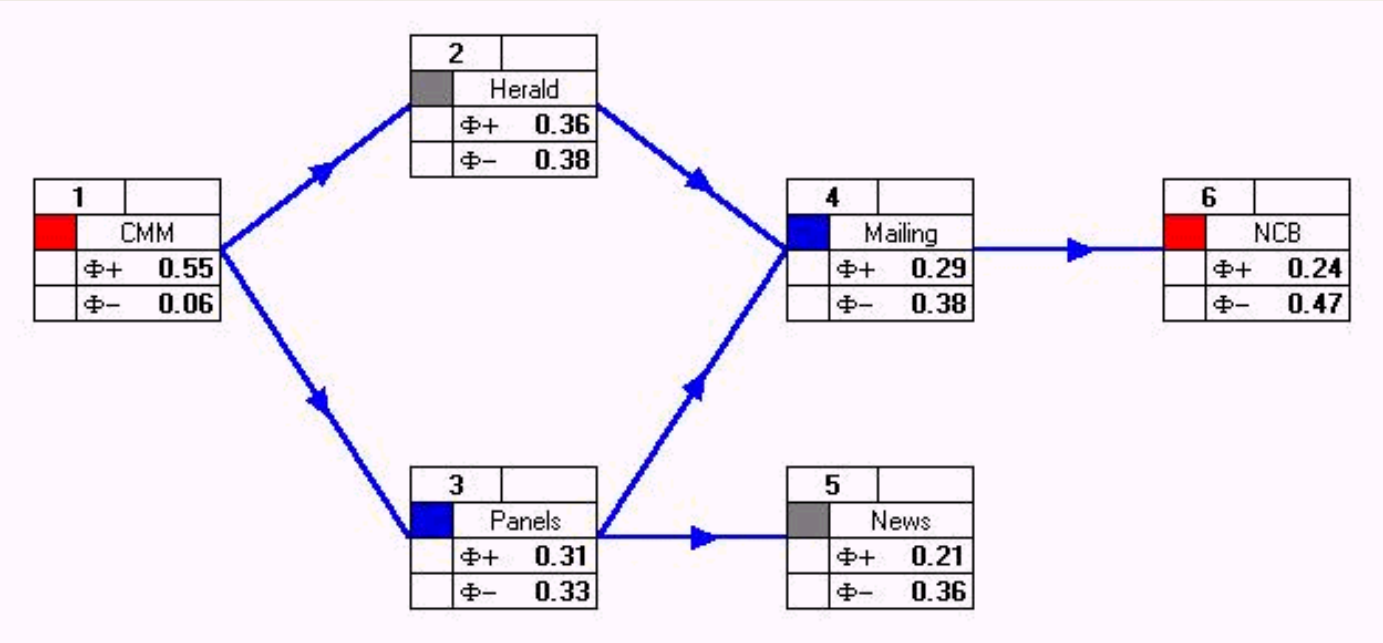
Six marketing actions are considered: Advertising in the international newspaper, *News; in the newspaper Herald; by mean of* advertising boards in large cities; of a personal mailing; by TV spots on channels *CMM or NCB.*

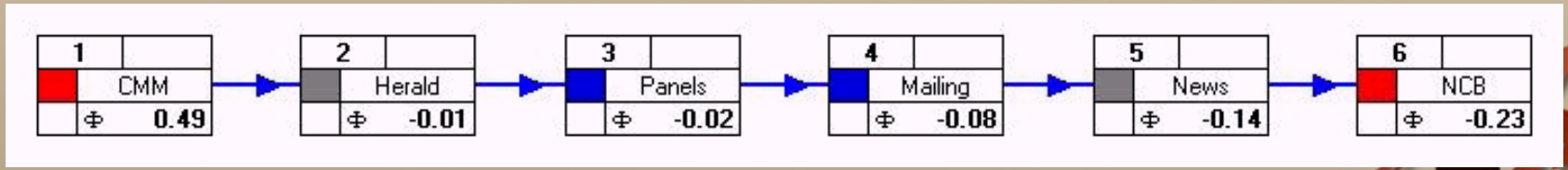
Criteria min/max	C1 cost min	C2 target max	C3 durat. max	C4 effic. max	C5 manp. min
News	60	900	22	51	8
Herald	30	520	31	13	1
Panels	40	650	20	58	2
Mailing	92	750	60	36	3
CMM	52	780	58	90	1
NCB	80	920	4	75	6

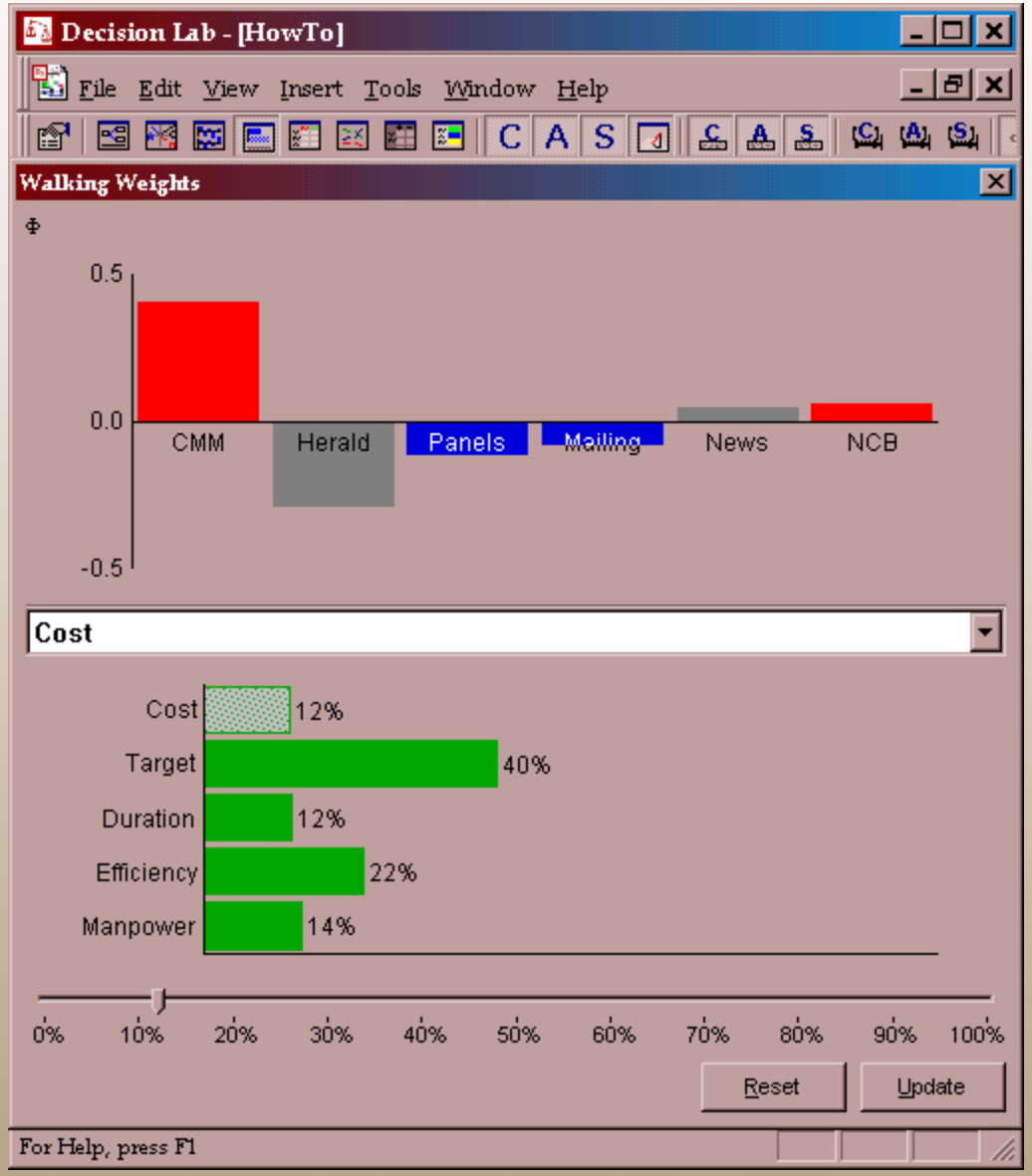
Units: Cost ($ 1,000), Target (10,000 people), Duration (days), Efficiency (0-100)
Manpower (# people involved in the company)



Partial anf full rankings with Promethee I and II







Ranking of the alternatives can be obtained for the selected weights

Including optimization: Additional constraints

- It is often necessary that several alternatives have to be selected subject to a set of goals.
- In this case an LP can be constructed with binary decision variables, which gives the selection of *r* actions, among *n* alternatives.

Let xi=1 if media i is selected and 0 otherwise, i=1,2,...,6.

φ(Ai) are the relative weight of media i, i=1,2,...,6.

Max φ(A1) *x1 + … + φ(A6) x6*

Subject to

x1 + x2 + x3 + x4 + x5 + x6 ≥ 2 (at least 2 media should be selected)

x1 + x2 + x3 + x4 + x5 + x6 ≤ 4 (at most 4 media should be selected.)

x1 + x2 = 1 (choose exactly one newspaper)

x5 + x6 = 1 ((choose exactly 1 TV channel)

625 *x1 + 550 x2 + 250 x3 + 150 x4 + 175 x5 +* 750 *x6 ≥ 1200 (min. expected return)*

- 60 *x1 - 30 x2 + 40 x3 + 92 x4 + 52 x5 +* 80 *x6 ≥ 0 (cost of advertising in newspapers should be less than 50% of total costs)*

AHP web tool

- https://bpmsg.com/ahp/
- Top link- Register (**for the leader of AHP evaluation**)
- After registration an Sign in- use - My AHP Projects
- For New project press “New hierarchy”. It will consist of two stages: 1 stage: criteria hierarchy is created and evaluated and saved (project type in My projects is “H”.
- 2 stage project will be generated automatically by listing alternatives after stage 1 evaluation phase. This will be shown in the My project list as project type A, same title as H.
- For creating project for stage 1, press “New hierarchy” and edit it (keeping the syntax signs- column, semicolumn. E.g.
- ATOSTOGOS:Saule,Gamta,Pramogos,Lankytini objektai;
- Hierarchy is extended by explaining criteria – e.g. „Pramogos“ has to be explained, so the new line is written in the same format:
- Pramogos: Jaunimui, Vaikams, Sportas;
- After creating (save, submit) hierarchy, copy the link for expert evaluation to all members (they must not be registered or signed to the system)

AHP web tool

- After receiving the link (or the session code), the experts can do evaluation. If they have a session code, they use AHP Group Session meniu. Each place where they see AHP, they have to do evaluation. They evaluate pairs of criteria : chose a dot for the more important criterion, assign value, how much more it is important.
- After submitting evaluation, the „consistency ratio“ is computed by system, and , if CR value > 10%, the suggestions for changing evaluation are highlighted. The experts are allowed to revise to reach CR<10 % (if this is not achieved, the evaluation will be excluded forom the decision).
- For initiating STAGE 2, project leader reviews the Stage 1 group results, selects "View results" press "Define alternatives" and writes the number of alternatives for discussion in the stage 2 etapui. Then the window opens for writing down the list of alternatives. After Save, the new code for voting is assigned. The new link with the new code is sent out to all experts.
- The same procedure for evaluation (following each AHP in red square) and adjusting CR is done.
- The team report will include the visualizations and tables about voting : criteria weights, alternative weights, table of voting by group members and their results . If some experts have CR>10, their results have to be excluded.

Decision making method: mindmapping

- Mind maps
- https://www.mindmeister.com/

It is an idea organization and visualization tool for grouping the ideas, summarizing them for further applying other (quantitative) decision making methods.

Decision alternatives – evaluation by “6 hats” method

- Present evaluation of alternatives by taking a role, expressed by the hat color:
- White hat – relies on information and facts;
- Yellow – positive support, discuss pros, potential;
- Red – based on emotional argumentation, convincing, taking perspective of market response;
- Black – reserved, critical, cautious thinking and argumentation;
- Green – creativity aspect, comparative evaluation of innovation and creativity influence for development;
- Blue- management aspects, control and summarizing all pros and cons

Delphi method

- Delphi is based on the principle that forecasts (or decisions) from a structured group of individuals are more accurate than those from unstructured groups.
- The experts answer questionnaires in **two or more rounds**. After each round, a facilitator or change agent provides an anonymised summary of the experts' forecasts from the previous round as well as the reasons they provided for their judgments.
- Thus, experts are encouraged to **revise their earlier answers** in light of the replies of other members of their panel.
- Finally, the process is stopped after a predefined stop criterion (e.g., number of rounds, achievement of consensus, stability of results), and the **mean or median scores of the final rounds determine the results**

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