$$\frac{1}{1-\alpha x} = \sum \alpha^k \cdot x^k \longleftrightarrow (1, \alpha, \alpha^2, \dots)$$

$$(a_0 + a_1 x + a_2 x^2 + \dots) + (b_0 + b_1 x + b_2 x^2 + \dots)$$

$$= (a_0 + b_0) + (a_1 + b_1) x + (a_2 + b_2) x^2 + \dots$$

$$a(x) + b(x) = c(x)$$

vytv. fce (a_k) (b_k) $(a_k + b_k)$

$$x^n (a_0 + a_1 x + a_2 x^2 + \dots) = a_0 x^n + a_1 x^{n+1} + a_2 x^{n+2} + \dots$$

$$= a_{-n} + a_{-n+1} x + \dots + a_{-1} x^{n-1} + a_0 x^n + \dots$$

$$\ln \frac{1}{1-x} = \sum \frac{1}{k} x^k \rightsquigarrow \frac{1}{1-x} \ln \frac{1}{1-x} = \sum H_k x^k$$

$$H_k = \frac{1}{1} + \dots + \frac{1}{k}$$

$$\underbrace{\frac{1}{1-x}}_{\text{sčítač}} \cdot \underbrace{\frac{1}{1-x} \ln \frac{1}{1-x}}_{H_k} = \underbrace{\frac{1}{(1-x)^2}} \cdot \ln \frac{1}{1-x}$$

$$\sum \underbrace{(k+1)}_{a_k} x^k \cdot \sum \underbrace{\frac{1}{k}}_{b_k} x^k$$

LHS $\quad H_1 + \cdots + H_k = \sum_n a_{k-n} \, b_n$

$$= \sum_n (k-n+1) \frac{1}{n}$$

$$= \sum \left((k+1) \frac{1}{n} - 1 \right)$$

$$= (k+1) H_k - k = \cdots$$

posl. $a_k = 2^k$ $\longleftrightarrow$ fce $\frac{1}{1-2x} = \sum 2^k \cdot x^k$

posl. část. součtů $2^0 + \dots + 2^k$ $\longleftrightarrow$ fce $\frac{1}{1-x} \cdot \frac{1}{1-2x}$

$$\|$$

$2 \cdot 2^k - 1$ $\longleftrightarrow$ $2 \frac{1}{1-2x} - \frac{1}{1-x}$

$$F_k = F_{k-1} + F_{k-2} \quad \text{pro } k \geq 2 \ ?$$

$F_0 = 0$
$F_1 = 1$

$$k=0: \ \underset{\substack{\| \\ 0}}{F_0} \stackrel{?}{=} \underset{\substack{\| \\ 0}}{F_{-1}} + \underset{\substack{\| \\ 0}}{F_{-2}} \quad \checkmark$$

$$k=1: \ \underset{\substack{\| \\ 1}}{F_1} \stackrel{?}{=} \underset{\substack{\| \\ 0}}{F_0} + \underset{\substack{\| \\ 0}}{F_{-1}} + 1 \quad \checkmark$$

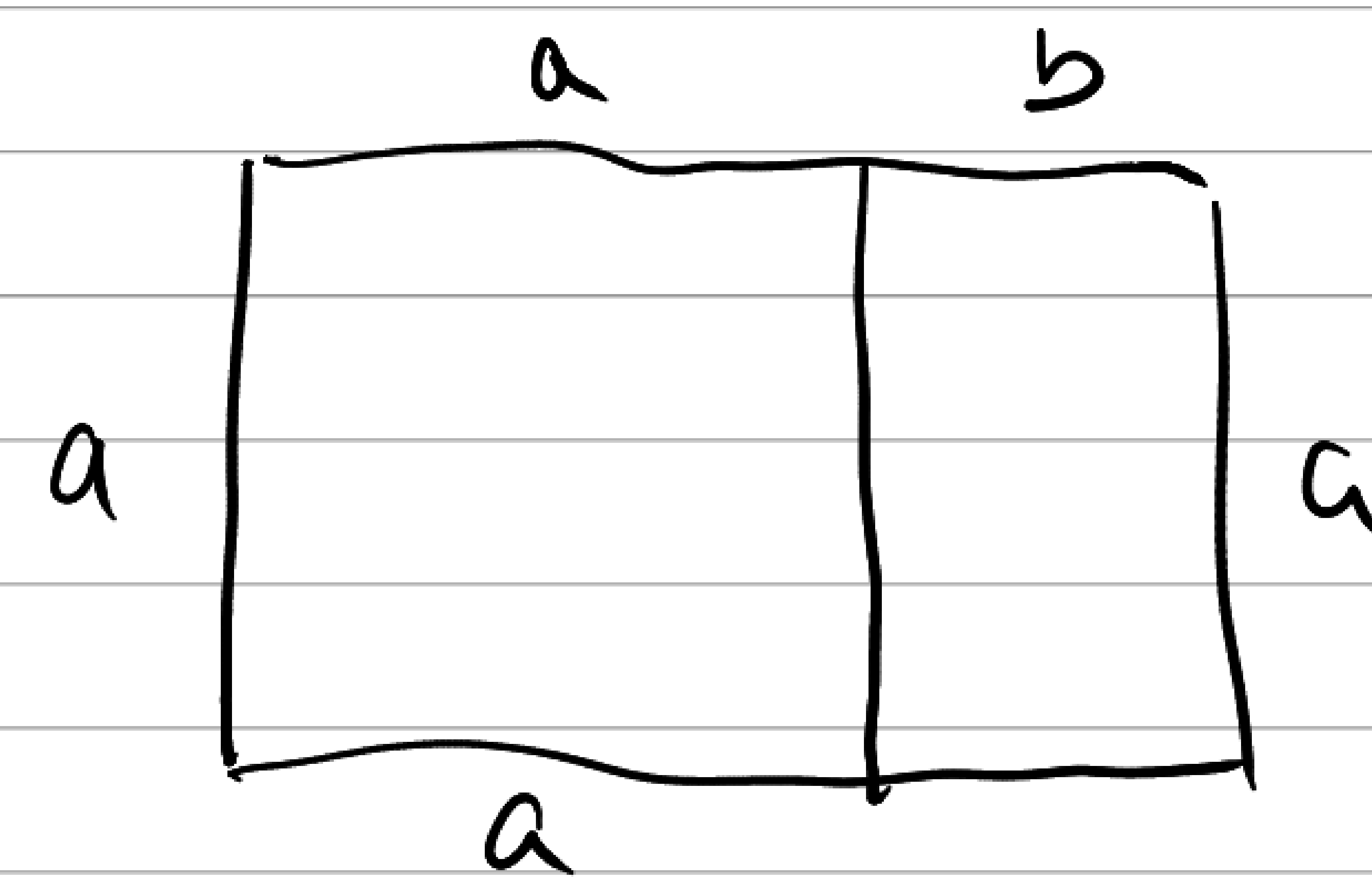
$\rightsquigarrow$ upravíme na $\quad F_k = F_{k-1} + F_{k-2} + [k=1] \quad \forall k$

$$\underset{\substack{\| \\ F(x)}}{\sum F_k \cdot x^k} = \underset{\substack{\| \\ x \cdot F(x)}}{\sum F_{k-1} x^k} + \underset{\substack{\| \\ x^2 \cdot F(x)}}{\sum F_{k-2} x^k} + \underset{\substack{\| \\ 0 + 1x + 0x^2 + \dots \\ \| \\ x}}{\sum [k=1] x^k}$$

vytvořující funkce posl. (F_k)

$$F(x) = x \cdot F(x) + x^2 \cdot F(x) + x$$

$$F(x) = x \cdot F(x) + x^2 \cdot F(x) + x$$



$$(1 - x - x^2) \cdot F(x) = x$$

$$F(x) = \frac{x}{1 - x - x^2}$$

$$\frac{a}{b} = \frac{a+b}{a} = \lambda$$

$$1 - x - x^2 = (1 - \lambda x)(1 - \mu x)$$

$$t^2 - t - 1 = (t - \lambda)(t - \mu)$$

$$\lambda, \mu = \frac{1 \pm \sqrt{5}}{2}$$

$$\lambda = \frac{1 + \sqrt{5}}{2}$$

$$\mu = \frac{1 - \sqrt{5}}{2}$$

$$A(1 - \mu x) + B(1 - \lambda x) = x$$

$$A = \frac{1}{\lambda - \mu} = \frac{1}{\sqrt{5}}$$

$$A + B = 0 \qquad -A\mu - B\lambda = 1$$

$$B = -A \qquad -A\mu + A\lambda = 1, \text{ d.h. } A(\lambda - \mu) = 1$$

$$a_k = 5a_{k-1} - 6a_{k-2} \qquad a_0 = 0 \qquad a_1 = 1$$

$$k=0: \quad \underset{0}{a_0} \overset{?}{=} 5\underset{0}{a_{-1}} - 6\underset{0}{a_{-2}} \quad \checkmark \qquad -2$$

$$k=1: \quad \underset{1}{a_1} = 5\underset{0}{a_0} - 6\underset{0}{a_{-1}} + 1 \qquad +3$$

$$\leadsto \quad a_k = 5a_{k-1} - 6a_{k-2} + [k=1] \qquad -2[k=0] + 3[k=1]$$

$$\sum a_k x^k = 5\sum a_{k-1}x^k - 6\sum a_{k-2}x^k + \sum [k=1]\, x^k$$

$$A(x) = 5x\,A(x) - 6x^2 A(x) + x$$

$$A(x)(1-5x+6x^2) = x$$

$$A(x) = \frac{x}{1-5x+6x^2} = \frac{A}{1-3x} + \frac{B}{1-2x}$$

$$1-5x+6x^2 = (1-2x)(1-3x)$$

$$\sum a_{k-1}x^k = x\sum a_{k-1}x^{k-1} = x\sum a_k x^k = x\,A(x)$$

$$\frac{x}{1-5x+6x^2} = \frac{A}{1-3x} + \frac{B}{1-2x} = \frac{1}{1-3x} - \frac{1}{1-2x}$$

$$x = A(1-2x) + B(1-3x)$$

$x = \frac{1}{2}$: $\frac{1}{2} = B\left(-\frac{1}{2}\right) \Rightarrow B = -1$

$x = \frac{1}{3}$: $\frac{1}{3} = A\,\frac{1}{3} \Rightarrow A = 1$

rozvinutí: $\sum (3x)^k - \sum (2x)^k = \sum (3^k - 2^k)\, x^k$

$$a_k = 3^k - 2^k$$

$$k C_k = k(k-1) + 2\sum_{i=1}^{k} C_{i-1} \qquad C_0 = C_1 = 0$$

není potřeba opravných členů

$$\sum k C_k x^k = \sum k(k-1) x^k + 2\sum\left(\sum_{i=1}^{k} C_{i-1}\right) x^k$$

$$C(x) = \sum C_k x^k \qquad C'(x) = \sum C_k \cdot k \cdot x^{k-1}$$

$$x C'(x) = \sum C_k \cdot k \cdot x^k$$

$$\frac{2}{(1-x)^3} = 2\sum \binom{k+2}{2} x^k$$

$$= \sum (k+2)(k+1) \cdot x^k$$

$$\frac{2x^2}{(1-x)^3} = \sum k(k-1) x^k$$

$$\frac{1}{1-x} \cdot C(x) = \sum \left(\sum_{i=0}^{k} C_i\right) x^k$$

$$\frac{x}{1-x} \cdot C(x) = \sum \left(\sum_{i=0}^{k-1} C_i\right) x^k$$

$$xC'(x) = \frac{2x^2}{(1-x)^3} + \frac{2x}{1-x}\, C(x)$$

$$C'(x) = \frac{2}{1-x} \cdot C(x) + \cancel{\frac{2x}{(1-x)^3}}$$

$$\frac{C'(x)}{C(x)} = \frac{2}{1-x} \qquad \Big/ \int dx$$

$$\| $$

$$(\ln C(x))'$$

$$\ln C(x) = -2\ln(1-x) + K \qquad / \, e^{()}$$

$$C(x) = e^k \cdot \left(e^{\ln(1-x)}\right)^{-2} = e^k \cdot (1-x)^{-2}$$

$$= L \cdot \frac{1}{(1-x)^2}$$

$$C'(x) = \frac{2}{1-x} \cdot C(x) + \frac{2x}{(1-x)^3}$$

$$C(x) = L(x) \frac{1}{(1-x)^2}$$

$$L'(x) \cdot \frac{1}{(1-x)^2} + \cancel{L(x) \frac{2}{(1-x)^3}} = \cancel{\frac{2}{1-x} L(x) \frac{1}{(1-x)^2}} + \frac{2x}{(1-x)^3}$$

$$L'(x) \cdot \frac{1}{(1-x)^2} = \frac{2x}{(1-x)^3}$$

$$L'(x) = \frac{2x}{1-x} \qquad / \int$$

$$L'(x) = -2 + \frac{2}{1-x} \rightarrow L(x) = -2x - 2\ln(1-x)$$

$$C(x) = \left(-2x - 2\ln(1-x)\right) \frac{1}{(1-x)^2}$$