Homework Sheet 1

Exercise 1 (5 points) Let A, B, C be propositional variables. Determine (with proof) whether there exists a formula φ such that the formula $A \to \varphi$ is equivalent to

(a) $\varphi \to A$;

(b) $\varphi \to B$;

(c) $B \to \varphi$.

In a second step, determine whether we can choose the formula φ such that it depends on the variable C (i.e., there exist two variable assignments v, v' that agree on all variables different from C and such that $v(\varphi) \neq v'(\varphi)$).

Exercise 2 (6 points) Let $\mathbb{N} = \{0, 1, 2, \dots\}$ be the set of natural numbers. Suppose that we have a propositional variable A_n, for every $n \in \mathbb{N}$. We call a variable assignment v an *n-assignment* if

$$v(A_k) = 0\,, \quad \text{for all } k \geq n\,.$$

(There therefore exist exactly 2^n n-assignments.) We call a formula φ an *n-formula* if it only contains implications $\to$ and (some of) the variables $A_0, \dots, A_{n-1}$.

(a) **(1 point)** Find a 3-formula that is true for exactly 7 3-assignments.

(b) **(2 points)** Prove (preferably by induction) that there is no 2-formula that is true for exactly 1 2-assignment.

(c)* **(1 point)** Find a 10-formula that is true for exactly 600 10-assignments.

(d)* **(2 points)** Determine (with proof) for which numbers $n, k \in \mathbb{N}$ there exists an n-formula that is true for exactly k n-assignments.